Quantal and semiclassical methods in relativistic electromagnetic excitation

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Two different quantal approaches to describe relativistic Coulomb excitation are compared and proven to be equivalent in their basic formulations. We reanalyze the $\Lambda\Sigma^0$ conversion experiment by means of the plane wave Born approximation and we also discuss the possible applications of this approach to ultrarelativistic Coulomb collision experiments.

I. INTRODUCTION

For a very long time electromagnetic excitation has been a subject of considerable theoretical and also experimental interest. Since the photon exchange amplitude is singular at four-momentum transfer $q_\mu q_\mu=0$, the virtual photon exchange makes a larger contribution to the amplitude for forward scattering angles than the exchange of strongly interacting particles.[1] The theoretical methods to study electromagnetic excitation can be grouped into quantal and semiclassical ones. In the Weizsäcker-Williams method[2] (originally developed by Fermi[3]), the field of a moving particle at the position of the target is replaced by a pulse of virtual radiation which is then absorbed by the target. If the projectile and target charge distributions do not overlap, such a procedure is rigorously correct. Quantal methods are mainly based on the plane wave Born approximation (PWBA) or Glauber theory.[4,5] In many problems, for example in heavy ion collisions, in addition to the electromagnetic interaction between the projectile and the target, there is also the strong interaction at close enough distances. We can take it into account only in a rough way.

In the semiclassical method,[6] the limitation due to strong nuclear absorption in close collisions is usually done quite simply: the strong absorption restricts straight-line paths to impact parameters larger than the sum of the nuclear radii $b \geq R_1+R_2$. In a recent publication[7] we have shown that such a restriction for the total cross section is related to the quantal calculations. We found that this semiclassical method is equivalent to a PWBA calculation provided that a cylindrical hole cutoff with $R=R_1+R_2$ is introduced. Thus the influence of strong absorption is entirely described by one rather well-determined parameter R. It was also seen that for $Z_1Z_2\alpha<1$, with α equal to the fine-structure constant, quantal diffraction effects dominate over the Coulomb repulsion in the angular distributions. Under such conditions semiclassical methods are not appropriate to describe angular distributions.

In this paper, we first compare in Sec. II the quantal approach in Ref. 7 with the Glauber-type method used in Ref. 5. We show that they are identical and that the minor differences in their final results must be due to small kinematics corrections in Ref. 5, like the inclusion of a Coulomb phase ϕ_C. Then we apply the present formalism to the $\Lambda\Sigma^0$ conversion in the nuclear Coulomb field (the Primakoff effect). With this simple method to take strong absorption into account we calculate angular distributions as well as total Coulomb production cross sections. Essentially we corroborate the value of the Σ^0 lifetime deduced from such an experiment. Finally we discuss the behavior of virtual photon numbers for ultrarelativistic collisions. We point out the importance of a hard component in the virtual phonon spectrum, which can lead to nuclear disintegration, π production, etc. in distant collisions of heavy ions, for future relativistic heavy ion accelerators. Conclusions are given in Sec. III.

II. RELATIVISTIC ELECTROMAGNETIC EXCITATION

A. Comparison of the two quantal methods

In Ref. 7 a quantal calculation of relativistic electromagnetic excitations was given. The PWBA amplitude for the excitation of a nucleus from the initial state $|I_iM_i\rangle$ to a final state $|I_fM_f\rangle$ is given by

$$a_{fi}=\frac{1}{c}\int d^3r\, A_\mu(\mathbf{r})\langle I_fM_f\,|\,j_\mu(\mathbf{r})\,|\,I_iM_i\rangle\ , \tag{1}$$

where $A_\mu(\mathbf{r})$ represents the four-potential created by the transition current of the projectile:

$$A_\mu(\mathbf{r})=\frac{1}{c}\int d^3r'\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\langle \mathbf{k}_f\,|\,J_\mu(\mathbf{r}')\,|\,\mathbf{k}_i\rangle\ , \tag{2}$$

where $k=\omega/c$ is the virtual photon wave number. We have

$$\langle \mathbf{k}_f\,|\,J_\mu(\mathbf{r}')\,|\,\mathbf{k}_i\rangle=Z_pev_\mu e^{i\mathbf{q}\cdot\mathbf{r}'} \tag{3}$$

with $v_\mu=(c,\mathbf{v})$. For more details see Ref. 7. We take nuclear absorption of the projectile into account by restricting the integration over $\mathbf{r}'$ in Eq. (2) to values of $\rho'\geq R$, with $\mathbf{r}'=(\rho',z')$. This means

$$\int d^3r'\cdots\rightarrow\int d^2\rho'\theta(\rho'-R)\int dz'\cdots\ , \tag{4}$$

and Eq. (1) becomes

$$a_{fi}=Z_p e v_\mu \int d^2\rho'\theta(\rho'-R)\int dz' e^{i\mathbf{q}\cdot\mathbf{r}'}\mathscr{A}_\mu(\mathbf{r}') , \quad (5)$$

where

$$\mathscr{A}_\mu(\mathbf{r}')=\frac{1}{c}\int d^3r\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\langle I_f M_f | j_\mu(\mathbf{r}) | I_i M_i\rangle . \quad (6)$$

