Multistep fragmentation of heavy ions in peripheral collisions at relativistic energies

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(Received 21 July 1986)

We study the electromagnetic excitation of high-lying nuclear states by means of peripheral relativistic heavy ion collisions. The present experimental evidence for the excitation of $E1$ and $E2$ giant resonances is discussed theoretically. The multiphonon giant dipole resonance excitation is explored. Absolute values of cross sections for N-phonon excitations are given using the harmonic oscillator model. The role of the damping of the giant resonance states is investigated.

I. INTRODUCTION AND GENERAL CONSIDERATIONS

The extremely strong electromagnetic fields which occur for a very short time scale in relativistic heavy ion collisions (RHIC's) open up new possible studies (see Ref. 1 and references therein). The electromagnetic excitation of giant resonances in RHIC's and their subsequent decay has been clearly observed and interpreted.[2,3] In these studies, the fragment formation by means of the nuclear interaction is also present. An AGS proposal[4] has been accepted that aims to study the extreme peripheral collisions in RHIC's in much greater detail. Of special interest will be the possibility of observation of multiphonon processes which can lead to the excitation of new states with possible exotic decay modes.[4] In view of this proposal it seems to be also of theoretical interest to apply the harmonic vibrator model of Ref. 5 to the cases which will now be studied experimentally and extend it further.

The passage of a particle with charge $Z_P e$, velocity v, and impact parameter b (larger than the nuclear interaction radius) by a nucleus at rest will predominantly cause a momentum change of the charged constituents of the nucleus, i.e., the protons. For not overly close collisions this momentum in the x direction (see Fig. 1) is given classically by (see, e.g., Ref. 6, p. 619)

$$\Delta p = 2\frac{Z_P Z_T e^2}{bv} . \tag{1.1}$$

From this we calculate the energy transferred to the nucleus as a whole as

$$\Delta E_A = \frac{(\Delta p)^2}{2m_A} = 2\frac{(Z_P Z_T e^2)^2}{A_T m_N b^2 v^2} , \tag{1.2}$$

where m_N is the nucleon mass. For very fast collisions we can assume the protons to move almost freely; the total amount of energy transferred to all protons is given by

$$\Delta E_z = 2\frac{(Z_P e^2)^2 Z_T}{m_N b^2 v^2} . \tag{1.3}$$

The difference gives the internal excitation energy of the nucleus,

$$\Delta E_{int} = 2\frac{N_T Z_T}{A_T}\frac{(Z_P e^2)^2}{m_N b^2 v^2} . \tag{1.4}$$

(This amounts to giving effective charges of Ne/A for protons and $-Ze/A$ for neutrons, respectively.) If the incident particle is also a nucleus the same Eq. (1.4) can be used for the determination of its internal excitation energy by exchanging the indices P and T. As an example, we consider the case of relativistic ($v \simeq c$) $^{238}U + ^{238}U$ collisions with $b = 15$ fm. We obtain

$$\Delta E_A \simeq 5 \text{ MeV}, \quad \Delta E_z \simeq 15 \text{ MeV}, \quad \text{and } \Delta E_{int} \simeq 10 \text{ MeV} .$$

This internal excitation energy corresponds to about the excitation energy of the giant quadrupole and dipole resonances in ^{238}U. From this simple classical estimate we can already deduce that there is a large probability for the excitation of giant resonances in peripheral RHIC's.

In Sec. II the direct excitation of giant resonances in RHIC's is studied (see also Ref. 1). Theoretical calculations are directly compared to the experimental results of Mercier *et al.*[3] Nuclear effects are also considered (see also Ref. 10). One has to envisage two nuclear modes which lead to fragment formation: direct nuclear knockout processes and the nuclear excitation of the giant resonances with subsequent particle decay.

The possibility of multiple excitation of giant resonances, especially of the giant dipole resonance (GDR), seems to be a unique domain of RHIC's. In nonrelativistic multiple Coulomb excitation the adiabaticity condition prevents the effective excitation of very high lying nuclear states. Nuclear excitation of multiphonon states (see, e.g., Ref. 7) will be strongly hindered because the decay time of these states is much faster than the collision time. In electron scattering, on the other hand, the interaction strength is not large enough to allow for appreciable multiphoton processes. The decay time of a GDR corresponding to a width of $\Gamma = 5$ MeV is given by

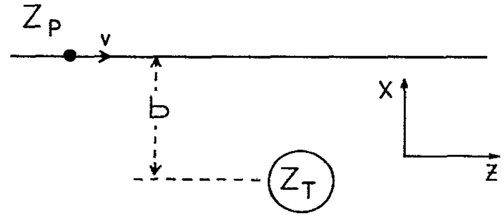

FIG. 1. Projectile with nuclear charge Z_P hitting a target Z_T with an impact parameter b. The coordinate system used in the text is given.

$$\tau_{\text{decay}}=\frac{\hbar}{\Gamma}\simeq 10^{-22}\text{ sec}\ . \tag{1.5a}$$

This time has to be compared to the electromagnetic interaction time in a relativistic collision,

$$\tau_{\text{coll}}\simeq\frac{b}{\gamma v}\simeq\frac{5}{\gamma}\times 10^{-23}\text{ sec}\ , \tag{1.5b}$$

for $b=15$ fm, where $\gamma=(1-v^2/c^2)^{-1/2}$ is the relativistic Lorentz factor. Thus we will have $\tau_{\text{decay}}>\tau_{\text{coll}}$ even for moderately high values of γ, and multiple excitation will, in principle, be possible.

In Sec. III the harmonic vibrator model (see Ref. 5) is studied in further detail. After a simple classical consideration the multiphoton excitation is studied quantum mechanically. It will be seen that a purely classical consideration already gives a remarkably accurate insight into the process. Numerical results are presented for experimentally relevant cases.

Some comments are also made about the expected angular distribution of the fragments, which can be of interest for future precision measurements in the extreme forward angular region. The influence of damping of the GDR states on the excitation and decay process will be of crucial importance. It is discussed in general terms in Sec. IV. Our conclusions are given in Sec. V.

