Physics Letters B 300 (1993) 205–209
North-Holland

PHYSICS LETTERS B

Differential cross sections of soft multipole states

C.A. Bertulani and H. Sagawa [1]
Cyclotron Laboratory, MSU, East Lansing, MI 48824-1321, USA

Received 5 October 1992; revised manuscript received 25 November 1992

We study the differential cross sections of the soft multipole excitations of ^{11}Li with C-target by using a reaction model in which all input parameters are determined from other sets of experimental data. The results describe properly the differential cross sections of giant resonances in ^{208}Pb excited by an α-projectile at an intermediate energy $E_{lab}=43$ MeV/u without introducing any free parameters. Predicted cross sections for soft multipole excitation are of order of 100 mb/sr at forward angles in heavy ion reactions at beam energies in the range 30–70 MeV/nucleon. It might be possible to distinguish the multipoles experimentally by the angular dependence at very forward angles.

It is now one of the main issues of nuclear physics to study the excitation spectra of halo nuclei, i.e., ^{11}Li, ^{11}Be and ^{14}Be [1–3]. Halo nuclei are characterized by extremely large radii compared with neighboring isotopes: the sizes of halo nuclei are substantially larger than that given by a standard formula $R=1.2A^{1/3}$ fm [4]. The loose binding of the neutrons is responsible for the large radii of these halo nuclei [5,6].

A unique feature of the halo nuclei is the excitation spectra at very low energy. Hansen and Jonson [5] predicted the existence of soft dipole mode in the halo nucleus ^{11}Li, which is characterized by an extremely low excitation energy and substantially large dipole transition strength, although it is described as a naive particle–hole excitation in terms of the shell model [7]. It was also pointed out [8,9] that possible soft multipole excitations other than the dipole exist at the very low excitation energy $E_x=1$–2 MeV, exhausting a significant portion of the energy-weighted sum rule transition strength (EWSR). The mechanism which leads to the large transition strength is due to a strong threshold effect of the halo nuclei [10].

During the last few years, several experiments have obtained evidence of the soft dipole mode, using the pion double-charge exchange reactions ^{11}B(π^-, π^+)^{11}Li [1], or measuring in triple coincidence the break-up reaction $^{11}\mathrm{Li}^* \to {}^9\mathrm{Li}+\mathrm{n}+\mathrm{n}$ [2,3]. These experiments find the transition strength of the soft dipole state at a very low excitation energy $E_x \simeq 1$ MeV with extremely large transition strength $B(\mathrm{E1}) \simeq 1$ $e^2\,\mathrm{fm}^2$, despite the fact that the low-energy isoscalar dipole excitation is forbidden. Results of microscopic calculations by the random phase approximation (RPA) [6,9] and also by the two-body Green's function method [11] are consistent with the experimental finding of the soft dipole state in ^{11}Li.

The RPA calculations show the existence of not only the soft dipole but also other excitation modes with the multipoles $L=0$ or 2 in the same energy region $E_x \simeq 1$–2 MeV. It is obvious that the Coulomb excitation is dominated by the dipole transitions when the transition strength is close to the single particle unit [12]. As is known, the monopole transition due to Coulomb excitation is forbidden. We need hadronic probes to obtain empirical information of the soft multipoles other than the dipole. An important difference between the Coulomb and the hadronic probes is that, being of long range, the Coulomb field measures the nuclear transition density as a whole, i.e., what enters the Coulomb excitation cross sections are the transition densities integrated over the coordinate space with a slowly varying weighting function, namely $r^\lambda Y_{\lambda\mu}(\theta,\phi)$. On the other hand, the

[1] Department of Physics, Faculty of Science, University of Tokyo, Hongo 7-3-1, Bunkyo-ku, Tokyo 113, Japan.

hadronic probes are more sensitive to the spatial distribution of the transition densities, especially at the surface of the nucleus. Therefore, hadronic probes would be relevant to test more closely the nuclear structure model of halo nuclei. Smaller isoscalar targets, like the α-nucleus or ^{12}C, have an advantage in exciting the non-dipole multipole excitations since their hadronic interactions will overcome the Coulomb force. In this paper we thus study soft multipole excitations in the reaction of $^{12}C+^{11}Li$ at medium energies.

