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Fermi and Gamow-Teller strength in charge exchange with radioactive beams

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Abstract

At forward angles, and bombarding energies $E > 200$ MeV, (p, n) and (n, p) reactions are thought to be directly proportional to the Gamow-Teller transition strengths in the nuclei. Assuming that this relationship also holds for charge exchange induced by high-energy heavy ions, this technique would be very useful in studies with radioactive beams. We show that the determination of Gamow-Teller and Fermi matrix elements from heavy-ion charge-exchange at forward angles is only useful for particular cases. © 1997 Elsevier Science B.V.

A major experimental and theoretical effort is under way to study the properties and applications of drip-line nuclei (for a review, see Ref. [1]). One of the main issues of this rapidly growing field is to understand the basic features of many unstable nuclei, which can only be assessed by using beams of radioactive nuclei. Of great relevance is the study of spin-isospin properties in unstable nuclei. A well-defined proportionality between heavy ion charge-exchange and beta-decay transition strengths would have important consequences. Such a relationship would be particularly useful to allow beta transition strength functions to be mapped out for drip-line nuclei in reactions with radioactive beams. An accurate link between heavy ion exchange reactions and the well understood beta-decay processes would also put our understanding of the nuclear structure and reaction mechanisms of unstable nuclei on a surer quantitative footing.

It is well known that (p, n) charge-exchange reactions at intermediate energies ($E > 100$ MeV) have been a powerful spectroscopic tool for measuring Gamow-Teller (GT) transition matrix elements [2,3]. This approach can also be used in reactions with radioactive beams, using a hydrogen gas target. However, due to the low luminosity of radioactive beams, this process yields low counting rates in inverse kinematics experiments on solid targets containing hydrogen. A more promising procedure would be the use of heavy ion charge exchange (HICEX) instead of (p, n) collisions. In Ref. [4] the possibility to use HICEX as a probe of Gamow-Teller matrix elements was suggested and a relation between the forward cross section and the Gamow-Teller matrix elements was found. A possible drawback of this approach is the complication originated by the internal structure of the target nucleus. This method was used as a guidance in Ref. [5] to draw conclusions on Gamow-Teller (and Fermi) matrix elements in ^{13}C and ^{13}N from the mir-

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for nuclei reaction $^{13}\text{C}(^{13}\text{N}, ^{13}\text{C})^{13}\text{N}$.

In this article we develop a strong absorption model of HICEX based on a single particle approach. We use this simple structure model for qualitative purposes only. Our aim is to access the reliability of the relationship of HICEX measurements and the Gamow-Teller and Fermi matrix elements. The key point to be discussed here is that if we cannot show the existence of a valid relationship for such a simple model and for rather simple reaction partners, then very probably it would not exist for more complicated nuclear models. Because of the use of mirror nuclei, and their isospin symmetry, the experimental data on the $^{13}\text{C}(^{13}\text{N}, ^{13}\text{C})^{13}\text{N}$ reaction is also an ideal test of the theory.

In heavy-ion charge exchange reactions it is important to know the underlying nature of the exchange mechanism. In Ref. [6] it was shown that in heavy ion collisions at high energies ($E > 100$ MeV·A) the exchange mechanism is dominated by the fundamental π - and ρ -exchange. The differential cross section for the process is given by

$$\frac{d\sigma}{d\Omega} = \frac{k'}{k} \left(\frac{\mu}{4\pi^2\hbar^2} \right)^2 (2J_1 + 1)^{-1} (2J_2 + 1)^{-1} \times \sum_{\alpha_1, \alpha_2} \left| T_{if}(\alpha_1, \alpha_2) \right|^2, \quad (1)$$

where μ is the reduced mass of the system (1+2), k (k') is the initial (final) center-of-mass momentum, J_1 and J_2 are the angular momenta in the entrance channel, and the sum is an average over initial angular momentum projections and a sum over final angular momentum projections, denoted respectively by α_1 and α_2 .