II. COULOMB AND NUCLEAR FRAGMENTATION IN PERIPHERAL RHIC'S

A group of experimentalists at Lawrence Berkeley Laboratory[2] and another at Ames Laboratory and Iowa State University[3] have presented clear evidence of Coulomb fragmentation in RHIC's. In the latest experiment[3] one obtained the cross sections for one-neutron removal of ^{59}Co, ^{89}Y, and ^{197}Au targets due to the irradiation by relativistic beams of ^{1}H, ^{12}C, ^{20}Ne (2.1 GeV/nucleon), ^{40}Ar (1.8 GeV/nucleon), and ^{56}Fe (1.7 GeV/nucleon). From the data on fragmentation cross sections of the same targets by means of relativistic proton beams (for which Coulomb effects are negligible), they were able to deduce the nuclear contribution to the one-neutron removal cross sections by RHI beams. A precise theoretical explanation of these data is complicated by the presence of nuclear contributions which can arise from a direct knockout of the neutrons or by means of a two-step process involving first the collective excitation of a giant resonance in the nuclei followed by the emission of one neutron. The nuclear contribution to this process is peaked at a certain impact parameter and decreases with increasing distances. It also decreases when the nuclei come closer together, since channels other than the one-neutron removal process become more important.[8] In this way one can reasonably assume that the probability of removing one neutron by means of the nuclear interaction in a RHIC is given by a Gaussian function of the impact parameter b, such as

$$P(b)=\beta\exp\left[-\left[\frac{b-R}{\delta}\right]^2\right]\ , \tag{2.1}$$

where 2δ is the thickness of the surface area contributing to that process and β is the maximum probability at an optimal impact parameter which we set to

$$R=1.2(A_P^{1/3}+A_T^{1/3})\text{ fm}\ . \tag{2.2}$$

Such a parametrization has also been found in theoretical calculations of fragmentation processes at nonrelativistic energies.[9] A justification of this surface peaked form is given in terms of a Glauber model in Ref. 8. The cross section will be

$$\sigma_{\text{N}}=2\pi\int_0^\infty bP(b)db\simeq 2(\pi)^{3/2}R\,\beta\delta\ . \tag{2.3}$$

In order to have an estimate of $\beta\delta$ we set the cross section given by (2.3) to the experimental values determined by Mercier *et al.*[3] We find the values of $\beta\delta$ as given in Table I, which are collected in Fig. 2 as a function of A_P+A_T. From that one infers an average value of

$$\delta\beta\simeq 1.1\pm 0.1\text{ fm}\ . \tag{2.4}$$

The question now arises about what the value of the maximum probability β should be. Clearly, there are other channels for fragmentation, such as, e.g., fission, two-nucleon removal, etc., in the peripheral collisions with small nuclear contact. In Ref. 10, a theoretical study has shown that there is an appreciable contribution to the fission channel in ^{238}U projectiles (1 GeV/nucleon) incident on nuclear emulsion. However, since the energy deposit in such collisions is small, the one-neutron removal process must be of greatest probability in most cases. If we use $\beta\simeq 1$ we get $\delta\simeq 1$ fm from (2.4). This means that the nuclear contribution is restricted within a small range of impact parameters in comparison to a much wider interval for the Coulomb contribution to the same process. In spite of the smaller energy deposit by means of the Coulomb interaction in a RHIC, its long range leads to

TABLE I. The values of $\delta\beta$ in units of fm [see Eqs. (2.1) and (2.3) for their definition] extracted from the experimental results of Ref. 3 for the various projectile and target combinations used in these experiments.

RHI	Reactions ^{59}Co(RHI,X)^{58}Co	^{89}Y(RHI,X)^{88}Y	^{197}Au(RHI,X)^{196}Au
^{12}C (2.1 GeV/nucleon)	1.00 ± 0.08	1.17 ± 0.11	0.95 ± 0.11
^{20}Ne (2.1 GeV/nucleon)	1.13 ± 0.09	1.22 ± 0.1	1.00 ± 0.12
^{40}Ar (1.8 GeV/nucleon)		1.43 ± 0.12	0.93 ± 0.12
^{56}Fe (1.7 GeV/nucleon)	1.02 ± 0.1	1.22 ± 0.12	0.82 ± 0.11

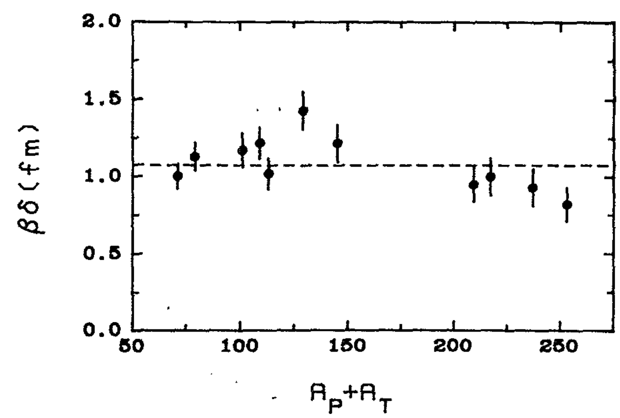

FIG. 2. The values of $\beta\delta$ [see Eqs. (2.1) and (2.3)] as a function of the sum of target and projectile mass numbers, A_T+A_P.

total cross sections which can be even larger than the geometrical cross section.[1]

The Coulomb contribution to the nuclear fragmentation in RHIC's is a two-step process involving the excitation of giant resonances followed by particle decay. We can write the cross sections for it as[11]

$$\sigma_C=\sum_{\pi\lambda}\int n_{\pi\lambda}(\omega)\sigma_\gamma^{\pi\lambda}(\omega)\frac{d\omega}{\omega}\ , \tag{2.5}$$

where $n_{\pi\lambda}(\omega)$ are the equivalent photon numbers with the excitation energy $E_\gamma=\hbar\omega$. They are given in analytical form in Ref. 11. The functions $\sigma_\gamma^{\pi\lambda}(\omega)$ are the photonuclear cross sections for the multipolarity $\pi\lambda$ and the total photonuclear cross section is given by

$$\sigma_\gamma(\omega)=\sum_{\pi\lambda}\sigma_\gamma^{\pi\lambda}(\omega)\ . \tag{2.6}$$

While, normally, the $\pi\lambda=E1$ contribution to the sum (2.5) is much larger than the others, it was shown in Ref. 11 that $n_{E2}\gg n_{E1}$ for beam energies around 1 GeV/nucleon. This leads to an appreciable contribution (5—20%) of the quadrupole multipolarity to the total Coulomb cross section at these energies. It is interesting to compare the experimental values of Ref. 3 with theoretical predictions based on Eq. (2.5) and on the sum rules for the photonuclear cross sections.