The DWBA is an appropriate tool for the investigation of inelastic scattering of hadronic probes. Two different approaches have been used in these calculations. One approach is the deformed potential model [13] in which one assumes that the transition density of the excited nucleus is strongly peaked at the nuclear surface. Here the full transition strength is probed in a peripheral collision. Although this assumption is reasonable for the excitation of heavy nuclei (e.g. ^{40}Ca, ^{208}Pb, etc.), it is rather crude for light nuclei, especially when the transition densities extend radially beyond the nuclear size. This is the case for the soft multipole excitations, for which the transition densities have very long tails, as shown in fig. 1. In these cases, the folding model for the transition densities is more appropriate. One advantage of this approach is that the potentials which generate the distorted waves and the interaction potential are constructed by using the same microscopic nucleon–nucleus potential. For the scattering phase, we adopt the eikonal approximation which is well justified in intermediate energy collisions, and renders the calculations simple and transparent.

The model that we describe here can be directly associated with the transition densities which are obtained by structure calculations such as the shell model or RPA. Moreover it could have predictive power for the differential cross sections of strongly excited states like giant resonances since ingredients of the calculations are all determined by other sets of experimental data at the same projectile energy. The eikonal approximation is well justified for the description of the cross sections at forward angles, high beam energies and small energy transfers. This is sufficient for the present purpose since the peculiar features of the soft multipole excitations will appear under these conditions. Our model is first applied to the inelastic scattering of ^{208}Pb with a medium energy α-projectile to test whether or not the absolute magnitude and the angular distributions of the cross sections of giant resonances are properly described.

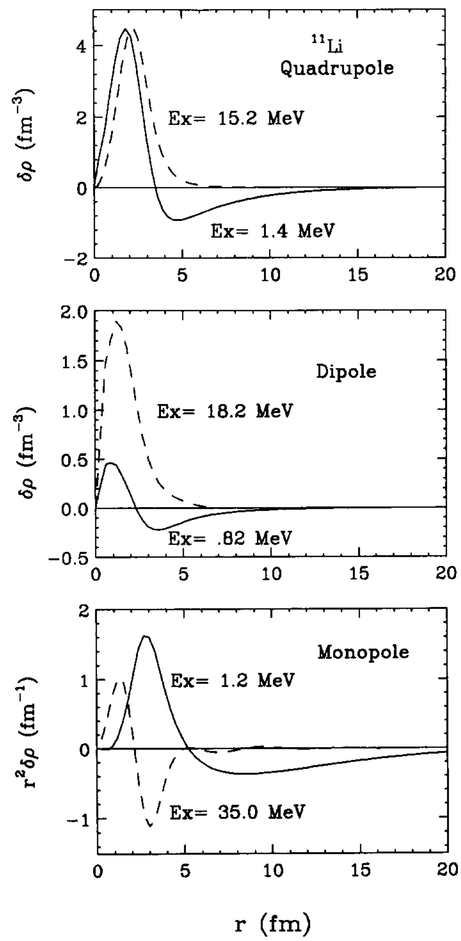

Fig. 1. RPA transition densities of soft multipole states (solid curves) and giant resonances (dashed curves) in ^{11}Li. The ground state of ^{11}Li is obtained by filling the HF single-particle orbits from the bottom of the potential. The spin–orbit coupling parameter W_0 of the Skyrme interaction SGII [14] is modified to be $W_0=160$ MeV fm^5 in order to simulate the loosely-bound halo neutrons in the $1p_{1/2}$-orbit.