The transition matrix T_{if} is given by [7]

$$T_{if} = \frac{1}{(2\pi)^3} \int d^3R \chi^{(-)*}(\mathbf{R}) \chi^{(+)}(\mathbf{R}) \times \int d^3q e^{iq\cdot\mathbf{R}} \mathcal{M}(\alpha_1, \alpha_2, \mathbf{q}), \quad (2)$$

where $\mathbf{R} \equiv (\mathbf{b}, z)$ is the relative coordinate between the nuclei, $\chi^{(-)}(\mathbf{R})$ [$\chi^{(+)}(\mathbf{R})$] is the incoming (outgoing) scattering wave obtained in terms of the optical potential for elastic scattering, and

$$\mathcal{M} = \int d^2r_1 d^3r_2 \delta\rho_1(\mathbf{r}_1) \delta\rho_2(\mathbf{r}_2) \times e^{iq\cdot(\mathbf{r}_1 - \mathbf{r}_2)} V_{\text{exch}}(\mathbf{q}), \quad (3)$$

where $\delta\rho_i$ is the transition density of the nucleus i , which includes spin, isospin and spatial transitions. The meson-exchange potential V_{OBEP} is a sum of π -, ρ -, and ω -exchange interaction. We use a modified meson-exchange interaction to include a non-spin-flip term: $V_{\text{exch}} = V_{\text{OBEP}} + V_\tau \tau_1 \cdot \tau_2$ [7,9].

Eq. (3) is separable in terms of the spin and isospin variables, yielding $\mathcal{M} = \mathcal{M}_{12} + \mathcal{M}_{2\leftrightarrow 1}$, where

$$\begin{aligned} \mathcal{M}_{12} = & \int d^2q \left\{ F_{\sigma\tau}(\mathbf{q}) \right. \\ & \times \sum_{\mu=1}^3 \langle \alpha_f(1) | \tau_1^{(+)} \sigma_\mu e^{iq\cdot r_1} | \alpha_i(1) \rangle \\ & \times \langle \alpha_f(2) | \tau_2^{(-)} \sigma_\mu e^{-iq\cdot r_2} | \alpha_i(2) \rangle \\ & + G_{\sigma\tau}(\mathbf{q}) \sum_{\mu, \mu'=1}^3 \hat{q}_\mu \hat{q}_{\mu'} \\ & \times \langle \alpha_f(1) | \tau_1^{(+)} \sigma_\mu e^{iq\cdot r_1} | \alpha_i(1) \rangle \\ & \times \langle \alpha_f(2) | \tau_2^{(-)} \sigma_{\mu'} e^{-iq\cdot r_2} | \alpha_i(2) \rangle \\ & + H_\tau(\mathbf{q}) \langle \alpha_f(1) | \tau_1^{(+)} e^{iq\cdot r_1} | \alpha_i(1) \rangle \\ & \left. \times \langle \alpha_f(2) | \tau_2^{(-)} e^{-iq\cdot r_2} | \alpha_i(2) \rangle \right\}. \quad (4) \end{aligned}$$

In the equation above the sums over μ and μ' run over the spherical components of the spin vector. $F_{\sigma\tau}(\mathbf{q})$, $G_{\sigma\tau}(\mathbf{q})$ and $H_\tau(\mathbf{q})$ are functions of the central ($L = 0$) and the tensorial ($L = 2$) component of the charge-exchange potential.