It is well known that heavy nuclei exhibit an electric dipole resonance at approximately $80/A^{1/3}$ MeV and a quadrupole resonance at $62/A^{1/3}$ MeV. We ascribe all strength in the Thomas-Reiche-Kuhn (TRK) sum rule to the electric dipole resonance

$$\int\sigma_\gamma^{E1}(E_\gamma)dE_\gamma\simeq 60\frac{NZ}{A}\ \text{MeV mb}\ , \tag{2.7a}$$

and in the energy weighted sum rule to the electric quadrupole resonance

$$\int\sigma_\gamma^{E2}(E_\gamma)\frac{dE_\gamma}{(E_\gamma)^2}\simeq 0.22ZA^{2/3}\ \mu\text{b/MeV}\ . \tag{2.7b}$$

Table II shows the theoretical values based on Eq. (2.5) and on the sum rules (2.7). One clearly sees the relevance of the $E2$ mode compared to $E1$. From the ratio between the experimental data and the theoretical predictions,

$$r=\frac{\sigma_C^{\text{expt}}}{\sigma_C^{E1}+\sigma_C^{E2}}\equiv\frac{\sigma_{\text{expt}}}{\sigma_{\text{SR}}}\ , \tag{2.8}$$

we obtain the values gathered in Fig. 3 as a function of A_P+A_T. On the average, $r\lesssim 1$, which is a reasonable result since σ_{SR} includes the total strength of the giant resonances which can decay by means other than one-neutron emission. In principle, one could also use the experimental photonuclear cross sections $\sigma(\gamma,\text{n})$ to do a more exact calculation of the one-neutron removal cross section by means of Eq. (2.5) (see, e.g., Ref. 12). However, the decomposition of $\sigma(\gamma,\text{n})$ into $E1$ and $E2$ (or other) multipolarities is not exactly known.

The only empirical parameter entering into Eq. (2.5) is the minimum impact parameter, which we set to R as given by (2.2). For impact parameters in the interval $R-\delta\le b\le R+\delta$, there is interference between the nuclear and the Coulomb interaction. By using $b_{\min}=R-\delta$ in Eq. (2.5), with $\delta=1$ fm, the theoretically estimated Coulomb cross sections increase by less than 10%. Because of our lack of knowledge of the nuclear and Coulomb interference effects, there exists even a greater uncertainty in the theoretical determination of the fragmentation cross section in peripheral RHIC's. The situation becomes simpler at higher energies and when both projectile and target are heavy nuclei, for which the Coulomb cross sections depend much less on the uncertainty in the minimum impact parameter. In that case the Coulomb interaction leads to much greater cross sections than the nuclear interaction and, for practical purposes, one can disregard the nuclear contributions in peripheral RHIC's. Multiple excitation of giant resonances will also

TABLE II. Theoretical electromagnetic excitation cross sections for $E1$ and $E2$ giant resonances for various projectile and target combinations. The incident projectile energy is given in parentheses and the cross section values are given in mb.

	Reactions					
	^{59}Co(RHI,X)^{58}Co		^{89}Y(RHI,X)^{88}Y		^{197}Au(RHI,X)^{196}Au	
RHI	$E1$	$E2$	$E1$	$E2$	$E1$	$E2$
^{12}C (2.1 GeV/nucleon)	8.70	1.88	15.5	3.39	46.5	10.3
^{20}Ne (2.1 GeV/nucleon)	22.9	4.65	41.1	8.45	124	26.2
^{40}Ar (1.8 GeV/nucleon)	63.0	12.7	114	23.4	354	74.6
^{56}Fe (1.7 GeV/nucleon)	121	24.2	221	45	694	145

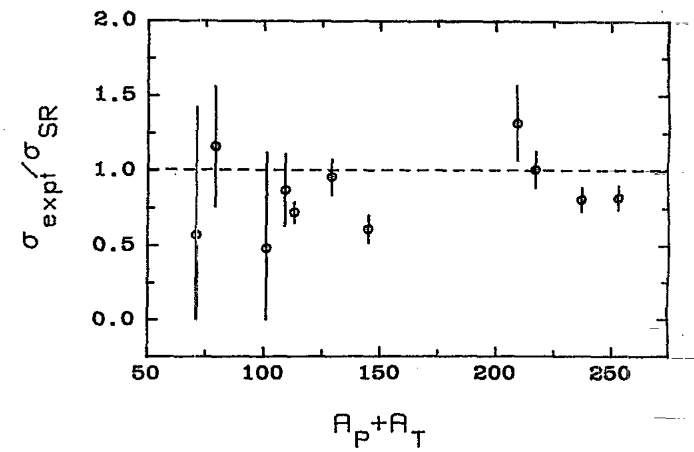

FIG. 3. The ratio of the experimentally determined Coulomb excitation cross section σ_{expt} and the theoretical value σ_{SR}, as derived from the sum rule model as a function of the sum of the target and projectile mass numbers, A_T+A_P.

be of importance at higher energies and in the following we present a theoretical analysis of it.

III. HARMONIC VIBRATOR MODEL

A. Excitation probabilities

The internal excitation energy of a nucleus by means of a relativistic charged particle as given by Eq. (1.4) does not take into account the binding energy of the nucleons, but we can account for it very easily if we use the harmonic vibrator model for the nucleus. The energy transferred to a harmonically bound particle, with charge e_i and mass M_i, by a relativistic particle with charge $Z_p e$ is given by (see Ref. 6, p. 623)

$$\Delta E_i(b)=2Z_P^2e^2\left[\frac{e_i^2}{M_i}\right]\frac{1}{v^2b^2}x^2\left[K_1^2+\frac{1}{\gamma^2}K_0^2\right], \quad (3.1)$$

where K_μ are the modified Bessel functions as function of $x=\omega b/\gamma v$.

