Elastic Coulomb scattering of heavy ions at intermediate energies

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Relativistic effects in two-body Coulomb collisions are investigated within a classical context. Special emphasis is given to heavy-ion collisions at intermediate energies. It is found that such corrections may be responsible for a 10–20 % increase of the elastic cross sections for a fixed scattering angle and kinetic energy. Comparisons with other methods are also made and some useful simple parametrizations of the cross sections are obtained. The separate contributions from kinematic and retardation corrections are also studied.

I. INTRODUCTION

The relativistic collision of two classical charged particles interacting electromagnetically is an unsolved problem. Only if one of the particles has infinite mass and emission of radiation is neglected does the problem have an analytical solution.[1] In any case, such a problem has almost no application in particle or nuclear physics, where the motion of the fundamental particles is suitably described by means of appropriate quantum equations. A customary exception is the collision of heavy ions which, due to their large mass, can be solved with a good degree of accuracy by means of classical, or semiclassical, methods. Elastic and inelastic scattering of heavy ions at low energies is a well-known application of such methods.[2]

An accurate knowledge of the elastic Coulomb cross section in intermediate energy collisions is of great relevance, e.g., as a calibration of nuclear reaction experiments at these energies, or to extract the excitation amplitudes induced by the Coulomb interaction. In fact, Coulomb excitation in intermediate energy collisions of heavy ions is an emergent field in experimental nuclear physics. With increasing projectile energies the hardness of the virtual photons exchanged in the collision is also increased. That is, states of high energy, like giant resonances, can be more easily excited in collisions at intermediate energies, and higher.[3] The excitation of giant resonances is indeed a promising tool to investigate nuclear structure properties in heavy collisions at intermediate energies. The cross sections for this process lie around 1–10 b/sr at the resonant energy. Moreover, Coulomb excitation is as strong for isovector states as it is for an entire class of giant resonances not yet available for systematic observation.[3]

The semiclassical theory of Coulomb excitation assumes that the inelastic cross section can be factorized as

$$\left[\frac{d\sigma}{d\Omega}\right]_{fi}=\left[\frac{d\sigma}{d\Omega}\right]_{\mathrm{el}}P_{fi}\ , \tag{1.1}$$

where $(d\sigma/d\Omega)_{\mathrm{el}}$ is the angular elastic cross section and P_{fi} is the probability amplitude for exciting a nuclear state f from an initial prescribed state i. These probability amplitudes are calculated taking the effect on a nucleus of the electromagnetic field generated by a projectile moving along a classical trajectory. Therefore, the direct proportionality between the inelastic and elastic Coulomb cross sections makes it necessary to have a good knowledge of the elastic cross section to extract from experiment useful information about the excitation probabilities. Recently, it has been shown in Ref. 4 that recoil and retardation corrections in Coulomb excitation at intermediate energies are responsible for 10–20 % of the variations in the inelastic cross sections.

The present situation demonstrates clearly that a closer analysis of relativistic corrections in Coulomb scattering at intermediate energies is needed. A work in this direction was done by Matzdorf *et al.*[5] They performed a numerical calculation based on a complete description of the two-body system interacting electromagnetically, except for noninclusion of radiative corrections. The effect of the retarded electromagnetic field of one particle on the other was calculated and the feedback of its corresponding motion on the electromagnetic field was considered. This led to a set of coupled equations which they solved numerically.

In this paper we make a study of the influence of relativistic corrections in Coulomb scattering at intermediate energies by means of an expansion of the Lagrangian in powers of $1/c$, truncated before the first term which includes radiation corrections. In two-body collisions, radiation terms appear only in the third-order approximation in $1/c$. In special cases, i.e., collisions between particles with the same charge-to-mass ratio, the appearance of radiation terms is in the fifth-order approximation in $1/c$. In this case, a Lagrangian up to the order c^{-4} can be constructed. Then it is possible to describe the problem only in terms of the relative coordinates of the two-body system. This permits us to make a simpler analysis of the relativistic corrections in intermediate energy collisions than that of Ref. 5. Although this is a quite simple problem, it has not yet been studied systematically with respect to the application in heavy-ion scattering at intermediate energies, except in Ref. 5. High quality data on the scattering of a light projectile by a heavy target are becoming available,[6] but it has not yet been shown to which extent the several relativistic corrections will contribute to the scattering.