Since the initial and final wavefunctions are decomposed into their spatial and spin-dependent parts, it is appropriate to expand the exponential functions in Eq. (4) into multipoles and use the Wigner-Eckart theorem, to obtain sums over products of Clebsch-Gordan coefficients and reduced matrix elements of the form

$$\begin{aligned} & \langle \phi_{j'}^{(f)} || j_I(qr) [\sigma \otimes Y_I]_{I'} \tau^{(\pm)} || \phi_j^{(i)} \rangle, \\ & \langle \phi_{j'}^{(f)} || j_I(qr) [\sigma \otimes Y_I]_{I'} || \phi_j^{(i)} \rangle, \\ & \langle \phi_{j'}^{(f)} || j_I(qr) \tau^{(\pm)} || \phi_j^{(i)} \rangle. \end{aligned}$$

These matrix elements can be calculated using Eqs. (A.2.19) and (A.2.24) of Ref. [8]. They involve geometric coefficients and overlap integrals of the form

$$\mathcal{I}_{jj'}(q) = \int R_{j'}(r) R_j(r) j_l(qr) r^2 dr, \quad (5)$$

in terms of the single-particle wavefunctions $R_j(r)$. The cross section results from interference of terms corresponding to several possibilities of angular momentum transfer.

From (4) it is straightforward to show the relationship between the heavy-ion charge-exchange cross sections and the Gamow-Teller and Fermi matrix elements. In beta-decay experiments a similar matrix element appears, but with the exponential factors replaced by $\exp i(\mathbf{p}_e + \mathbf{p}_\nu) \cdot \mathbf{r}$, where \mathbf{p}_e and \mathbf{p}_ν are the electron and the neutrino momentum, respectively. This exponential factor can be approximated by the unity, if $|\mathbf{r}| < \hbar/p_e$ and \hbar/p_ν . Since the energies of the light particles involved in beta-decay are of the order of a few MeV, the Compton wavelengths of the electron and the neutrino are approximately equal to 10^3 fm, i.e., about hundred times larger than the nuclear sizes over which the integration occurs. This contrasts with charge exchange experiments, where the exponential factor in (4) involves the Compton wavelength of π and ρ . Thus, the relation $\mathbf{q} \cdot \mathbf{r} \ll 1$ is not well justified for HICEX.

Assuming that the “low- q ” (or long-wavelength) approximation is also valid for HICEX, i.e., inserting Eq. (4), with the exponentials replaced by the unity in Eqs. (1)–(4), one gets

$$\frac{d\sigma}{d\Omega} \simeq c_1 B_1(\text{GT}) B_2(\text{GT}) + c_2 B_1(\text{F}) B_2(\text{F}) + c_3 \left[B_1(\text{F}) B_2(\text{F}) B_1(\text{GT}) B_2(\text{GT}) \right]^{1/2}, \quad (6)$$

where

$$B(\text{GT}) = \frac{1}{2j+1} \sum_{\alpha', \alpha} \left| \langle \alpha' | \sigma \tau^{(\pm)} | \alpha \rangle \right|^2, \\ B(\text{F}) = \frac{1}{2j+1} \sum_{\alpha', \alpha} \left| \langle \alpha' | \tau^{(\pm)} | \alpha \rangle \right|^2 \quad (7)$$

are the Gamow-Teller and Fermi strengths, respectively. The coefficients c_1 , c_2 , and c_3 are products of

geometric factors and integrals over the q - and R -coordinates. The “low- q ” approximation (6) also includes a sum over all single-particle transitions, which is not explicitly shown for simplicity.

Eq. (6) is the result that one would like to have as a reliable approximation to analyze the experimental data. Assuming that the charge-exchange potential (corrected for medium effects, etc.) is accurately known and that the same is true for the scattering waves, then the coefficients c_i could also be reliably calculated. An experimental measurement of HICEX would then provide the magnitude of $B(\text{GT})$'s and $B(\text{F})$'s for several nuclei. This would be a specially important technique for studies of dripline nuclei. We now investigate if this relation is valid using the $^{13}\text{C}(^{13}\text{N}, ^{13}\text{C})^{13}\text{N}$ reaction at 105 MeV/nucleon [5] for comparison.