We now apply this result to the excitation of GDR's in nuclei. In this case we assume that all nucleons vibrate with the same frequency $\omega=E_{GDR}/\hbar$ and, to disregard the center of mass motion, we use the effective charge of a nucleon as $(e_i)_{eff}=(N/A)e$ for protons and $(e_i)_{eff}=-(Z/A)e$ for neutrons. Summing for all nucleons,

$$\sum_i\left[\frac{e_i^2}{M_i}\right]=\frac{e^2}{m_N}\left[\sum_{i=1}^{Z}\left[\frac{N}{A}\right]^2+\sum_{i=Z+1}^{A}\left[\frac{Z}{A}\right]^2\right]$$
$$=\frac{NZ}{A}\frac{e^2}{m_N}, \quad (3.2)$$

we obtain

$$\Delta E(b)=\sum_i\Delta E_i(b)$$
$$=\frac{2E_{GDR}^2}{m_Nc^2}\alpha^2\frac{Z_P^2N_TZ_T}{A_T}\left[\frac{c}{v}\right]^4\frac{1}{\gamma^2}\left[K_1^2+\frac{1}{\gamma^2}K_0^2\right], \quad (3.3)$$

where α is the fine structure constant.

One can easily see that (3.3) reduces to (1.4) in the limit $x=\omega b/\gamma v\ll 1$, corresponding to the low frequency limit.

We can also interpret $\Delta E(b)/E_{GDR}$ as the probability $\phi(b)$ of exciting a GDR in a collision with impact parameter b, i.e.,

$$\phi(b)=\frac{2E_{GDR}}{m_Nc^2}\alpha^2\frac{Z_P^2N_TZ_T}{A_T}\left[\frac{c}{v}\right]^4\frac{1}{\gamma^2}\left[K_1^2+\frac{1}{\gamma^2}K_0^2\right]. \quad (3.4)$$

By taking $E_{GDR}=80\ \text{MeV}/A^{1/3}$, we obtain

$$\phi(b)=a_0^2+a_1^2+a_{-1}^2\ , \quad (3.5a)$$

where

$$a_0=0.41\alpha\frac{Z_P\sqrt{N_TZ_T}}{A_T^{2/3}}\left[\frac{c}{\gamma v}\right]^2K_0(x) \quad (3.5b)$$

and

$$a_{\pm1}=0.29\alpha\frac{Z_P\sqrt{N_TZ_T}}{A_T^{2/3}}\left[\frac{c}{v}\right]^2\frac{1}{\gamma}K_1(x)\ . \quad (3.5c)$$

We observe that this purely classical derivation perfectly agrees with the previous semiclassical and quantal calculations of Refs. 13 and 11, respectively, where one used the TRK sum rule to determine the nuclear dipole transition strength [$B(E1)$ value]. In those references, $a_\mu=|a_{fi}^\mu|$ were identified as the probability amplitude of exciting a nucleus by transferring to it an amount $\mu\hbar$ of angular momentum in the beam direction. Classically, the amplitude a_0 corresponds to the action of the parallel electric field E_z (see Fig. 4), which generates vibrations along the beam direction. These vibrations correspond to an angular momentum perpendicular to the beam direction, i.e., $\mu=0$. The field E_x will generate $\mu=\pm1$ vibrations and the excitation probability by symmetry must be equally distributed between $\mu=+1$ and -1. Since E_x dominates for $\gamma\gg1$, the target (or the projectile) will gain essentially internal vibrations perpendicular to the beam direction in that limit.

From the dynamics of the electromagnetic excitation process, the angular distribution of the fragments can be directly calculated. For the sake of simplicity of presenta-

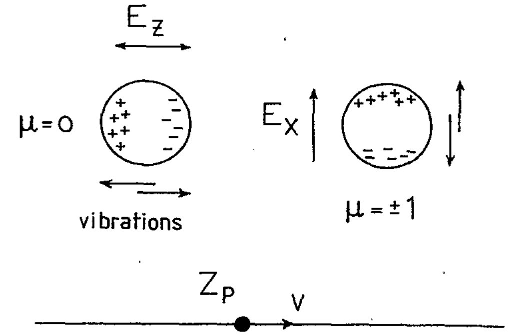

FIG. 4. Proton and neutron vibrations induced by the passage of a relativistic heavy ion.

tion, we illustrate the essential points for spinless projectiles and fragments. We consider the projectile fragmentation process $A \to B + C$ in the system of the projectile. The transition from the projectile's ground state,

$$\psi_i = \frac{1}{r} f(r) Y_{00}(\hat{\mathbf{r}}) , \tag{3.6a}$$

described by a $B+C$ cluster wave function to the final state characterized by the relative momentum $\mathbf{k}_f$ given by the wave function

$$\psi_{\mathbf{k}_f} = \sum_{lm} Y^*_{lm}(\hat{r}) Y_{lm}(\hat{\mathbf{k}}_f) g_l(r,k_f) \frac{1}{r} , \tag{3.6b}$$

is determined by the excitation amplitude

$$a_{fi} = \frac{2Z_T e^2}{\hbar b v} \left[x K_1(x) D^x_{fi} + i \frac{x}{\gamma} K_0(x) D^z_{fi} \right] , \tag{3.7}$$

where $x = \omega b / \gamma v$. The x and z components of the nuclear dipole matrix elements are denoted D^x_{fi} and D^z_{fi}, respectively. As usual, these matrix elements can be separated into a geometrical part determined entirely by the angular momentum quantum numbers and an overall strength factor, which gives the $B(E1)$ value [in the simplified model given here it is determined by the radial dipole matrix element $R(k_f) = \int_0^\infty dr\, g_{l=1}(r,k_f) r f(r)$]. One finds, for a_{fi},

$$a_{fi} = \frac{2Z_T e^2}{\hbar b v} \left[x K_1(x)(-\sin\theta\cos\phi) + i \frac{x}{\gamma} K_0(x) \cos\theta \right] \frac{R(k_f)}{\sqrt{4\pi}} , \tag{3.8}$$

where θ and ϕ denote the polar angles of $\hat{\mathbf{k}}_f$. The $\mu = \pm 1$ excitations are proportional to $xK_1(x)$, the $\mu = 0$ excitation of $i(x/\gamma)K_0$. For $x \ll 1$ this leads to a very strong alignment to the final fragment state, as has already been seen above (cf. Fig. 4). Because of the phase difference there is no interference of $\mu = \pm 1$ and 0 excitations for the angular distributions. Averaging over the azimuthal angle ϕ, one obtains

$$|a_{fi}|^2 = \frac{Z_T^2 \alpha^2}{\pi} \left[\frac{c}{bv} \right]^2 [R(k_f)]^2 \times \left[x^2 K_1^2(x) \sin^2\theta + \frac{x^2}{\gamma^2} K_0^2(x) \cos^2\theta \right] , \tag{3.9}$$

i.e., for $x \ll 1$, as will usually be the case, there is a strong tendency of emission perpendicular to the beam axis.