The ideal hadronic probe for our purpose is an α or ^{12}C target for a radioactive beam. For such nuclei, the nucleon–nucleus potential could be given by a gaussian form,

$$U_{NA}(r)=-(v_0+iw_0)e^{-r^2/a^2}, \tag{1}$$

where the potential depth v_0 and w_0 are isospin-averaged values. The transition matrix element for the excitation of the nucleus A in peripheral collisions with another nucleus a is given by

$$T_{\mathrm{if}} = \int \mathrm{d}^3R \int \prod_j \mathrm{d}^3r_j\, \Psi_{aA}^{(-)*}(\boldsymbol{R})\phi_{\mathrm{f}}^*(\boldsymbol{r}_1, ..., \boldsymbol{r}_A) \times \sum_j U_{NA}(|\boldsymbol{R}-\boldsymbol{r}_j|)\, \Psi_{aA}^{(+)}(\boldsymbol{R})\phi_{\mathrm{i}}(\boldsymbol{r}_1, ..., \boldsymbol{r}_A) = \int \mathrm{d}^3R \int \mathrm{d}^3r\, \Psi_{aA}^{(-)*}(\boldsymbol{R})\, \Psi_{aA}^{(+)}(\boldsymbol{R}) \times U_{NA}(|\boldsymbol{R}-\boldsymbol{r}|)\delta\rho(\boldsymbol{r}) , \qquad (2)$$

where $\boldsymbol{R}$ is the relative coordinate between the nuclei and $\boldsymbol{r}_j$ is the internal coordinate of the jth nucleon in the nucleus A. The scattering and the intrinsic wave functions are denoted as $\Psi_{aA}(\boldsymbol{R})$ and $\phi(\boldsymbol{r}_1, ..., \boldsymbol{r}_A)$, respectively. The transition density $\delta\rho(\boldsymbol{r})$ is defined by

$$\delta\rho(\boldsymbol{r}) \equiv \int \prod_j \mathrm{d}^3r_j\, \phi_{\mathrm{f}}^*(\boldsymbol{r}_1, ..., \boldsymbol{r}_A) \times \sum_j \delta(\boldsymbol{r}-\boldsymbol{r}_j)\phi_{\mathrm{f}}(\boldsymbol{r}_1, ..., \boldsymbol{r}_A) . \qquad (3)$$

Using the eikonal approximation for the scattering waves $\Psi^{(\pm)}$ and expanding $U_{NA}(|\boldsymbol{R}-\boldsymbol{r}|)$ and $\delta\rho(\boldsymbol{r})$ into multipoles, we find the amplitude to excite the λ multipole mode in the nucleus A as

$$T_{\lambda\mu} = 8\pi^{5/2}a(v_0+\mathrm{i}w_0)i^{\lambda+\mu}Y_{\lambda\mu}(\theta=\tfrac{1}{2}\pi, 0) \times \int \mathrm{d}b\, bJ_\mu(2\sqrt{kk'}\, b\sin(\tfrac{1}{2}\theta))\, \mathrm{e}^{\mathrm{i}\chi(b)}\, \mathcal{O}_\lambda(b) , \qquad (4)$$

where

$$\mathcal{O}_\lambda(b) = \mathrm{e}^{-b^2/a^2} \int \mathrm{d}r\, r^2\delta\rho_\lambda(r)\mathrm{e}^{-r^2/a^2} j_\lambda\left(\mathrm{i}\frac{2rb}{a^2}\right), \qquad (5)$$

and the eikonal phase $\chi(b)$ is

$$\chi(b) = -\frac{1}{\hbar v} \int_{-\infty}^{\infty} \mathrm{d}z\, U_{\mathrm{opt}}(\sqrt{b^2+z^2}) + 2\frac{Z_aZ_Ae^2}{\hbar v}\ln(kb) . \qquad (6)$$

In the equations above, $J_\mu(x)$ is the Bessel function of integer order, $j_\lambda(\mathrm{i}x)$ is the spherical Bessel function of imaginary argument, and the optical potential U_{opt} is constructed by folding the nucleon–nucleus $(N+a)$ potential with the nuclear density of nucleus A. A detailed derivation of the above results will be presented elsewhere. The differential cross section for the excitation to a given multipole is given by

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_\lambda = \left(\frac{\mu_{aA}}{2\pi\hbar^2}\right)^2 \sum_\mu |T_{\lambda\mu}|^2 . \qquad (7)$$

where μ_{aA} is the reduced mass.