In Sec. II we present the classical approach to the solution of the electromagnetically interacting two-body problem including relativistic corrections up to order c^{-2}. For symmetrical systems corrections up to order c^{-4} are also included. In Sec. III we compare the scattering angles and Coulomb elastic cross sections obtained under the several approximations presented in Sec. II. Section IV deals with the scattering of a light particle by a heavy particle. It is shown that if magnetic and retardation corrections are neglected, keeping only the kinematic corrections, analytical expressions can be obtained. A comparison with the approximations of Sec. II is also performed. Our conclusions are given in Sec. V.

II. RELATIVISTIC CORRECTIONS IN THE TWO-BODY ELECTROMAGNETIC SCATTERING

A system of two point charges interacting electromagnetically and moving at low velocities can be described by an approximate Lagrangian which depends only on the degrees of freedom of the particles, neglecting those related to the electromagnetic field (the Darwin Lagrangian).[1] In this approximation, it is possible to separate the degrees of freedom associated with the relative position $\mathbf{r}$ and relative velocity $\mathbf{v}$ of the particles from the center of mass degrees of freedom. For a system of particles with different masses, this approximation is only possible until the c^{-2} order, whereas for a system with equal charge-to-mass ratio ($Z_1/m_1=Z_2/m_2$) the approximation goes up to order c^{-4}.

In the center of mass frame of reference the approximate Lagrangian for a system of two point charges Z_1e and Z_2e with masses m_1 and m_2, respectively, can be written as

$$\mathcal{L}=\mathcal{L}^{(0)}+\mathcal{L}^{(2)} , \tag{2.1}$$

where the zeroth-order term is given by

$$\mathcal{L}^{(0)}=\tfrac{1}{2}\mu v^2-\frac{Z_1Z_2e^2}{r} , \tag{2.2}$$

and the c^{-2} term is

$$\mathcal{L}^{(2)}=\frac{\mu^4}{8c^2}\left[\frac{1}{m_1^3}+\frac{1}{m_2^3}\right]v^4-\frac{\mu^2Z_1Z_2e^2}{2m_1m_2c^2r}(v^2+v_r^2) \tag{2.3}$$

with $v_r=\mathbf{v}\cdot\mathbf{r}/r$. In these expressions $\mathbf{r}$ ($\mathbf{v}$) is the relative position (velocity) and

$$\mu=m_1m_2/(m_1+m_2)$$

is the reduced mass of system. The first terms of (2.2) and (2.3) are simply the usual relativistic Lagrangian of a free particle expanded to order c^{-2}; the second term in (2.3) represents the combined effect of magnetic and retarded Coulomb interactions.

With the help of the standard canonical procedure one can define a Hamiltonian and obtain the equations of relative motion, which, up to order c^{-2}, are

$$\frac{d\mathbf{r}}{dt}=\frac{\mathbf{p}}{\mu}-\frac{p^2}{2c^2}\left[\frac{1}{m_1^3}+\frac{1}{m_2^3}\right]\mathbf{p}+\frac{Z_1Z_2e^2}{m_1m_2c^2r}\left[\mathbf{p}+\frac{p_r}{r}\mathbf{r}\right] , \tag{2.4a}$$

$$\frac{d\mathbf{p}}{dt}=\frac{Z_1Z_2e^2}{r^3}\mathbf{r}+\frac{Z_1Z_2e^2}{2m_1m_2c^2r^2}\left[(p^2+3p_r^2)\frac{\mathbf{r}}{r}-2p_r\mathbf{p}\right] , \tag{2.4b}$$

where $p_r=\mathbf{p}\cdot\mathbf{r}/r$, and the canonical momentum $\mathbf{p}$ is given by

$$\mathbf{p}=\mu\mathbf{v}+\frac{\mu^4}{2}\left[\frac{1}{m_1^3}+\frac{1}{m_2^3}\right]\frac{v^2}{c^2}\mathbf{v}-\frac{\mu^2Z_1Z_2e^2}{m_1m_2c^2r}\left[\mathbf{v}+v_r\frac{\mathbf{r}}{r}\right] . \tag{2.5}$$

