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Relativistic Coulomb excitation by a deformed projectile

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The effects of a deformed projectile charge on the relativistic Coulomb excitation theory are investigated. The effects of relevance are (a) the variation of the minimum impact parameter as a function of the orientation of the projectile and (b) the modification of the time dependent electromagnetic field due to the projectile deformation.

Coulomb excitation by a relativistic projectile is well described theoretically if the charge distribution of the projectile is spherically symmetric [1,2]. The theory gives the amplitudes and cross sections for the monopole–multipole excitations. However, the effects of a deformed projectile distribution have not yet been calculated. As suggested by Justice et al. [3] these effects could be of relevance for the experiments involving, e.g., uranium projectiles (or targets). We shall investigate these effects in this article. To obtain a qualitative insight of the effects we shall consider a prolate deformed projectile with a variable deformation.

In the frame of reference of the projectile the Coulomb field at a position $\boldsymbol{r}$ with respect to the center-of-charge of the distribution is given by

$$\phi(\boldsymbol{r}) = 4\pi \sum_{l,m} \frac{1}{2l+1} \frac{1}{r^{l+1}} Y_{lm}^{*}(\theta,\phi)\, M_{lm}\,, \tag{1}$$

where

$$M_{lm} = \int \rho(\boldsymbol{r}')\, r'^{l}\, Y_{lm}(\hat{r}')\, \mathrm{d}^3 r'\,, \tag{2}$$

with $\rho(\boldsymbol{r})$ equal to the ground state charge distribution of the projectile. For simplicity we will consider a uniform spheroidal charge distribution with the z-axis along the symmetry axis. The charge distribution drops to zero for distances to the center greater than the angle-dependent radius

$$R(\theta) = R_0[1+\beta Y_{20}(\theta)]\,. \tag{3}$$

In lowest order in the multipole expansion, eq. (1) becomes

$$\phi(\boldsymbol{r}) = \frac{Ze}{r} + \sqrt{\frac{\pi}{5}}\frac{1}{r^3} Y_{20}(\theta)\, Q_0^{(c)}\,, \tag{4}$$

where $Q_0^{(c)}$ is the quadrupole moment of the charge distribution,

$$Q_0^{(c)} = \frac{3}{\sqrt{5\pi}} Ze\, R_0^2 \beta\,(1+0.16\beta) + \mathcal{O}(\beta^2)\,. \tag{5}$$

To obtain the (time-dependent) field in the frame of reference of the target we perform a Lorentz transformation of eq. (4). For a straight-line trajectory one finds

$$\phi(r',t) = \frac{\gamma Ze}{r'} + \gamma\sqrt{\frac{\pi}{5}}\frac{1}{r'^3} Y_{20}(\theta')\, Q_0^{(c)}\,, \tag{6}$$

where $r' = \sqrt{b^2+\gamma^2 v^2 t^2}$, with b equal to the impact parameter, v the projectile velocity, and $\gamma = (1-v^2/c)^{-1/2}$.

The first term in the above equation is the well-known Liénard–Wiechert potential of a relativistic charge. It gives rise to monopole–multipole excitations of the target [1]. The second term accounts for quadrupole–multipole excitations of the target and is a correction due to the deformation of the projectile. This field will depend on the orientation of the projectile with respect to its trajectory (see fig. 1). We can separate the orientation angles from the angular

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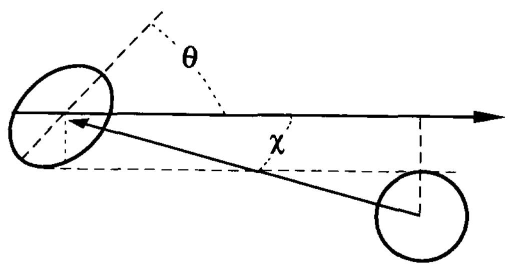

Fig. 1. A nuclear target is Coulomb excited by a fast moving deformed projectile. Besides the angle θ, the orientation of the projectile also includes an azimuthal angle ϕ which can rotate its symmetry axis out the scattering plane. For simplicity, this is not shown. χ is the angular position of the c.m. of the projectile with respect to the target.

position of the projectile (along its trajectory) with respect to the target by using the identity

$$Y_{20}(\theta') = \sqrt{\frac{4\pi}{5}} \sum_m Y_{2m}(\theta,\phi)\, Y_{2m}(\chi',0)\,, \tag{7}$$

where (θ,ϕ) denotes the orientation of the projectile symmetry axis with respect to the bombarding axis and $\chi' = \gamma vt/r'(t)$.

