Electromagnetic production of heavy leptons in relativistic heavy ion collisions

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Using the equivalent photon method, the cross sections for muon- and tau-pair production in relativistic heavy ion collisions are calculated. A simple analytical formula is obtained, valid for the relativistic Lorentz factor $\gamma \lesssim 16$ for muon-pair and $\gamma \lesssim 270$ for tau-pair production. The absolute values of the pair production cross sections are found to be very small, quite in contrast to the case of electron-positron pair production.

The cross section for electron-positron pair production in relativistic heavy ion (RHI) collisions has been calculated in Ref. 1 using the method of equivalent photons. The cross section turned out to be quite large, of the order of kilobarns for U-U collisions. Based on a simple formula originally of Landau and Lifshitz,[2] e^+e^- pair production is discussed in a review by Anholt and Gould.[3] It is proposed there to use pair production as a real-time nondestructive luminometer in RHI colliders.

As an application of the equivalent photon method, the electromagnetic pair production cross section in the collision of two relativistic heavy ions of charges Z_1 and Z_2

$$Z_1+Z_2 \rightarrow Z_1+Z_2+l^+ +l^- \tag{1}$$

is calculated, where $l=\mu$ and τ. Due to the much heavier mass ($m_\mu=105$ MeV, $m_\tau=1784$ MeV) of these particles, the physics of the process turns out to be quite different as compared to the e^+e^- production. The equivalent photon spectrum extends approximately up to the value of

$$E^{\max} \simeq \gamma \frac{\hbar c}{R} , \tag{2}$$

where the adiabatic cutoff sets in. The Lorentz factor γ is given by

$$\gamma = 1 \Big/ \left[1 - \left[\frac{v}{c} \right]^2 \right]^{1/2} ,$$

where v is the relative velocity of the ions and $R=R_1+R_2$ is the minimal distance of approach where nuclear interaction effects dominate. We can take $R_i=1.2 \text{ fm}\, A_i^{1/3}$ where A_i is the mass number of the heavy ion ($i=1,2$). For the efficient production of lepton pairs we must have

$$E^{\max} > 2m_l c^2 . \tag{3a}$$

This condition leads to

$$\gamma_l > 2 \frac{e^2}{\hbar c} \frac{R}{r_l} , \tag{3b}$$

where

$$r_l = \frac{e^2}{m_l c^2}$$

is the classical lepton radius ($r_\mu=0.014$ fm, $r_\tau=0.8\times 10^{-3}$ fm). With $R=15$ fm we obtain from Eq. (3b) the conditions $\gamma_\mu > 16$ and $\gamma_\tau > 270$. These are very high values. In the approximation used in Ref. 1 there would be no heavy lepton production below these values for γ_μ and γ_τ. On the other hand, Eq. (3b) leads to no restriction for e^+e^- production ($r_e=2.82$ fm). On these grounds we already expect a much stronger suppression of μ and τ production as could be expected from the simple scaling with r_l^2, as was discussed in Ref. 3.

Since the range of γ values given by condition (3b) is rather unrealistic for heavy ion machines at present, we want to calculate the cross sections for heavy lepton production in the opposite limit, i.e.,

$$\gamma_l < 2 \frac{e^2}{\hbar c} \frac{R}{r_l} . \tag{4a}$$

With $R=15$ fm we have

$$\gamma_\mu < 16 \tag{4b}$$

and

$$\gamma_\tau < 270 . \tag{4c}$$

In this range of γ values we can approximate the equivalent photon spectrum by its exponential tail:

$$n(E) = \frac{Z_1^2 \alpha}{2} e^{-2(\omega R/\gamma c)} , \tag{5}$$

where we set $v \simeq c$ [see Eq. (4.7a) of Ref. 4, in the limit of $\omega R/\gamma c \rightarrow \infty$]. The Coulomb production cross section for reaction (1) is then calculated as

$$\sigma = \int_{2m_l c^2}^{\infty} dE \frac{n(E)}{E} \sigma_{\gamma Z_2 \rightarrow Z_2 l^+ l^-} , \tag{6}$$

where $\sigma_{\gamma Z_2 \rightarrow l^+ l^- Z_2}$ is the lepton pair photoproduction cross section on a nucleus with charge Z_2. For the photoproduction cross section we use the nonrelativistic Born approximation result (for a point nucleus) given in Ref. 5:

$$\sigma_{\gamma Z_2 \rightarrow l^+ l^- Z_2} = \frac{\pi}{24} Z_2^2 \alpha r_l^2 \left[\frac{E-2m_l c^2}{m_l c^2} \right]^3 . \tag{7}$$

It is valid for

$$Z_2\alpha \ll \left[\frac{E-2m_lc^2}{E}\right]^{1/2} \ll 1 .$$

This approximation for the total photoproduction cross section seems fairly realistic and has the virtue of simplicity. This in turn leads to a rather simple expression for the total Coulomb production cross section [Eq. (6)]

$$\sigma=\frac{\pi}{3}Z_1^2Z_2^2\alpha^2r_l^2e^{-4\mu}I(\mu) , \tag{8}$$

where the integral I can be expressed in terms of the exponential integral $Ei(x)$ (see Ref. 6, p. 312).