Since the projectile and target charge distributions are assumed to be nonoverlapping, one can express the nuclear transition matrix element in terms of the usual multipole moments which describe the interaction with photons (Eq. 2.14 of Ref. 7).

On the other hand, a Glauber-type model was used in Ref. 5 to describe relativistic electromagnetic excitation. They start from the profile function

$$\Gamma_{\text{Born}}(\mathbf{b},q_l)=\frac{1}{2\pi i k_i}\int d^2q_t e^{-i\mathbf{q}_t\cdot\mathbf{b}}T_{\text{Born}}(\mathbf{q}_t,q_l) , \quad (7)$$

where

$$T_{\text{Born}}(\mathbf{q}_t,q_l)=\mathscr{F}_\mu(\mathbf{q})\int d^3r' e^{i\mathbf{q}\cdot\mathbf{r}'}\mathscr{A}_\mu(\mathbf{r}') , \quad (8)$$

with $\mathscr{A}_\mu(\mathbf{r}')$ given by (6). In the above expression $\mathscr{F}_\mu(\mathbf{q})$ is related to the form factor of the projectile. For a point projectile or one with a spherical symmetric charge distribution, $\mathscr{F}_\mu(\mathbf{q})=Z_p e v_\mu$.

In order to take strong absorption into account the profile function (7) is modified by a cutoff

$$\Gamma_\gamma=\theta(b-R)\Gamma_{\text{Born}} , \quad (9)$$

and the transition probability will be

$$T_{fi}=\frac{ik_i}{2\pi}\int d^2b' e^{i\mathbf{q}_t\cdot\mathbf{b}'}\Gamma_\gamma$$

$$=Z_p e v_\mu \int d^2\rho'\theta(\rho'-R)\int dz' e^{i\mathbf{q}\cdot\mathbf{r}'}\mathscr{A}_\mu(\mathbf{r}') \quad (10)$$

for a point projectile. In the last step we used the Dirac delta function

$$\frac{1}{(2\pi)^2}\int e^{i\mathbf{q}_t\cdot(\mathbf{b}-\mathbf{b}')}d^2q_t=\delta(\mathbf{b}-\mathbf{b}') .$$

This proves that both approaches should give the same results for a point projectile.

In Ref. 8 it is mentioned that the results of Ref. 5 ought to be equivalent to the Weizsäcker-Williams method (see, e.g., Ref. 9) for the limit of a point charge. Since the PWBA with cylindrical hole cutoff prescription is now shown to be equivalent to the one of Ref. 5, and since the PWBA results agree with that of Weizsäcker-Williams, this statement is further strengthened. Of course, the roles of the projectile and target can be interchanged in the above description.

B. Determination of the Σ^0 lifetime by means of $\Lambda\Sigma^0$ conversion in the nuclear Coulomb field

In this subsection the "Primakoff effect" is reinvestigated by means of our quantal method. In an experiment at CERN,[10] a highly relativistic Λ beam was scattered on a nuclear target, where Σ^0 hyperons were produced at forward angles in the nuclear Coulomb field:

$$\Lambda+Z\to\Sigma^0+Z . \quad (11)$$

The Σ^0 were detected through their decay $\Sigma^0\to\Lambda\gamma$, which is by far the dominant decay mode of the Σ^0 particle. The cross section for the Σ^0 Coulomb production [Eq. (11)] can be expressed in terms of the magnetic transition moment $\mu_{\Lambda\Sigma^0}$ or the Σ^0 lifetime. This is especially interesting since it allows for a test of the $SU(3)_{\text{flavor}}$ properties of the strong and electromagnetic interactions. Coleman and Glashow[11] found that

$$\mu_{\Lambda\Sigma^0}=-\frac{\sqrt{3}}{2}\mu_n , \quad (12)$$

where

$$\mu_n=-1.91\frac{e\hbar}{2m_n c}$$

is the neutron magnetic moment. The Primakoff effect seems to be the only practicable way up to now to measure the Σ^0 lifetime.