The dipole excitation of the target is the most relevant and we shall restrict ourselves to this case only [2]. At a point $\boldsymbol{r} \equiv (x,y,z)$ from the center of mass of the target the field is obtained by replacing $\boldsymbol{r}' = (b,0,\gamma vt)$ by $[b-x, y, \gamma(vt-z)]$ in eq. (6). The excitation amplitude is given by

$$a_{\rm fi} = \frac{1}{i\hbar} \int_{\infty}^{\infty} e^{i\omega t} [\rho_{\rm fi}(\boldsymbol{r})\phi(\boldsymbol{r},t) - (\boldsymbol{v}/c^2)\cdot \boldsymbol{j}_{\rm fi}\phi(\boldsymbol{r},t)]\, d^3r\, dt\,, \tag{8}$$

where $E_{\rm fi} = \hbar\omega$ is the excitation energy and $\rho_{\rm fi}(\boldsymbol{r},t)$ and $\boldsymbol{j}_{\rm fi}(\boldsymbol{r},t)$ is the charge transition density and current transition density matrix elements, respectively.

Using the continuity equation $\nabla\cdot \boldsymbol{j}_{\rm fi} = -i\omega\rho_{\rm fi}$ and expanding (8) to lowest order in $\boldsymbol{r}$ we find

$$a_{\rm fi} = a_{\rm fi}^{(1)} + a_{\rm fi}^{(2)}\,, \tag{9}$$

where

$$a_{\rm fi}^{(1)} = -i\frac{2Ze}{\hbar v}\frac{\xi}{b}\left\{K_1(\xi)D_{\rm fi}^{(x)} + i\frac{1}{\gamma}K_0(\xi)D_{\rm fi}^{(z)}\right\} \tag{10}$$

and

$$a_{\rm fi}^{(2)} = -i\frac{4\pi}{5}\frac{Q_0^{(c)}}{\hbar v}\frac{\xi^2}{b^3}\left\{K_2(\xi)D_{\rm fi}^{(x)} + i\frac{1}{\gamma}K_1(\xi)D_{\rm fi}^{(z)}\right\} \times \sum_m Y_{2m}(\theta,\phi)\, Y_{2m}(\tfrac{1}{2}\pi,0)\,, \tag{11}$$

where $\xi = \omega b/\gamma v$ and K_i is the modified Bessel function of order i. In the expression (11) we have used the approximation $Y_{2m}(\chi,0) \simeq Y_{2m}(\frac{1}{2}\pi,0)$ which is valid for high energy collisions since the quadrupole field is strongly peaked at $t=0$, corresponding to the distance of closet approach. The dipole matrix elements for the nuclear excitation are given by $D_{\rm fi}^{x(z)} = \langle f|x(z)|i\rangle$.

Eqs. (10) and (11) allow us to calculate the dipole excitation cross section by integrating their absolute squares over impact parameter, starting from a minimum impact parameter for which the strong interaction sets in. Neglecting the diffuseness of the matter distribution of the nuclei we can write (see fig. 1)

$$b_{\min}(\theta) \simeq R_T + R_P[1 + \beta Y_{20}(\tfrac{1}{2}\pi + \theta)]\,, \tag{12}$$

with the nuclear radii given by $R_i = 1.2A_i^{1/3}$. The total cross section is

$$\sigma = 2\pi\left\langle \int_{b_{\min}(\theta)} db\, b\, |a_{\rm fi}(b,\Omega)|^2 \right\rangle, \tag{13}$$

where the $\langle\ \rangle$ sign means that an average over all the possible orientations of the projectile, i.e., over all angles $\Omega = (\theta,\phi)$, is done.