Following Ref. [9] we normalize the interaction so that at zero momentum transfer $V_{\text{OBEP}} \equiv V_{\sigma\tau} = 265$ MeV·fm³ and $V_\tau = 100$ MeV·fm³, appropriate for proton laboratory energies of order of 100 MeV. We describe the scattering waves in terms of eikonal wavefunctions. Since an optical potential for this reaction at 100 MeV/nucleon is not known, we use an optical potential which fits the reaction $^{12}\text{C} + ^{12}\text{C}$ at 85 MeV/nucleon [11,7]. To calculate the single-particle wavefunctions we use a potential of the form

$$V = V_0 f(r) + V_{SO}(\mathbf{L} \cdot \boldsymbol{\sigma}) (df/dr)/r,$$

with

$$f(r) = \left[1 + \exp(r - R_0)/a \right]^{-1},$$

for the neutrons. For the protons we add the Coulomb potential of a uniformly charged sphere of radius R_0 . For both ^{13}C and ^{13}N we choose the set of parameters $V_0 = -53$ MeV, $R_0 = 2.86$ fm, $a = 0.65$ fm, and $V_{SO} = -15.5$ MeV. In this simple model, the relevant transitions are $1p_{1/2}^{\pi'} \rightarrow 1p_{1/2}^{\pi}$ ($\pi = n$, or p , i.e., one proton in ^{13}N goes to a neutron in ^{13}C , and vice versa) involving the operator τ (Fermi transitions). For Gamow-Teller transitions ($\sigma\tau$ operator) the relevant transitions are $1p_{1/2}^{\pi'} \rightarrow 1p_{1/2}^{\pi}$ and $1p_{1/2}^{\pi'} \rightarrow 1p_{3/2}^{\pi}$.

In Table 1 we show the matrix elements of the single particle transitions: $B_{1/2-} \equiv B_{1/2-} \rightarrow B_{1/2-}$ and $B_{3/2-} \equiv B_{1/2-} \rightarrow B_{3/2-}$ for Fermi and Gamow-Teller transitions. For comparison, we also give the values

Table 1

Modified Fermi and Gamow-Teller strengths for single particle transitions and as a function of q . The Gamow-Teller and Fermi operators are replaced by their respective product with a spherical Bessel function $j_0(qr)$. For $q = 0$, $j_0(qr) = 1$, and the usual Fermi and Gamow-Teller matrix elements are restored

| | $q = 0$ | $q = 0.5 \text{ fm}^{-1}$ | $q = 1 \text{ fm}^{-1}$ |
|--------------------------|---------|---------------------------|-------------------------|
| $B_{1/2^-} \text{ (F)}$ | 1 | 0.65 | 0.14 |
| $B_{1/2^-} \text{ (GT)}$ | 2.66 | 1.73 | 0.37 |
| $B_{3/2^-} \text{ (GT)}$ | 0.33 | 0.18 | 0.02 |

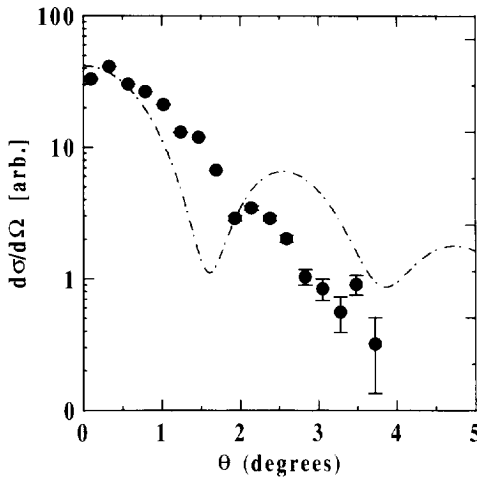


Fig. 1. Differential cross section of the charge exchange reaction $^{13}\text{C}(^{13}\text{N}, ^{13}\text{C})^{13}\text{C}$ at 105 MeV/nucleon for the double excitation case (target and projectile go to the $3/2^-$ state). Experimental data [5] and calculation are given in arbitrary units.

of these matrix elements, in the case that the overlap integral includes a spherical Bessel function of zeroth-order, $j_0(qr)$, for additional two values of q . The matrix elements are drastically reduced for large values of q , especially for $1p_{1/2} \rightarrow 1p_{3/2}$ transitions.