The $\Lambda\Sigma^0$-conversion cross section was calculated by Dreitlein and Primakoff[12] and by Pomeranchuk and Shmushkevich.[1] In these calculations, nuclear form factors and absorption are taken into account in a rather complicated method.[13,14] We include nuclear absorption from the outset, and no nuclear form factors enter any longer.

From Ref. 7 the angular distribution for the process (11) can be expressed in terms of the $B(M1)$ value of the $\Lambda\to\Sigma^0$ transition as

$$\frac{d\sigma}{d\Omega}=\frac{16\pi}{9}q_\Lambda^2\left[\frac{Z\alpha\omega}{\gamma v}\right]^2[\chi_1(R)]^2 B(M1)/e^2 , \quad (13)$$

where $q_\Lambda=p_\Lambda/\hbar$ is the momentum of the incident Λ beam, $\gamma=(1-v^2/c^2)^{-1/2}$ with v equal to the beam velocity, and $\hbar\omega$ is the energy of the virtual photon absorbed by the Λ particle in its rest frame:

$$\hbar\omega=\frac{(m_{\Sigma^0}^2-m_\Lambda^2)c^2}{2m_{\Sigma^0}} . \quad (14)$$

The $B(M1)$ value is related to the transition magnetic moment $\mu_{\Lambda\Sigma^0}$ and to the lifetime τ_{Σ^0} by

$$B(M1)=\frac{9\mu_{\Lambda\Sigma^0}^2}{4\pi}=\frac{9\hbar}{16\pi\left[\dfrac{\omega}{c}\right]^3\tau_{\Sigma^0}} . \quad (15)$$

The angular distribution is given by quantal diffraction effects through the function[7]

$$\chi_1(R)=\int_R^\infty J_1(q_t x)K_1(q_l x)x\,dx$$

$$=\frac{R^2}{\eta^2+\xi^2}[\xi J_1(\eta)K_2(\xi)-\eta J_2(\eta)K_1(\xi)] , \quad (16)$$

where

$$\eta=q_t R=q_\Lambda R\sin\vartheta \quad (17a)$$

and

$$\xi = q_l R = \frac{\omega R}{\gamma v} . \tag{17b}$$

The momenta q_t and q_l are, respectively, the transversal and the longitudinal momentum transfer in the laboratory frame of reference.

By means of Eqs. (16) and (17) we can rewrite (13) as

$$\frac{d\sigma}{d\eta} = \frac{32\pi^2}{9}(Z\alpha\xi R)^2 \frac{B(M1)}{e^2}\frac{df(\eta,\xi)}{d\eta} . \tag{18}$$

The function

$$\frac{df}{d\eta} = \eta[\chi_1(R)/R^2]^2$$

is plotted in Fig. 1 for $\xi = 0.1$, 0.2, 0.5, and 1. The values are normalized so that $df/d\eta = 1$ for $\eta = 1$. To obtain the real values one must multiply $df/d\eta$ by the corresponding factors for each ξ. Since for relativistic collisions $q_\Lambda R \gg 1$, the peaks of the angular distribution will occur for $\eta \cong \xi$, which means a maximum scattering angle $\vartheta_m \cong \xi / q_\Lambda R \ll 1$, so that the cross section will be strongly forward peaked. Nonetheless, for exact forward scattering ($\vartheta = 0$) the angular distribution vanishes. This is a characteristic of all magnetic multipole excitations in relativistic Coulomb collisions, as was demonstrated in Ref. 7.

The total cross section is obtained by integrating (18) over all η. The final result is (see also Ref. 7)

$$\sigma(M1) = \frac{32\pi^2}{9}(Z\alpha)^2 \left[\xi K_0 K_1 - \frac{\xi^2}{2}(K_1^2 - K_0^2) \right] \frac{B(M1)}{e^2} , \tag{19}$$

where the modified Bessel functions K_ν are functions of ξ. The present result can be directly applied to the measurements of Dydak *et al.*[10] The only parameter which enters into our calculation is the nuclear absorption radius, which we assume to be $R = 1.2A^{1/3}$ fm. The results of these calculations are shown in Fig. 2(a) for $Z = 92$ (^{238}U), and for $Z = 28$ (^{58}Ni) in Fig. 2(b), together with the experimental results of Ref. 10. We used Eq. (15) with the value of $\mu_{\Lambda\Sigma^0}$ given by (12), as predicted by Coleman and Glashow. From these figures one can see that we are in agreement with the analysis of Dydak *et al.* We also feel that even a more careful analysis of the experimental data with our formulation could not result in a change of the value $\tau = (5.8 \pm 1.3) \times 10^{-20}$ sec given by Dydak *et al.*