Let us compare the momentum of the fragment obtained from the decay of the excited resonance state to the momentum obtained from the Coulomb repulsion of the whole projectile during the collision. The momentum due to the Coulomb collision is perpendicular to the beam and is given by Eq. (1.1). The momentum due to the decay of the resonant state is given by $p_d = \sqrt{2m_0 \Delta E}$, where ΔE is the decay energy and m_0 is the reduced mass of $B+C$. As seen above, the main component of p_d is also perpendicular to the beam axis. As an example, for $Z_T = 92$, $Z_P = 8$, $b = 15$ fm, and $v \simeq c$, we obtain, from Eq. (1.1), $p_\perp \simeq 150$ MeV/c for the momentum due to Coulomb repulsion of the projectile. If we assume a decay energy of $\Delta E \simeq 10$ MeV (i.e., excitation energy above the threshold for $A \to B + C$) and a reduced mass $m_0 \simeq 1$ GeV (which is about the reduced mass in the case of one-nucleon emission), then $p_d \simeq 140$ MeV/c. Compared to the incident momentum

$$p_{\text{lab}} \simeq \frac{E_{\text{lab}}}{c} \simeq (\gamma - 1) A \text{ GeV}/c ,$$

the above quantities are only a small percent of it. This means that a study of the angular distribution of the fragments can only be achieved in very high precision experiments.

Quantum mechanically, (3.4) corresponds to the result of a first order perturbation theory. If this excitation probability approaches the value of unity, first order perturbation theory will, of course, break down and multiple excitation occurs. In the exact theory of multiple excitation of a harmonic oscillator (see, e.g., Ref. 14), one obtains a Poisson distribution for the excitation probability of an N-phonon state

$$P_N = \frac{1}{N!} \phi^N e^{-\phi} . \tag{3.10}$$

This result can also be interpreted classically. The probability P_N to excite an oscillator by an energy amount $N\hbar\omega$ is equivalent to the probability to excite N uncoupled oscillators from a given ensemble, each by an energy amount $\hbar\omega$. In the limit that this ensemble possesses an infinite number of oscillators, P_N will be given by a Poisson distribution of the probability to excite only one oscillator.[15] One interesting feature is that, for the mean excitation energy, we obtain

$$\overline{\Delta E(b)} = \sum_N N \hbar\omega P_N(b) = \hbar\omega\, \phi(b) . \tag{3.11}$$

This means that the energy transfer, calculated in first order perturbation theory, gives the correct value even in the case where first order excitation calculations are not justified [e.g., if $\phi(b) \gtrsim 1$]. This is a special property of the harmonic oscillator model.

B. Ultrarelativistic limit ($\gamma \gg 1$)

As quoted above in the high energy limit $\gamma \gg 1$, $a_0 \ll a_{\pm 1}$ and a good approximation, as long as $b < \gamma c / \omega$, is

$$a_{\pm 1} = 0.29 \alpha \frac{Z_P \sqrt{N_T Z_T}}{A_T^{2/3}} \frac{c}{\omega b} , \tag{3.12}$$

and (3.10) becomes

$$P_N(b) \simeq \frac{1}{N!} \left[\frac{S}{b^2} \right]^N e^{-S/b^2} , \tag{3.13a}$$

where

$$S = 5.45 \times 10^{-5} \frac{Z_P^2 N_T Z_T}{A_T^{2/3}} \text{ fm}^2 . \tag{3.13b}$$

The total cross section is obtained by integrating over the impact parameter, starting from a minimum impact parameter $b_{\min}=R$, where nuclear absorption sets in

$$\sigma_C^{(N)}=2\pi\int_R^\infty bP_N(b)db \ . \tag{3.14}$$

If we use the approximation (3.13), then for $N=1$ it is necessary to introduce the adiabatic cutoff radius $b_{\max}\simeq\gamma c/\omega$ in order to have a convergent integral. For $N\geq 2$ the excitation probability decreases fast enough to ensure convergence. We obtain

$$\sigma_C^{(N=1)}\simeq 2\pi S \ln\left[\frac{\gamma c}{\omega R}\right] \tag{3.15a}$$

and

$$\sigma_C^{(N\geq 2)}\simeq\frac{\pi S}{N(N-1)}\left[1-e^{-u}\sum_{k=0}^{N-2}\frac{u^k}{k!}\right]\simeq\frac{\pi S u^{N-1}}{N!(N-1)} , \tag{3.15b}$$

Where $u=S/R^2$ and the last approximation is valid for $u\ll 1$, which is generally the case for light ions.