We will apply the above formalism to the Coulomb excitation of ^{208}Pb by ^{238}U projectiles. We will give the ^{238}U an artificial deformation in the range β = 0–1 to check the dependence of the cross sections with this parameter. The cross section given above contains three terms: $\sigma = \sigma_1 + \sigma_2 + \sigma_{12}$. σ_1 is due to the monopole–dipole excitation amplitude, σ_2 is due to the quadrupole–dipole excitation amplitude, and σ_{12} is the interference between them.

In fig. 2 we present the results for the numerical calculation of the quantity

$$\Delta = 100 \times \frac{\sigma_1 - \sigma_1^{\beta=0}}{\sigma_1^{\beta=0}}\,, \tag{14}$$

which is the percent correction of dipole excitations in ^{208}Pb by a uranium projectile due the average over

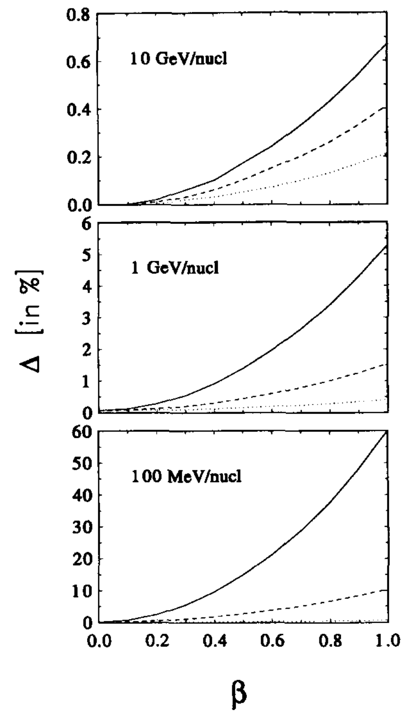

Fig. 2. Percent increase of the Coulomb excitation cross section of dipole states in ^{208}Pb due to the dependence of the minimum impact parameter on the deformation. The effect is shown for ^{238}U projectiles at 100 MeV/nucleon, 1 GeV/nucleon and 10 GeV/nucleon, respectively, and as a function of the deformation parameter β. The solid (dashed) [dotted] line corresponds to an excitation energy of 1 (10) [25] MeV. For the actual deformation of ^{208}U, $\beta \simeq 0.3$, the effect is small.

the orientation of the projectile; $\sigma_1^{\beta=0}$ is the cross section for $\beta = 0$. We present results for three bombarding energies, 10 GeV/nucleon, 1 GeV/nucleon and 100 MeV/nucleon, and as a function of β. The quantity defined by eq. (14) is independent of the nature of the state excited, since the dipole matrix elements cancel out. They depend on the energy of the state. In order to see how the effect depends qualitatively on the energy of the state we used three different excitation energies $E_{\mathrm{fi}} = 1$, 10 and 25 MeV, respectively. These correspond to the dotted, dashed and solid lines in fig. 1, respectively.

One observes from fig. 2 that the deformation effect accounted for by an average of the minimum impact parameter which enters eq. (13) increases the magnitude of the cross section. Thus the average is equivalent to a smaller "effective" impact parameter, since the cross sections increase with decreasing values of $b_{\min}$. The effect is larger the greater the excitation energy is. This effect also decreases with the bombarding energy. For very high bombarding energies it is very small even for the largest deformation. These results can be explained as follows. The Coulomb excitation cross section at very high bombarding energies, or very small excitation energies, is proportional to $\ln[\omega b_{\min}(\theta)/\gamma v)]$. Averaging over orientation of the projectile means an average of $\ln(b_{\min})$ due to the additivity law of the logarithm. One can easily do this average and the net result is a rescaling of $b_{\min}$ as $f b_{\min}$, with f smaller, but very close to one.

For high excitation energies, or small bombarding energies, the cross section is proportional to $\exp[-2\omega b_{\min}(\theta)/\gamma v]$ due to the adiabaticity condition [1]. Thus, in these situations, the cross section is strongly dependent on the average over orientation due to the strong variation of the exponential function with the argument.