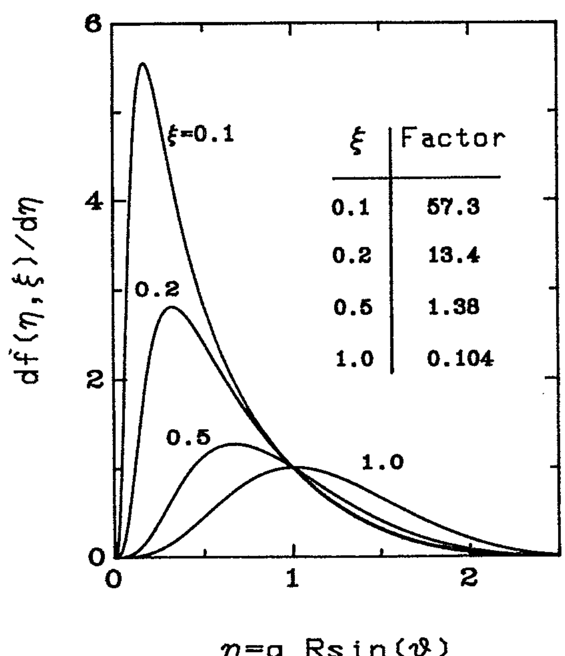

FIG. 1. Angular distribution of the inelastically scattered particles after a magnetic dipole excitation. The values are normalized so that $df/d\eta = 1$ for $\eta = 1$. To obtain the absolute values one must multiply $df/d\eta$ by the respective factors for each ξ.

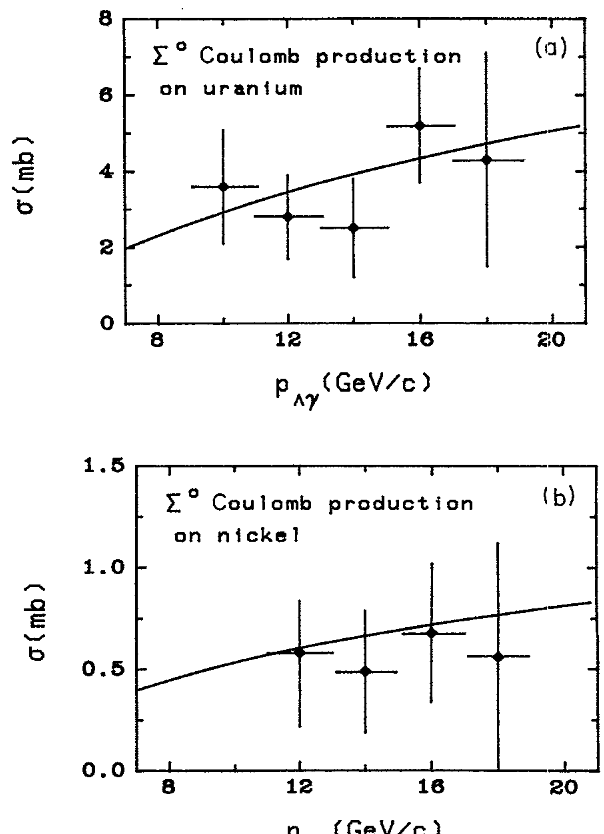

FIG. 2. Total cross section of the Coulomb production $\sigma(\Lambda \to \Sigma^0)$ as a function of the momentum of the $\Lambda\gamma$ pair of the Σ^0 decay for (a) a uranium target and (b) a nickel target. The full line corresponds to Eq. (19) with $\tau_{\Sigma^0} = 0.7 \times 10^{-19}$ sec. Compare with Fig. 19 of Ref. 10.

The essential reason for overcoming the large excitation energy $m_{\Sigma^0}-m_\Lambda=76.86$ MeV/c^2 is the high value of γ. For $\gamma\cong 20$, as was the case in the CERN experiment, the distance d where the adiabacity parameter $\xi=\omega d/\gamma v$ becomes equal to 1 is given by $d\cong 50$ fm; i.e., the area which contributes to the electromagnetic excitation cross section is much larger than the nuclear geometric cross section. Similarly, for heavy ion collisions with $\gamma\gg 1$ (heavy ion accelerators with $\gamma\cong 10$–100 will be operating in the next few years), we can expect large effects from the hard component of the virtual photon spectrum. Among other possibilities we cite the following: for $E_\gamma=\hbar\omega\cong 10$–$20$ MeV the excitation of giant resonances, with subsequent nucleon emission as already observed by Olson *et al.*;[8] for $E_\gamma\cong 20$–100 MeV the quasideuteron effect which corresponds to a photon absorption of a correlated N-N pair; and for $E_\gamma>100$ Mev pion production through Δ-isobar excitation which has a maximum at $E_\gamma\cong 200$ MeV. The virtual photon spectrum extends up to a value of the order of magnitude of