The term of next order in $1/c$, which is not included in the Lagrangian (2.1), is a contribution from the dipole emission of radiation. The treatment of such a term is quite involved and the damping caused on the particle trajectories is generally handled within the Abraham-Lorentz formalism.[7] One obtains a third-order differential equation, which needs specification of position, velocity, and acceleration at the initial time to be solved. The solutions diverge mostly (runaway solutions). In the case that the particles have equal charge-to-mass ratio, the dipole moment of the system is conserved. Since the damping force is proportional to the time derivative of the dipole moment, it vanishes and the Lagrangian can be extended to higher order in $1/c$ without the effective influence of the emission of radiation. To simplify we shall consider systems with $Z_1=Z_2=Z$ and $m_1=m_2=m$, for which the correction $\mathcal{L}^{(4)}$ to be added to the Lagrangian (2.1) is[1]

$$\mathcal{L}^{(4)}=\frac{1}{512c^4}mv^6+\frac{Z^2e^2}{16c^4r}\left[\frac{1}{8}(v^4-3v_r^4+2v_r^2v^2)+\frac{Z^2e^2}{mr}(3v_r^2-v^2)+4\frac{Z^4e^4}{m^2r^2}\right] . \tag{2.6}$$

In this case the canonical equations of motion become

$$\frac{d\mathbf{r}}{dt}=\left[2-\frac{p^2}{m^2c^2}-\frac{9}{4}\frac{p^4}{m^4c^4}\right]\frac{\mathbf{p}}{m}+\frac{Z^2e^2}{m^2c^2r}\left[\left[1-\frac{1}{2}\frac{p^2}{m^2c^2}\right]\mathbf{p}+\left[1+\frac{3}{2}\frac{p_r^2}{m^2c^2}\right]\frac{p_r}{r}\mathbf{r}\right]+\frac{Z^4e^4}{m^3c^4r^2}\mathbf{p} \tag{2.7a}$$

and

$$\frac{d\mathbf{p}}{dt}=\frac{Z^2e^2}{r^3}\mathbf{r}+\frac{Z^2e^2}{2m^2c^2r^2}\left[p^2+3p_r^2-\frac{1}{4}\frac{p^4}{m^2c^2}+\frac{15}{4}\frac{p_r^4}{m^2c^2}\right]\frac{\mathbf{r}}{r}-\frac{Z^2e^2p_r}{m^2c^2r^2}\left[1+\frac{3}{2}\frac{p_r^2}{m^2c^2}\right]\mathbf{p}+\frac{Z^4e^4p^2}{m^3c^4r^4}\mathbf{r}-\frac{3}{4}\frac{Z^6e^6}{m^2c^4r^5}\mathbf{r} \tag{2.7b}$$

with the canonical momentum given by

$$\mathbf{p}=\frac{1}{2}\left[1+\frac{1}{8}\frac{v^2}{c^2}+\frac{3}{128}\frac{v^4}{c^4}\right]m\mathbf{v}-\frac{Z^2e^2}{4c^2r}\left[\left[1-\frac{1}{8}\frac{v^2}{c^2}-\frac{1}{8}\frac{v_r^2}{c^2}+\frac{Z^2e^2}{2mc^2r}\right]\mathbf{v}+\left[1-\frac{1}{8}\frac{v^2}{c^2}+\frac{3}{8}\frac{v_r^2}{c^2}-\frac{3Z^2e^2}{2mc^2r}\right]\frac{v_r}{r}\mathbf{r}\right]. \quad (2.8)$$

The equations of motion (2.7) with corrections up to c^{-4} order are very useful in order to compare with the results of Eqs. (2.4) and to test the convergence of the expansion of the Lagrangian, at least for particles with equal charges and masses.