In Fig. 1 we compare the experimental [5] and the theoretical calculation of the differential charge exchange cross section for the double excitation case, i.e. when both target (^{13}C) and projectile (^{13}N) go to the $3/2^-$ state. Both experimental data and the calculation are given in arbitrary units. Thus, only the angular dependence of the cross section can be discussed. The angular dependence is rather insensitive to the structure model for the nuclei, but strongly sensitive on the parameters of the optical potential used to calculate the scattering wavefunctions. Only when

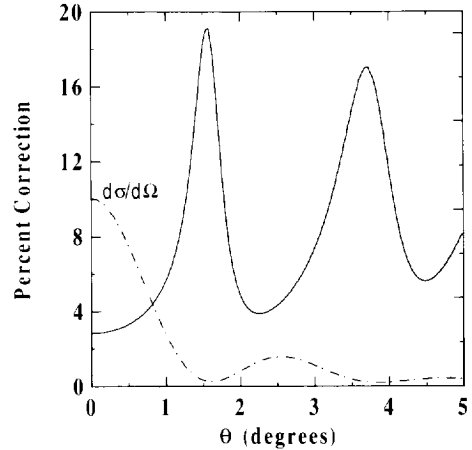


Fig. 2. Ratio between the exact calculation and the approximation 6 for the charge exchange reaction $^{13}\text{C}(^{13}\text{N}, ^{13}\text{C})^{13}\text{C}$ at 105 MeV/nucleon, as a function of the scattering angle.

such parameters are known from, e.g., elastic scattering one can have a better knowledge of the influence of the Gamow-Teller and Fermi matrix elements on the charge exchange cross section. This, of course, assuming that the exchange potential is well known. In fact, the form of the exchange potential is crucial for this aim.

In Fig. 2 we plot the ratio between the exact calculation and the low- q approximation, Eq. (6). With the single particle basis the expansion can be stopped at the $l = 5$ term. In fact, the term with $l = 0$ dominates. This is shown in the figure where the contribution of the $l = 0$ (dashed curve) term of the expansion is shown. The total sum includes terms up to $l = 5$ (solid curve). However, note that the ratio between the exact calculation and the low- q approximation amounts to a factor of 5 at the forward direction. This difference can be understood in a simple way by using Eq. (2). If strong absorption and final state interactions could be neglected, $\chi^{(-)*}(\mathbf{R}) \chi^{(+)}(\mathbf{R}) \approx \exp\{i\mathbf{Q} \cdot \mathbf{R}\}$, where $\mathbf{Q} = \mathbf{k}' - \mathbf{k}$ is the momentum transfer to the c.m. scattering. Thus, the integral over \mathbf{R} yields a delta function, $\delta(\mathbf{q} - \mathbf{Q})$. That is, $\mathbf{Q} = 0$ (forward scattering) would imply $q = 0$, too. Very forward scattering would then justify replacing the exponentials in Eq. (4) by unity, and an equation of the form of Eq. (6) would be obtained. However, strong absorption modifies this scenario: \mathbf{Q} is not easily related to \mathbf{q} , and Eq. (6) is a bad approximation. The larger the absorption radius,

the worse this approximation should be. This may explain why for (p, n) reactions the low- q approximation is better justified, since the absorption radius is smaller in that case. However, even for (p, n) reactions the low- q approximation seems to be ill-defined, as was shown recently [12], especially for the weak transitions.