$$(\hbar\omega)_{\max}=\gamma\frac{\hbar c}{d}\cong\gamma\frac{200}{d}\,\text{MeV fm}\;, \tag{20}$$

which for $\gamma=100$ and $d=10$ fm gives $(\hbar\omega)_{\max}\cong 2$ GeV, and for $\gamma=100$ and $d=100$ fm gives $(\hbar\omega)_{\max}=200$ MeV, which is still in the peak area for the pion production reactions

$$\gamma+\mathrm{p}\to\Delta\to\begin{matrix}\mathrm{p}\pi^0\\ \mathrm{n}\pi^+\end{matrix}\;. \tag{21}$$

C. Virtual photon numbers for large γ

With the above possibilities in mind let us finally discuss the behavior of the virtual photon numbers for all multipolarities πl in the ultrarelativistic limit $\gamma\to\infty$. It will be shown that they become all equal.

The virtual photon number (integrated from a minimum impact parameter R to ∞) is given by[7]

$$n_{\pi l}=Z^2\alpha\frac{l[(2l+1)!!]^2}{(2\pi)^3(l+1)}\sum_m |G_{\pi lm}|^2 g_m(\xi)\;. \tag{22}$$

For $\gamma\to\infty$ we have[6]

$$G_{Elm}=iG_{Mlm}=i^{l+m}\frac{m}{l}\frac{\sqrt{4\pi}}{m!(2l+1)!}\times\left[\frac{(l+m)!}{(l-m)!}\right]^{1/2}\frac{1}{(2\gamma)^{m-1}}\;, \tag{23a}$$

for $|m|\geq 1$, and

$$G_{El0}=i^l\frac{(l+1)}{(2l+1)!!}\frac{\sqrt{4\pi}}{\gamma},\quad G_{Ml0}=0\;. \tag{23b}$$

Also, in this limit

$$\begin{aligned} g_{-m}(\xi)=g_m(\xi)&=\pi(m-1)[(m-2)!]^2\left[\frac{2}{\xi}\right]^{2m-2} && \text{for } m>1\\ &=\pi\ln\left[\left[\frac{\delta}{\xi}\right]^2+1\right] && \text{for } m=1\\ &=\pi && \text{for } m=0\;, \end{aligned} \tag{24}$$

where $\delta=0.681\,085\ldots$.

In the sum over m the leading term for $\gamma\to\infty$ is the one with $m=1$ which gives a logarithmic rise with γ, since for $m>1$ there is no dependence on γ. In this case the virtual photon numbers are equal to

$$n_{\pi l}=Z^2\alpha\frac{1}{\pi}\ln\left[\left[\frac{\delta}{\xi}\right]^2+1\right] \tag{25}$$

valid for all multipoles. Since $\xi=\omega R/\gamma v\to 0$, we have a logarithmic rise of the cross section for all multipolarities with γ. The impinging projectile acts like a spectrum of plane wave photons with helicity $m=\pm 1$. Such a photon spectrum contains equally all multipolarities πl.

For $l>1$ and a not too large value of γ, the $m=l$ term can still be substantial. For a comparison we retain only the terms $m=l$ in the sum (22), obtaining

$$n_{\pi l}(m=l\text{ contribution})=Z^2\alpha\frac{1}{2\pi}\frac{(2l)!}{(l+1)l(l-1)}\times(kR)^{2-2l}\;. \tag{26}$$

For $kR\ll 1$, as is the case for low lying excited levels, this term dominates over the $m=1$ term (25), unless γ is extremely large. However, it must be kept in mind that with relativistic particles it is possible to excite states with $kR\cong 1$, or as we have seen in the example of the $\Lambda\Sigma^0$ conversion, even $kR>1$ is possible. In this case the term with $m=1$ dominates, and it is just the logarithmic increase of the cross section with the beam energy which one sees in Fig. 2.

III. CONCLUSIONS

We saw that a simple and transparent quantum mechanical theory based on the PWBA plus a cutoff approximation to account for strong interactions is successful in describing the electromagnetic excitation in relativistic Coulomb collisions. The total cross section is equivalent to that obtained by semiclassical methods, but the diffraction patterns characteristic of each multipolarity of transition can only be obtained by a quantal method and are useful for analysis of experiments.

The illustrative example of the $\Lambda\Sigma^0$ conversion can be reliably and transparently calculated within the present approach. Also, for the experiments with relativistic heavy ion collisions at already existing accelerators (see, e.g., Ref. 8) or future accelerators, that approach could be of great usefulness.

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