III. NUMERICAL CALCULATIONS AND RESULTS

The solution of Eqs. (2.4) and (2.5) was carried out numerically by means of predictor-corrector methods.[8] We applied it to several heavy-ion systems with initial conditions fixed in terms of the impact parameter b and asymptotic kinetic energy $E_{\rm lab}$ (from which one derives the asymptotic momentum by unfolding the corresponding Hamiltonian). The center of mass scattering angle was calculated through

$$\Theta=\arccos\left[\frac{\mathbf{v}_i\cdot\mathbf{v}_f}{v_i v_f}\right], \quad (3.1)$$

where $\mathbf{v}_i$ and $\mathbf{v}_f$ are the initial and final relative velocities, calculated at a large cutoff radius.

In Fig. 1 we plot the difference (in percent) between Θ and the scattering angle $\Theta_{\rm NR}$ calculated with nonrelativistic dynamics, for the system ^{208}Pb+^{208}Pb colliding with laboratory energy $E_{\rm lab}=100$ MeV per nucleon. The nonrelativistic scattering angle is given by

$$\Theta_{\rm NR}=2\arctan(a_0/b)$$

where $a_0=Z_1Z_2e^2/\mu v^2$ is half the distance of closest approach in a head-on collision. The solid line corresponds to the solution up to c^{-2} and the dashed line corresponds to the solution up to c^{-4}. The deviation from the nonrelativistic solution is smaller for small impact parameters, increasing rapidly up to an approximate constant behavior above 10 fm. But the physical impact parameters which correspond to a pure Coulomb collision, without nuclear contact, are larger than $b_{\rm min}\approx R_1+R_2=15$ fm. Therefore, the relativistic corrections to the angles of relevance in heavy-ion scattering at intermediate energies are about 6–7%. The convergence of higher-order corrections is fast. As one sees, the difference between the solutions of the Lagrangian of order up to c^{-4} and the one of order up to c^{-2} is much smaller than their absolute difference with $\Theta_{\rm NR}$.

In Fig. 2 is shown the same relative difference,

$$\delta\Theta=(\Theta-\Theta_{\rm NR})/\Theta_{\rm NR},$$

as in Fig. 1, but for a collision at grazing impact parameter $b_{\rm min}$, and as a function of the laboratory energy per nucleon. The relative difference increases steadily with energy and at large energies the c^{-4} corrections become

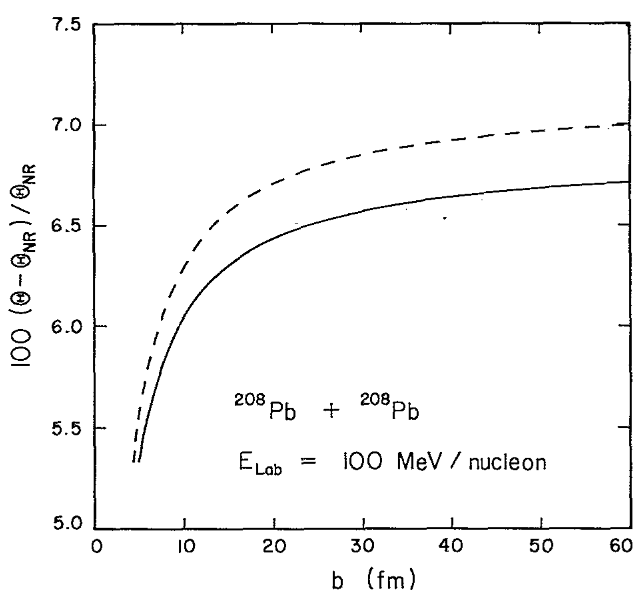

FIG. 1. Relative difference $\delta\Theta$ (in percent) between the center of mass scattering angle of ^{208}Pb+^{208}Pb at a laboratory energy of 100 MeV per nucleon, calculated with nonrelativistic kinematics, and the scattering angle calculated with relativistic corrections up to order c^{-2} (solid line) and up to order c^{-4} (dashed line). The horizontal axis represents the impact parameter b (in fm).