It is worthwhile to mention that in Ref. [13], which apparently motivated Ref. [4], the calculated cross section at small q turned out to be closely proportional to the $B(\text{GT})$ values calculated with the same sd shell wavefunctions used in the DWBA reaction model. Other cases have also been studied and reasonable agreement with the experimental data has been found. However, we cannot take these results as a general proof of the validity of the low- q approximation. In fact, we see from expressions (2) and (3) that all depends on the Fourier transforms of the product of the transition densities and the exchange potential: if this product is either always positive, or always negative, then the maximum contribution of the q -integral occurs at $q = 0$. This is well explained in Ref. [4], in connection with their Fig. 5. However, if the product is oscillating, with approximately equal positive and negative parts, the maximum contribution comes from a particular $q \neq 0$, inversely proportional to the wavelength of the oscillation. Besides, as shown in Eq. (4), the several parts of the exchange interaction leads to interference terms which often leads to the oscillations in the q -transform of the transition densities. This is further Fourier transformed (now in the variable Q) folded with the an absorption profile. The maximum of this transform does not necessarily comes out at $Q = 0$ either.

Another possibility to get direct information on the Fermi and Gamow-Teller strengths from HICEX reactions would be in the case that the expansion of the exponential function in (4) can be stopped at the term with $l = 0$. One gets matrix elements of the form

$$\langle \phi_{j' m'}^{\pi'} | \sigma_{\mu} \tau^{\nu} j_0(qr) | \phi_{j m}^{\pi} \rangle = \hat{j}(j 1 m \mu | j' m') \mathcal{F}_{jj'}(q) \left[B(\text{GT}) \right]^{1/2}, \quad (8)$$

and a similar expression for Fermi transitions. The function $\mathcal{F}(q)_{jj'} = |\mathcal{I}_{0jj'}(q) / \mathcal{I}_{0jj'}(q=0)|^2$ has to be calculated with accuracy in this case. Similar arguments are given in Ref. [3] for (p, n) reactions.

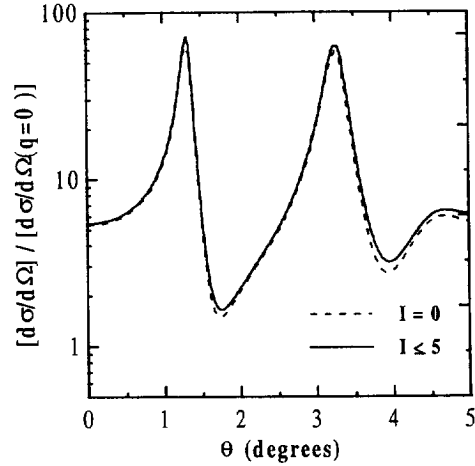


Fig. 3. Percentage deviation from the exact calculation and the approximation obtained by keeping only the $l = 0$ term of the multipole expansion (4) for the charge exchange reaction $^{13}\text{C}(^{13}\text{N}, ^{13}\text{C})^{13}\text{C}$ at 105 MeV/nucleon, as a function of the scattering angle.

In Fig. 3 we show the percentage deviation between the exact calculation and a calculation done with only the $l = 0$ term of the expansion. Also shown (dashed curve) is the differential cross section in arbitrary units. At $\theta = 0$ the difference is of about 2% and increases largely close to the minima of the cross section. This is because, close to the minima (which are basically determined by the l_0 term) the spherical Bessel functions $j_{l>0}$ contribute more to the integrals.

Although the ($l = 0$)-approximation is not bad at $\theta = 0$ compared to the full calculation, it cannot be considered as a useful tool to obtain the Fermi and Gamow-Teller strengths from HICEX, unless a good model for the overlap integrals can be done. This is, however, just what one would like to extract from the experiments, and not from theory. As one sees from Table 1 the matrix elements involved in HICEX are strongly modified by the short range property of the exchange interaction. Only a small part of the transition densities, close to the surface of the nucleus, is relevant for the matrix elements. In contrast, in beta-decay the whole transition density is relevant. To show this we decrease artificially the masses of the pion and the rho by a factor α . These masses enter into the form of the exchange interaction and are responsible for its range [9,7]. As we see from Fig. 4, as α increases,