Fig. 2 shows the differential cross sections of monopole and quadrupole giant resonances in ^{208}Pb excited by an α-projectile at $E_{\mathrm{lab}}=172$ MeV. Experimental data and parameters for the alpha–nucleon potential are taken from ref. [15]. The transition densities of both states are the Tassie ones calculated by using the HF density of ^{208}Pb and assuming 100% of the energy-weighted sum rules. It is remarkable that both the angular distributions and the absolute magnitudes of the cross sections at forward angles, $\Theta_{\mathrm{cm}} \leqslant 15°$, are well described by using the established

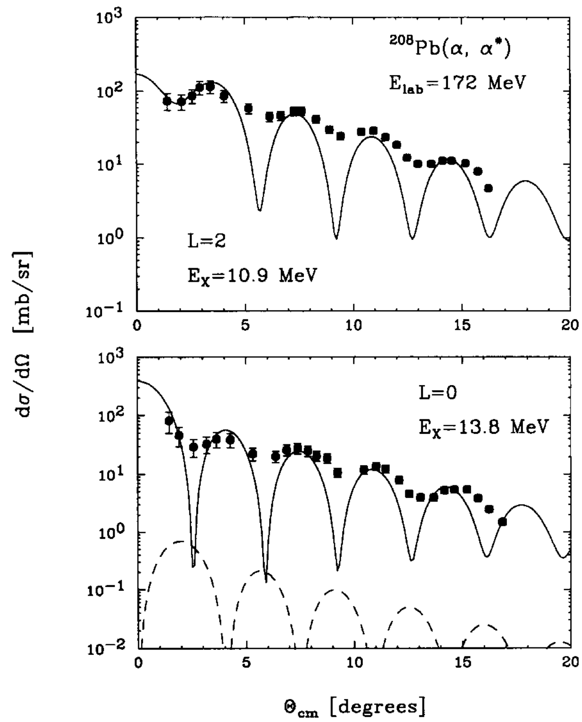

Fig. 2. Differential cross sections for giant resonances in ^{208}Pb excited by an α-projectile at $E_{\mathrm{lab}}=172$ MeV. The calculated cross sections are shown by the solid curves for $L=0$ and $L=2$, while the dashed curve in the lower part of the figure shows that of the giant dipole state. Experimental data are taken from ref. [15].

optical potentials for the nucleon–nucleus scattering.

We apply our reaction model to the soft multipole excitations of ^{11}Li. The transition densities of the soft multipole excitations are calculated by using the self-consistent HF+RPA method [16]. The dominant peaks of all multipoles appears at $E_x = 1-1.5$ MeV, having narrow widths of $\Gamma_{FWHM} \leqslant 1$ MeV. The transition strengths for the soft modes are calculated to be $B(E0) = 61.4\ e^2\,fm^4$, $B(E1) = 0.82\ e^2\,fm^2$ and $B(E2) = 31.5\ e^2\,fm^4$, respectively, exhausting 11%, 2% and 7% of the EWSR values. Although the fraction of the EWSR is small, the transition strengths of the soft multipoles are larger than those of the giant resonances in the same nucleus, because of the very low excitation energies of the soft multipoles. In the dipole case, the calculated value is very close to the empirical value, which was recently obtained in a triple coincidence experiment [3].