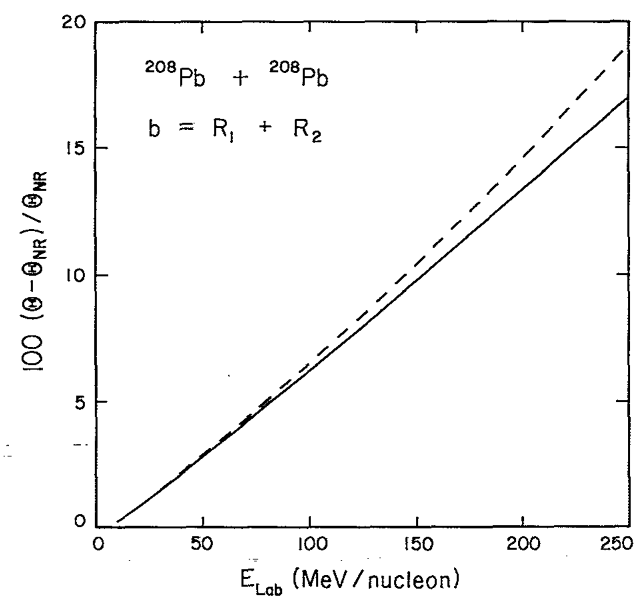

FIG. 2. Same as in Fig. 1, but for a collision at grazing impact parameter $b=R_1+R_2$, and as a function of the laboratory energy $E_{\rm lab}$ (in MeV per nucleon).

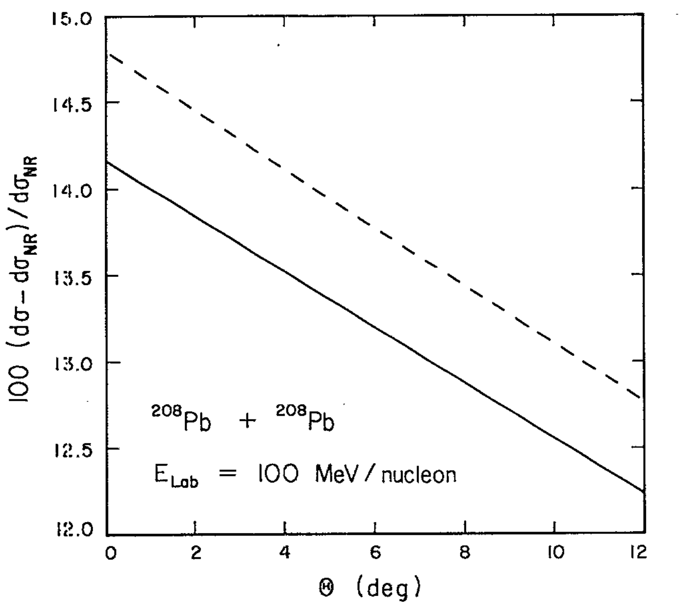

FIG. 3. Relative difference $\delta\sigma$ (in percent) between the Rutherford elastic scattering differential cross section of $^{208}Pb+^{208}Pb$ at a laboratory energy of 100 MeV per nucleon and the differential elastic cross section calculated with corrections up to order c^{-2} (solid line) and up to order c^{-4} (dashed line). The horizontal axis represents the center of mass scattering angle Θ (in deg).