$$I(\mu)=\int_0^\infty \frac{x^3e^{-4\mu x}}{1+x}dx$$
$$=e^{4\mu}Ei(-4\mu)+\sum_{k=1}^{3}(k-1)!\frac{(-1)^{3-k}}{(4\mu)^k} \tag{9a}$$

with

$$\mu=\frac{Rm_lc}{\gamma\hbar}=\frac{m_lc^2}{E^{\max}} . \tag{9b}$$

In the limit of $\mu \gg 1$ the integral I [Eq. (9a)] reduces to

$$I\simeq\frac{3!}{(4\mu)^4} . \tag{9c}$$

In this approximation, in which the threshold γ energy is much larger than the value $E^{\max}$ given by Eq. (2), the Coulomb production cross section, Eq. (8), attains the simple form

$$\sigma=\frac{\pi}{128}Z_1^2Z_2^2\alpha^2r_l^2e^{-4m_lcR/\gamma\hbar}\left[\frac{\gamma\hbar}{Rm_lc}\right]^4 . \tag{10}$$

Numerical values are plotted in Fig. 1 for U-U collisions as a function of the Lorentz factor γ. The cross sections are drastically smaller as compared to the e^+e^- production cross sections. They appear to be negligible in practice. This is due to the rather severe limitation imposed by the adiabatic cutoff [see Eq. (2)] which strongly inhibits the excitation of high-lying states, i.e., the production of heavy lepton pairs. For $\mu^+\mu^-$ pair production for $\gamma \gg 16$ [see Eq. (3b)] (heavy ion machines which attain such γ values are proposed; see, e.g., Ref. 3), the formula (6.3) of Ref. 1 can be used (however, a factor of 2 must be taken out), in which the classical electron radius r_0 [see Eqs. (6.2a) and (6.2b) of Ref. 1] is replaced by the classical muon radius r_μ.

As a final remark, let us compare the two-photon production mechanism as discussed here for heavy ions with

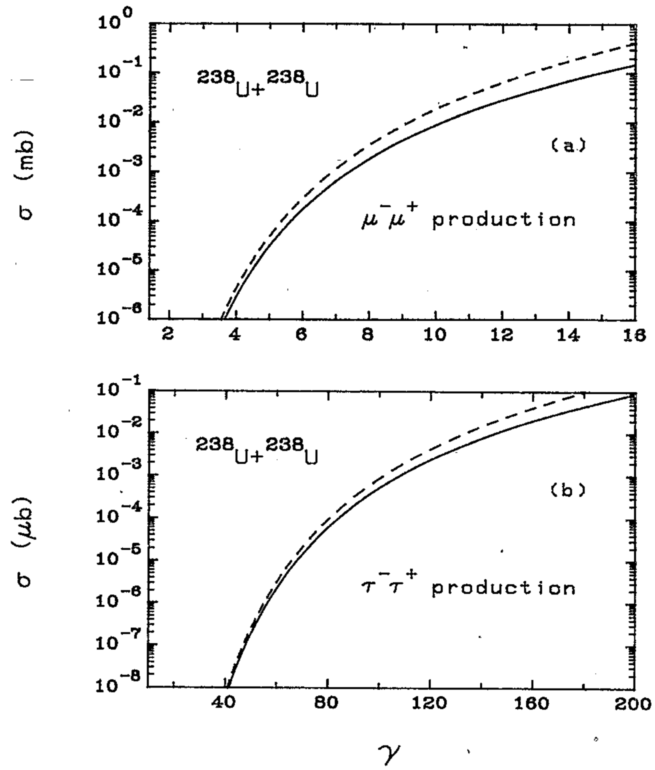

FIG. 1. The Coulomb production cross section of $\mu^+\mu^-$ and $\tau^+\tau^-$ pairs as a function of γ for U-U collisions. The minimal impact parameter is chosen to be $R=15$ fm. The solid lines correspond to the use of Eq. (9a), whereas for the dashed lines we used the approximation given by Eq. (10).

the two-photon production process at e^-e^+ colliders. For a detailed review see Ref. 7. In such machines, the γ values achieved are much higher than in the heavy ion case, therefore the adiabatic cutoff [see Eq. (2)] is of less importance in these cases; also the minimal impact parameter R is much less as compared to the heavy ion case. We have astonishingly large e^+e^- production cross sections in RHI collisions, due to the large charge factor $Z_1^2\cdot Z_2^2$; however, high-lying states ($\mu^+\mu^-$,$\tau^+\tau^-$ pairs) are practically not reached. Also, the Coulomb production of heavy quark-antiquark states [like the η_c, which was recently studied with the PLUTO detector at the Positron Electron Tandem Ring Accelerator (PETRA) in high energy e^+e^- collisions[8]] will be negligible.

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