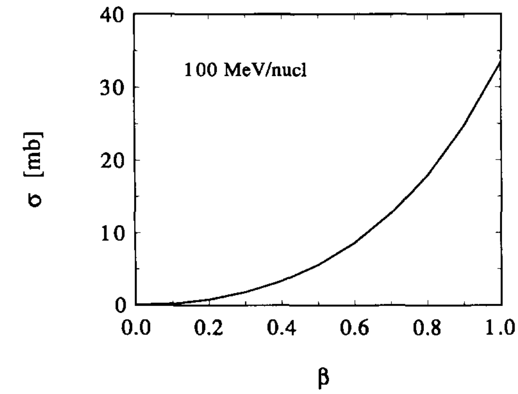

Fig. 3. Coulomb excitation cross section of a giant dipole resonance in ^{208}Pb due to the quadrupole–dipole interaction with 100 MeV/nucleon uranium projectiles, as a function of the deformation parameter β. These cross sections are averaged over all possible orientations of the projectile.

Now we consider the effect of the second term of eq. (6), namely of the quadrupole–dipole excitations. In fig. 3 we show the excitation of a giant resonance dipole state in lead ($E_{\mathrm{fi}} = 13.5$ MeV) due to the second term of eq. (6), as a function of the deformation parameter β and for a bombarding energy of 100 MeV/nucleon. We assume that the giant dipole state

Table 1
Cross sections (in mb) for Coulomb excitation of the dipole giant resonance in ^{208}Pb by ^{238}U projectiles at 100 MeV/nucleon. In the second (third) column the cross sections are due to the monopole (quadrupole)–dipole interaction. The last column is the total cross section. An average over the orientation of the projectile was done. A realistic value of the deformation of ^{238}U corresponds to $\beta \simeq 0.3$. But, a variation of β is used to obtain an insight of the magnitude of the effect.

β	σ_1 [mb]	σ_2 [mb]	σ [mb]
0	1171	0	1171
0.1	1173	0.179	1174
0.2	1179	0.748	1184
0.3	1189	1.773	1200
0.4	1202	3.34	1224
0.5	1220	5.57	1242
0.6	1241	8.61	1291
0.7	1265	12.6	1335
0.8	1294	17.9	1389
0.9	1326	24.7	1446
1	1362	33.3	1522

exhausts fully the TRK sum-rule [4] in lead. Now the average over orientation also includes the dependence of the quadrupole–dipole interaction on $\Omega = (\theta, \phi)$. As expected the cross section increases with β. But it is small as compared to the monopole–dipole excitations even for a large deformation. At this beam energy the monopole–dipole excitation is of order of 1 b.

The total cross section contains an interference between the amplitudes $a_{\rm fi}^{(1)}$ and $a_{\rm fi}^{(2)}$. This is shown in table 1 for 100 MeV/nucleon for which the effect is larger. The second column gives the cross sections for monopole–dipole excitations of a giant resonance dipole state in lead. The effect of the orientation average can be seen as an increase of the cross section as compared to the value in the first row (zero deformation). For $\beta = 0.3$ which is approximately the deformation parameter for ^{238}U the correction to the cross section is negligible. In the third column the cross section for quadrupole–dipole excitation are given. They are also much smaller than those for the monopole–dipole excitations. The total cross sections, given in the last column, are also little dependent on the effect of the deformation. For $\beta = 0.3$ it corresponds to an increase of 3% of the value of the original cross section (first row). This effect also decreases with the bombarding energy. For 1 GeV/nucleon, $\sigma^{\beta=0} = 5922$ mb, while $\sigma = 5932$ mb for $\beta = 0.3$, with all effects included.

In conclusion, the effects a deformation of the charge distribution of the projectile on the Coulomb excitation of a nuclear target was studied. The effect of averaging over the projectile orientation is to increase slightly the cross sections. The inclusion of the quadrupole–dipole interaction increases the cross section, too. However these corrections are small for realistic deformations. They cannot be responsible for the large deviations of the experimental values of the Coulomb fragmentation cross sections from the standard theory [1,2], as has been observed recently [3,5] for deformed projectiles.

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