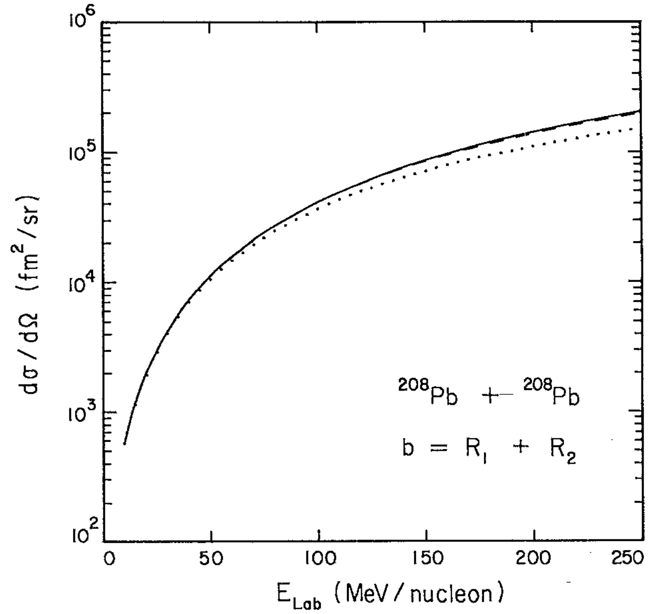

FIG. 5. Absolute value of the Coulomb differential elastic cross section $d\sigma/d\Omega$ (in fm^2/Sr) for a $^{208}Pb+^{208}Pb$ collision at grazing impact parameter as a function of the laboratory energy E_{lab} (in MeV per nucleon). The dotted curve is the Rutherford cross section, the dashed and solid curves were obtained with relativistic corrections up to c^{-2} and c^{-4} order, respectively.

important, as expected. One also observes that the corrected scattering angle is always larger than what one obtains by using nonrelativistic kinematics.

From the solutions of the scattering angle as a function of b one readily obtains the elastic scattering differential cross section by means of

$$\frac{d\sigma}{d\Omega}=\frac{b}{\sin\Theta}\left|\frac{db}{d\Theta}\right| . \tag{3.2}$$

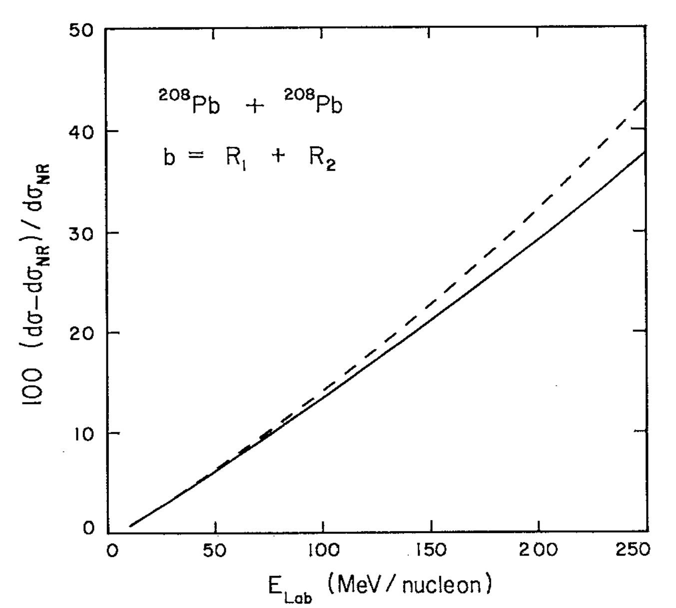

FIG. 4. Same as in Fig. 3, but for a collision at grazing impact parameter $b=R_1+R_2$ and as a function of the laboratory energy E_{lab} (in MeV per nucleon).

The relative difference,

$$\delta\sigma=(d\sigma-d\sigma_{NR})/d\sigma_{NR} ,$$

with $d\sigma_{NR}$ equal to the Rutherford differential cross section, is plotted in Fig. 3 as a function of Θ in the collision $^{208}Pb+^{208}Pb$ at 100 MeV per nucleon. In this collision, the maximum Coulomb scattering angle is about 4 deg. One observes that the difference diminishes slowly and almost linearly with the scattering angle. On average one has a 13–14% difference between the corrected elastic cross section and the Rutherford cross section. Also, corrections of order c^{-4} are much smaller than the ones of order c^{-2}.

Figure 4 shows the same function as in Fig. 3, but for a collision at grazing impact parameter and as a function of the laboratory beam energy per nucleon. The solid line corresponds to corrections of c^{-2} order in the Lagrangian, and the dashed line corresponds to corrections up to c^{-4} order. At 200 MeV per nucleon the corrections can reach 30% above the nonrelativistic value. The corrections tend always to increase the cross section, as can be seen by looking at Fig. 5, where the differential elastic cross section at the grazing angle is plotted versus the laboratory energy per nucleon for the same system. This is due to the fact that on increasing the impact parameter, the corrected scattered angle decreases more slowly than what is expected from nonrelativistic kinematics.