With these values the maximum possible cross section $\sigma_C^{(N)}$ can be immediately calculated. The cross sections for the excitation of relativistic ^{16}O, ^{32}S, and ^{238}U projectiles in the collision with ^{238}U targets are given in Table III. We also show in Fig. 5 the N-phonon Coulomb fragmentation cross sections of ^{16}O projectiles incident on ^{238}U as a function of the laboratory energy per nucleon. The solid lines correspond to the use of Eqs. (3.4), (3.10), and (3.14), and the dashed lines correspond to the approximations (3.15). As is expected from the increase of S with the mass, N-phonon states are excited with larger cross sections with increasing mass. On the other hand, the amplitudes of the collective motion of all protons against all neutrons are larger for light nuclei than for heavier ones. This can be readily seen from the simple model adopted for the GDR. The dipole operator is given by

$$\hat{D}=\sum_{i=1}^{A}(\hat{\rho}_i-\hat{R})=\frac{NZ}{A}\hat{\rho} , \tag{3.16a}$$

where $\hat{R}$ is the center of mass and

$$\hat{\rho}=\frac{1}{Z}\sum_{i=1}^{Z}\hat{\rho}_i-\frac{1}{N}\sum_{i=Z+1}^{A}\hat{\rho}_i \tag{3.16b}$$

is the difference between the center of mass of all protons with respect to all neutrons. Assuming that the TRK sum rule is exhausted by the GDR, one obtains

TABLE III. Limiting value ($\gamma\to\infty$) of total cross sections for N-phonon GDR excitation of ^{16}O, ^{32}S, and ^{238}U projectiles with ^{238}U as the target nucleus.

N	^{16}O	^{32}S	^{238}U
2	3.1 mb	17 mb	1.28 b
3	22 μb	0.25 mb	0.14 b
4	0.16 μb	4 μb	15 mb

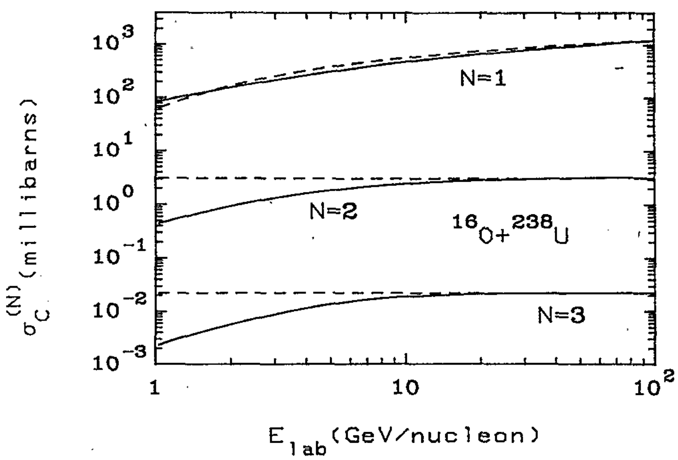

FIG. 5. The total cross sections $\sigma_C^{(N)}$ for the excitation of N-phonon GDR states in ^{16}O + ^{238}U collisions as a function of the incident energy $E_{\rm lab}$. The exact results are given by a solid line; the dashed lines correspond to the approximation [Eq. (3.15a)] for $N=1$ and the limit [Eq. (3.15b)] for $N\geq 2$.

$$D^2=|\langle\psi_{\rm GDR}|\hat{D}|\psi_0\rangle|^2=\frac{NZ}{A}\left[\frac{\hbar^2}{2m_{\rm N}}\right]\frac{1}{E_{\rm GDR}}$$
$$=0.26\ {\rm fm}^2\frac{NZ}{A^{2/3}} . \tag{3.17}$$

In terms of the collective coordinate ρ, one has

$$\rho=\frac{A}{NZ}D=\frac{0.51}{\sqrt{NZ}}A^{2/3}\ {\rm fm} , \tag{3.18}$$

which decreases likes $A^{-1/3}$ with A. Thus neutrons and protons are more effectively separated in low mass nuclei. However, the excitation cross sections are smaller. For ^{16}O one finds an average p-n separating distance in a GDR of about $\rho\simeq 0.4$ fm. It would be interesting to know about the response of the nuclear system to an $N\geq 2$ phonon state. If we assume a linear dependence of ρ with $\sqrt{N}$, we would obtain an average p-n separation distance around $\rho\simeq 0.8$ fm for $N=4$, which is quite a high value. Indeed, the excitation energy of such a state would be $E^{(N=4)}=4E_{\rm GDR}\simeq 127$ MeV, which is exactly the energy necessary to separate all protons and neutrons in ^{16}O. In the simple harmonic model, the maximum separation distance of the p-n vibrations, i.e., the amplitude of the vibration, is given by $d=\sqrt{2}\rho$, which implies that in an $N=4$ state the protons and neutrons would separate beyond the range of the nuclear forces. Since the cross sections for excitation of this state by means of the electromagnetic interaction in a RHIC with a heavy target are of orders of millibarns, this process could be of great importance for producing neutron-rich fragments, as was pointed out in Ref. 4.

The possible signatures of the $N\geq 2$ phonon states remain speculation, especially regarding what the specific decay widths and decay channels will be like, and regarding the probability of formation of polyneutrons and other exotic phenomena, as discussed in Ref. 4. Yet it is interesting and necessary to discuss the influence of the damping of the GDR motion on the excitation process in more general terms.

IV. THE INFLUENCE OF DAMPING: A DISSIPATIVE QUANTUM VIBRATOR UNDER EXTERNAL FORCES

The giant dipole state is a very short-lived state. Being high in the continuum, it couples strongly to other more complicated states and eventually decays mainly statistically by particle (neutron) emission. A typical width of $\Gamma=5$ MeV corresponds to a lifetime of $\tau_{\rm decay}\simeq 10^{-22}$ sec [see Eq. (1.5a)]. The width of the N-phonon ($N\geq 2$) GDR states is expected to be even larger. In a situation where the lifetime of a state is comparable to or even smaller than the collision time, an essential modification of the usual description of Coulomb excitation has to be introduced. This was accomplished by Weidenmüller and Winther.[16] The nuclear states are divided into bound and continuum states; direct excitation of continuum states as well as continuum-continuum coupling is neglected. In this case, the usual coupled equations for the time dependent amplitudes $C_N(t)$ read[16]

$$i\hbar\dot{C}_N(t)=\sum_M \langle N\,|\,V(t)\,|\,M\rangle e^{(i/\hbar)(E_N-E_M)t}C_M(t)+\int_{-\infty}^{\infty}dt'K_N(t-t')C_N(t')\ , \qquad (4.1)$$

where the function K_N takes the coupling to the more complicated channels into account (in our example, the N-phonon states are identified with the bound states of the nucleus; all the complicated decay channels of these states correspond to the continuum; which is assumed to be excited only via the GDR-doorway states). This function is given in terms of the width $\Gamma_N(E)$ by

$$K_N(t-t')=-\frac{i}{4\pi}\int_{-\infty}^{\infty}d\omega\, e^{i\omega(t-t')}\Gamma_N\left[\omega+\frac{E_N}{\hbar}\right] . \qquad (4.2)$$