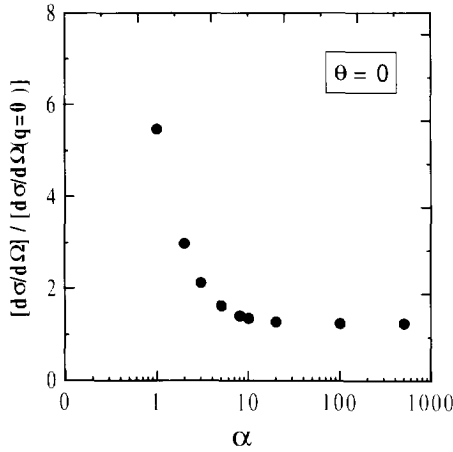


Fig. 4. Ratio between the exact calculation and the approximation 6 as a function of a scaling factor α for the charge exchange reaction $^{13}\text{C}(^{13}\text{N}, ^{13}\text{C})^{13}\text{C}$ at 105 MeV/nucleon. The scaling factor α is used to reduce the pion and the rho masses in the charge exchange interaction.

increasing the range of the interaction, the ratio between the exact calculation and the low- q approximation decreases, reaching unity at $\alpha \approx 10$. This means that the relationship of Eq. (6) would be valid for an interaction of range $r_0 \approx 10 \times (\hbar/m_\pi c) \approx 10$ fm. For an interaction with this range, or longer, the matrix elements are quite insensitive to the form of the interaction, justifying the use of the low- q approximation.

In conclusion, based on this simple reaction case, we have shown that the extraction of Gamow-Teller or of Fermi strength from HICEX experiments not always produces good results. This might be surprising, since weak interaction strengths determined from (p, n) reactions are astonishingly good, although they have been shown to be inaccurate for important Gamow-Teller transitions whose strengths are a small fraction of the sum rule limit [12].

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References

- [1] Proc. of the Int. Symposium on the Physics of Unstable Nuclei, Nucl. Phys. A 588 (1995) 1.
- [2] C.D. Goodman, C.A. Goulding, M.B. Greenfield, J. Rappaport, D.E. Bainum, C.C. Foster, W.G. Love and F. Petrovich, Phys. Rev. Lett. 44 (1980) 1755.
- [3] T.N. Tadeucci, C.A. Goulding, T.A. Carey, R.C. Byrd, C.D. Goodman, C. Gaarde, J. Larsen, D. Horen, J. Rappaport and E. Sugarbaker, Nucl. Phys. A 469 (1987) 125.
- [4] F. Osterfeld, N. Anantaraman, S.M. Austin, J.A. Carr and J.S. Winfield, Phys. Rev. C 45 (1992) 2845.
- [5] M. Steiner et al., Phys. Rev. Lett. 76 (1996) 26.
- [6] H. Lenske, H.H. Wolter and H.G. Bohlen, Phys. Rev. Lett. 62 (1989) 1457.
- [7] C.A. Bertulani, Nucl. Phys. A 554 (1993) 493.
- [8] R.D. Lawson, Theory of the Nuclear Shell Model (Clarendon, Oxford, 1980).
- [9] G.E. Brown, J. Speth and J. Wambach, Phys. Rev. Lett. 46 (1981) 1057.
- [10] D.E. Greiner, P.J. Lindstrom, H.H. Heckman, B. Cork and F.S. Bieser, Phys. Rev. Lett. 35 (1975) 152.
- [11] M. Buenerd et al., Phys. Rev. C 26 (1982) 1299.
- [12] S.M. Austin, N. Anantaraman and W.G. Love, Phys. Rev. Lett. 73 (1994) 30.
- [13] N. Anantaraman, J.S. Winfield, S.M. Austin, J.A. Carr, C. Djilili, A. Gilbert, W. Mittag, J.A. Nolen Jr. and Z.W. Long, Phys. Rev. C 44 (1991) 398.