We now consider the ^{11}Li–^{12}C reaction at $E_{lab} = 30$ and 60 MeV/n. The interaction is deduced by the global optical potential of ref. [17]. The parameters in our gaussian potential, eq. (1), are chosen to give approximately the same depth and width as the the Woods–Saxon potential of ref. [17]. We also ignore the spin–orbit part of the potential. For example, the parameter values at the energy 30 MeV are $v_0 = 43.9$ MeV, $w_0 = 3.29$ MeV, and $a = 2.93$ fm for neutrons, and $v_0 = 44.9$ MeV, $w_0 = 2.88$ MeV, and $a = 2.93$ fm for protons.

The calculated differential cross sections are shown in fig. 3. There are substantial differences between the monopole and the quadrupole excitations. The first crucial point is the steeper slope of the monopole cross section at very forward angles, $\Theta \sim 0°$; the difference is clearly seen in the ratio between the first and the second peaks of the cross sections, which is almost 1 for $L=2$ but greater than 10 for $L=0$. This difference originates in the fact that three Legendre polynomials with different μ contribute to the cross section in the $L=2$ case, as seen in eq. (4), while only one Legendre polynomial appears in the $L=0$ case. This could be a key feature for distinguishing the monopole excitation from the quadrupole one in experimental cross sections. A second difference can be found in the dips in the cross sections. The first deep minimum for the $L=0$ case is found at $\Theta \simeq 1.6$ (1.0)°, while a shallow one appears at $\Theta \simeq 1.0$ (0.7)° for the $L=2$ case, at the projectile energy $E_{lab} = 330$ (660) MeV. These differences were certainly an important clue in finding the giant monopole states in many heavy nuclei [15]. For $E_{lab} = 660$ MeV the dips of all multipole excitations occur in shorter intervals because of the larger wave number, but the magnitudes of the cross sections do not change appreciably. This is because the nucleon–carbon potentials are not very different at these two bombarding energies [16].

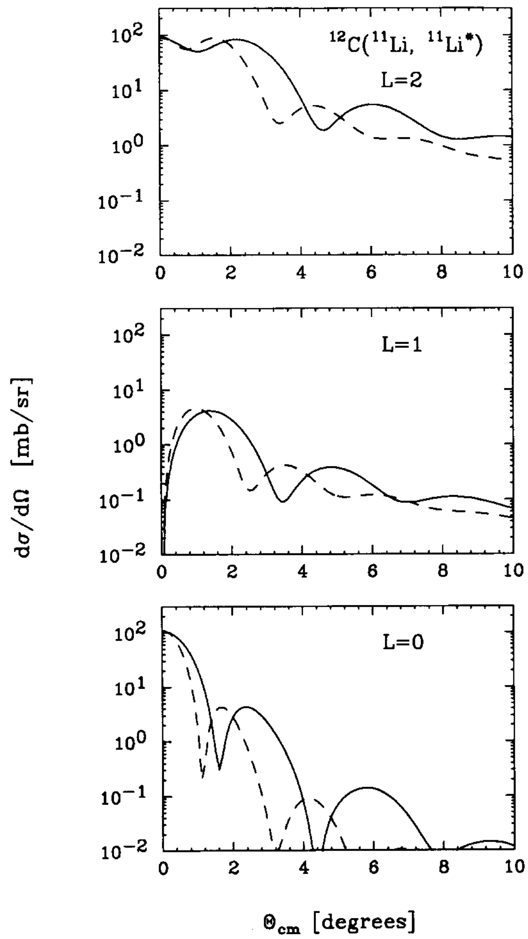

Fig. 3. Differential cross sections of soft multipole states in ^{11}Li excited by the ^{12}C target at $E_{lab} = 330$ MeV (solid curves) and at $E_{lab} = 660$ MeV (dashed curves).