For $\Gamma_N=$const, one obtains

$$K_N(t-t')=-i\frac{\Gamma_N}{2}\delta(t-t')$$

and the coupled equations (4.1) become

$$i\hbar\dot{C}_N(t)=\sum_M \langle N\,|\,V(t)\,|\,M\rangle e^{(i/\hbar)(E_N-E_M)t}C_M(t)-i\frac{\Gamma_N}{2}C_N(t)\ . \qquad (4.3)$$

Since $V(t)$ is very well known for the Coulomb interaction and the nuclear states $|\,N\rangle$ are assumed to be solutions of the harmonic oscillator with energies $E_N=N\hbar\omega$, the excitation amplitudes $C_N(t)$ can be calculated from (4.3) and the initial condition $C_N(-\infty)=\delta_{N0}$. To accomplish this, more about the values of the widths Γ_N should be known. Until now we only know that $\Gamma_0=0$ and $\Gamma_1=\Gamma_{\rm GDR}$. The solution for $\Gamma_N=0$ ($N=0,1,2,\ldots$) was given in Sec. III.

As a consequence of having $\Gamma_N\neq 0$, the total probability $P_{\rm tot}=\sum_N |\,C_N(t)\,|^2$ is no longer conserved because flux is now put into the decay channels. Multiplying Eq. (4.3) by $C_N^*(t)$ and its complex conjugate by $C_N(t)$ and subtracting the results, we obtain, for the change of the occupation probability $\widetilde{P}_N(t)=|\,C_N(t)\,|^2$,

$$\frac{d\widetilde{P}_N(t)}{dt}=\frac{2}{\hbar}{\rm Im}\left[\sum_M \langle N\,|\,V(t)\,|\,M\rangle e^{(i/\hbar)(E_N-E_M)t}C_MC_N^*\right]-\frac{\Gamma_N}{\hbar}\widetilde{P}_N(t)\ . \qquad (4.4)$$

The first part of the right-hand side of Eq. (4.4) describes the redistribution of flux in the various channels N during the collision. If only this term were present, we would have conservation of the total probability $P_{\rm tot}(t)=\sum_N \widetilde{P}_N(t)$, since $V(t)$ is Hermitian. This term leads to a change of the occupation probability given by

$$G_N(t)=\frac{2}{\hbar}{\rm Im}\left[\sum_M \langle N\,|\,V(t)\,|\,M\rangle\times e^{(i/\hbar)(E_N-E_M)t}C_M(t)C_N^*(t)\right] . \qquad (4.5a)$$

The non-Hermitian part of the interaction leads to a loss out of channel N, given by

$$L_N(t)=\frac{\Gamma_N}{\hbar}\widetilde{P}_N(t)\ , \qquad (4.5b)$$

i.e., we have the balance equation

$$\frac{d\widetilde{P}_N(t)}{dt}=G_N(t)-L_N(t)\ . \qquad (4.6)$$

This equation can also be written as the integral equation

$$\widetilde{P}_N(t)=\int_{-\infty}^{t}e^{-(\Gamma_N/\hbar)(t-t')}G_N(t')dt'+\delta_{N0}\ , \qquad (4.7)$$

where we used the initial condition $\widetilde{P}_N(-\infty)=\delta_{N0}$. A further insight into Eq. (4.6) can be obtained by summing it over all states:

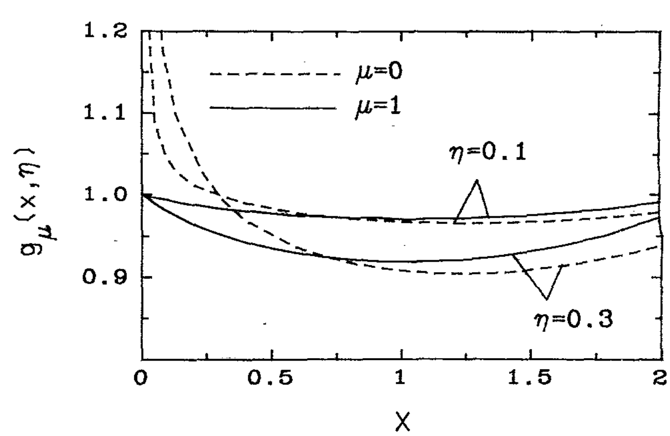

FIG. 6. The adimensional function $g_\mu(x,\eta)$ as defined by Eq. (4.12b). In the limit $\eta=\Gamma/E_{\rm GDR}\to 0$, we have $g_\mu(x,\eta)\to 1$.

TABLE IV. Cross sections for N-phonon Coulomb excitation of ^{16}O in the reaction $^{16}O+{}^{238}U$. The values corresponding to $\tilde{\sigma}_C^{(N)}$ ($\sigma_C^{(N)}$) take (do not take) into account the widths of the states (see text).

$E_{\rm lab}$ (GeV/nucleon)	$\tilde{\sigma}_C^{(1)}$	$\sigma_C^{(1)}$	$\tilde{\sigma}_C^{(2)}$	$\sigma_C^{(2)}$	$\tilde{\sigma}_C^{(3)}$	$\sigma_C^{(3)}$
0.5	36 mb	34 mb	0.13 mb	0.12 mb	0.5 μb	0.43 μb
2	0.14 b	0.13 b	0.81 mb	0.67 mb	4.9 μb	3.9 μb
10	0.43 b	0.41 b	2.4 mb	2.2 mb	18 μb	16 μb
100	1.0 b	1.0 b	3.1 mb	3.1 mb	23 μb	22 μb

$$\sum_N \frac{d\tilde{P}_N(t)}{dt} = \sum_N L_N(t) . \tag{4.8}$$

Defining the flux function

$$F_N(t) = \int_{-\infty}^{t} L_N(t')dt' = \frac{\Gamma_N}{\hbar} \int_{-\infty}^{t} \tilde{P}_N(t')dt' , \tag{4.9}$$

the integration of (4.8) can be written as

$$1 = \sum_N \tilde{P}_N(t) + \sum_N F_N(t) . \tag{4.10}$$

Due to the exponential decay of the states with $N \geq 1$, we have, for $t \to \infty$, the limit $\tilde{P}_N(\infty) = \delta_{N0}\tilde{P}_0(\infty)$ and

$$1 = \tilde{P}_0(\infty) + \sum_N F_N(\infty) . \tag{4.11}$$

This means that for $t \to \infty$ there is a probability to find the system in the ground state given by $\tilde{P}_0(\infty)$ and a probability that it has been excited and decayed through channel N, which is given by $F_N(\infty)$. If the widths Γ_N are known, Eq. (4.3) can be solved, and from Eq. (4.9) the contribution to the fragmentation through channel N can be deduced.

An approximate solution can be found in the case of linearly increasing widths with increasing energy, i.e., $\Gamma_N = N\Gamma$. Following the classical interpretation leading to the Poisson distribution as discussed in Sec. III, the excitation probability of the states $|N\rangle$ is equal to the excitation probability of N uncoupled oscillators, each having a decay width of Γ. Instead of (3.3), the energy transferred to a damped oscillator will be given by [see Ref. 6, Eq. (13.24)]

$$\begin{aligned}\widetilde{\Delta E}(b) &= \frac{2\hbar\Gamma}{\pi m_N c^2}\alpha^2 \frac{Z_P^2 N_T Z_T}{A_T}\left[\frac{c}{v}\right]^4 \frac{1}{\gamma^2}\left[\int_0^\infty \frac{\Omega^4 K_1^2(\Omega b/\gamma v)}{(\Omega^2-\omega^2)^2+\Omega^2\Gamma^2/4\hbar^2}d\Omega + \frac{1}{\gamma^2}\int_0^\infty \frac{\Omega^4 K_0^2(\Omega b/\gamma v)}{(\Omega^2-\omega^2)^2+\Omega^2\Gamma^2/4\hbar^2}d\Omega\right] \\ &= \frac{2E_{\rm GDR}^2}{m_N c^2}\alpha^2 \frac{Z_P^2 N_T Z_T}{A_T}\left[\frac{c}{v}\right]^4 \frac{1}{\gamma^2}\left[g_1(x,\eta)K_1^2(x) + \frac{1}{\gamma^2} g_0(x,\eta) K_0^2(x)\right] ,\end{aligned} \tag{4.12a}$$

where

$$g_\mu(x,\eta) = \frac{\eta^2}{\pi K_\mu^2(x)} \int_0^\infty \frac{y^4 K_\mu^2(\eta x y)dy}{(y^2-1/\eta^2)^2+y^2/4} , \tag{4.12b}$$

with $x = \omega b/\gamma v$ and $\eta = \Gamma/\hbar\omega \equiv \Gamma/E_{\rm GDR}$. In terms of $g_\mu(x,\eta)$ the excitation probability in first order is, as in (3.5), given by

$$\tilde{\phi}(b) = \tilde{a}_0^2 + \tilde{a}_1^2 + \tilde{a}_{-1}^2 , \tag{4.13a}$$

with

$$\tilde{a}_0^2 = \frac{2E_{\rm GDR}}{m_N c^2}\alpha^2 \frac{Z_P^2 N_T Z_T}{A_T}\left[\frac{c}{\gamma v}\right]^4 g_0(x,\eta)K_0^2(x) \tag{4.13b}$$

and

$$\tilde{a}_1^2 = \frac{E_{\rm GDR}}{m_N c^2}\alpha^2 \frac{Z_P^2 N_T Z_T}{A_T}\left[\frac{c}{v}\right]^4 g_1(x,\eta)K_1^2(x) . \tag{4.13c}$$

The functions $g_\mu(x,\eta)$ are plotted in Fig. 6 for $\eta = 0.1$ and 0.3. When $\eta \to 0$, then $g_\mu \to 1$ and we obtain the same results as given by Eqs. (3.5). For $\eta \neq 0$, then we observe that g_0 will have the greater influence, especially for $x \ll 1$. Since, as we saw in Sec. III, $a_0 \ll a_1$ in the limit of high energies of collision, we expect that in this limit the influence of the widths of the states in the cross sections calculated in Sec. III will be very small. Inserting Eqs. (4.13a)–(4.13c) into Eqs. (3.10) and (3.14), we find the results given in Table IV for the reaction $^{16}O+{}^{238}U$ as a function of the laboratory energy. One observes that the inclusion of the widths of the states modifies appreciably the previous calculations only for low energies and for large N. In the limit $\gamma \gg 1$, the interaction is very sudden and the widths of the states have practically no influence on the excitation process. In that limit the theoretical results of Sec. III are sufficiently accurate for application in RHIC's. However, if the widths of the states are too large, then the experimental detection of them will be very difficult.

V. CONCLUSION

Rather simple classical and quantal considerations show the importance of giant dipole excitations in peripheral RHIC's. The present experimental status is compared to theoretical calculations using a sum rule approach for the nuclear states. We find completely satisfactory agreement. In view of recently proposed detailed experimental studies of extreme peripheral collisions, the possibility of multiphonon giant dipole excitations is discussed. Using a harmonic oscillator model, absolute values of total cross sections can be obtained with a simple formula. The cross sections are found to be quite appreciable. Whereas the cross sections for heavy projectile excitation are larger than those for light projectile excitation, such as ^{16}O or ^{32}S, the separation amplitude of neutrons from protons will be larger for the lighter projectiles. This could prove to be a means of producing new and exotic nuclei, perhaps, e.g., polyneutrons. Finally, we try to include—in a phenomenological way—the effects of damping of the giant dipole collective motion in the theory. A qualitative study indicates that, for the extremely short collision times which occur in the RHIC's, the excitation of these states is still possible. However, we do not know the actual properties of these new nuclear states. Our theoretical study has indicated quite safely the possibility of excitation of such states with appreciable values for the cross sections, and we are excited about the future experimental and theoretical developments in this area.

ACKNOWLEDGMENTS

We are grateful to Professor P. Braun-Munzinger for sending us a copy of Ref. 4. This work was partially supported by the Deutscher Akademischer Austauschdienst.

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