The average slope of the differential cross section is essentially governed by the surface diffuseness of the nuclei involved in the reaction. It is interesting to compare the results in figs. 2 and 3. Since the surface is sharp in ^{208}Pb, the slope decreases very slowly in the case of $^{208}Pb+\alpha$, while it drops quickly in the $^{12}C+^{11}Li$ case because of the very diffuse surface of ^{11}Li. This behavior could be a common feature in the reaction cross section involving any halo nucleus [18]. It should be noticed that the absolute magnitude of the differential cross section in fig. 3 is of the order of 100 mb/sr for the monopole and quadrupole excitations which is almost the same as the observed magnitude of the $Pb+\alpha$ reaction in fig. 2. It is also seen that the soft dipole state has a smaller cross section and a different angular dependence than those of the other two multipoles. Although a secondary beam always has lower intensity than ordinary beams, the soft multipole excitations could be tested experimentally with modern high sensitive detector systems.

In conclusion, we studied the differential cross sections of the soft multipole excitations in ^{11}Li with a ^{12}C target at the intermediate energies by using the reaction model based on the eikonal approximation. Our model described well the differential cross sections at forward angles $\Theta_{cm} \leqslant 15°$ of the monopole and quadrupole giant resonances in ^{208}Pb excited by an α-projectile with $E_{lab}=172$ MeV, both qualitatively and quantitatively. One of the advantages of our model is that the ingredients of the calculations are physically tested by other experiments. Thus, we have no adjustable parameters in the calculations, which is important for making predictions for future experiments. The inelastic differential cross sections of ^{11}Li with a ^{12}C-target are studied by using the microscopic transition densities calculated by the HF+RPA model. We find that there are substantial differences among the angular distributions of the multipole excitations with $L=0$, 1 and 2, especially at forward angles. These could be experimentally distinguished. The typical cross sections at forward angles are found to be 100 mb/sr for the monopole and the quadrupole excitations and is almost the same as the observed cross sections of the giant resonances in ^{208}Pb with an α-projectile.

One of the authors (H.S.) would like to thank the kind hospitality of the Cyclotron Laboratory at MSU. This work was partially supported by the US National Science Foundation under grant PHY-9017077.

References

[1] T. Kobayashi, Nucl. Phys. A 538 (1992) 343c.
[2] T. Shimoura, in: Proc. Intern. Symp. on Structure and reactions of unstable nuclei, eds. K. Ikeda and Y. Suzuki (World Scientific, Singapore, 1991) p. 132.
[3] K. Ieki et al., to be published.
[4] I. Tanihata, H. Hamagaki, O. Hashimoto, Y. Shida, N. Yoshikawa, K. Sugimoto, O. Yamakawa, T. Kobayashi and N. Takahashi, Phys. Rev. Lett. 55 (1985) 2676.
[5] P.G. Hansen and B. Johnson, Europhys. Lett. 4 (1987) 409.
[6] H. Sagawa, Phys. Lett. B 286 (1992) 7.
[7] G.F. Bertsch and J. Foxwell, Phys. Rev. C 41 (1990) 1300; C 42 (1990) 1159.
[8] H. Sagawa, Nucl. Phys. A 538 (1992) 619c.
[9] S.A. Fayans, Phys. Lett. B 267 (1991) 443.
[10] H. Sagawa, N. Takigawa, Nguyen van Giai and M. Ishihara, to be published.
[11] G.F. Bertsch and H. Esbensen, Ann. Phys. (NY) 209 (1991) 327.
[12] C.A. Bertulani and A. Sustich, Phys. Rev. C, in press.
[13] G.R. Satchler, Direct nuclear reactions (Oxford U.P., Oxford, 1983).
[14] Nguyen van Giai and H. Sagawa, Phys. Lett. B 106 (1981) 379.
[15] H.P. Morsch et al., Phys. Rev. C 28 (1983) 1947.
[16] G.F. Bertsch and S.F. Tsai, Phys. Rep. 18 (1975) 126; K.F. Liu and Nguyen van Giai, Phys. Lett. B 65 (1976) 23.
[17] R.L. Varner et al., Phys. Rep. 201 (1991) 57.
[18] N. Takigawa, M. Ueda, M. Kuratani and H. Sagawa, Phys. Lett. B 288 (1992) 244.