IV. SCATTERING OF A LIGHT PARTICLE BY A HEAVY PARTICLE

In order to have a better understanding of the separate influences of the magnetic and retardation effects and the relativistic increase of the mass of the particles we can compare our previous results with those obtained when one of the particles is much heavier than the other. This problem has an analytical solution when one of the particles has infinite mass and radiation is neglected (see Ref. 1 , p. 127). In such a case, there is no retardation effect, since the field of the infinitely heavy particle has a static instantaneous value at all points in space. The only relativistic effect considered is the dynamical increase of the mass of the light particle. The scattering angle will be given by

$$\Theta=\pi-2\frac{\eta}{\sqrt{\eta^2-\beta^2}}\arctan\sqrt{\eta^2-\beta^2}\ , \tag{4.1a}$$

where

$$\eta=\frac{vL}{Z_1Z_2e^2}\ , \tag{4.1b}$$

with L equal to the angular momentum of the system, and $\beta=v/c$.

Expanding (4.1a) up to order β^2, one reaches the result

$$\eta(\Theta)=\eta_{\rm NR}(\Theta)[1-f(\Theta)\beta^2+\mathcal{O}(\beta^4)]\ , \tag{4.2a}$$

where

$$f(\Theta)=\frac{1}{2}\left[\left[\frac{\pi-\Theta}{2}\right]\sec^2\frac{\Theta}{2}-\tan\frac{\Theta}{2}\right]\tan\frac{\Theta}{2} \tag{4.2b}$$

and

$$\eta_{\rm NR}(\Theta)=\cot\frac{\Theta}{2}$$

is the nonrelativistic limit of (4.1a).

Since $L=pb$, where p is the particle's momentum, one obtains

$$\frac{d\sigma}{d\Omega}=\left[\frac{Z_1Z_2e^2}{pv}\right]^2\frac{\eta}{\sin\Theta}\left|\frac{d\eta}{d\Theta}\right|\ . \tag{4.3}$$

Inserting (4.2a) into this equation and keeping the lowest order correction, we get

$$\frac{d\sigma(v,\Theta)}{d\Omega}=\left[\frac{Z_1Z_2e^2}{2mv^2\sin^2(\Theta/2)}\right]^2[1-h(\Theta)\beta^2+\mathcal{O}(\beta^4)]\ , \tag{4.4a}$$

where

$$h(\Theta)=1+\tfrac{1}{2}[1+(\pi-\Theta)\cot\Theta]\tan^2\frac{\Theta}{2}\ , \tag{4.4b}$$

and m is given by the reduced mass to account for recoil correction. Formula (4.4) is given in terms of the initial projectile velocity v. If, instead, we consider the corrections as a function of kinetic energy K, the corresponding expression is

$$\frac{d\sigma(K,\Theta)}{d\Omega}=\left[\frac{Z_1Z_2e^2}{4K\sin^2(\Theta/2)}\right]^2\times\left\{1+g(\Theta)\frac{K}{mc^2}+\mathcal{O}\left[\left[\frac{K}{mc^2}\right]^2\right]\right\}, \tag{4.5a}$$

where

$$g(\Theta)=3-2h(\Theta)\ . \tag{4.5b}$$

In both cases the factorized term is, of course, the nonrelativistic Rutherford cross section.

In Fig. 6 we plot the relative differences between the Rutherford cross section and the elastic cross sections calculated by solving the equations of motion (2.4), based on corrections up to c^{-2} order (solid line), and also by means of Eqs. (4.5) (dotted line). When we turn off the magnetic and retardation corrections in the Lagrangian (2.3) we obtain the results shown by the dashed line. We see that the agreement with (4.5) is very good, meaning that (4.5) is a reliable way of obtaining a quick estimate for the elastic Coulomb cross section when magnetic and retardation effects are neglected. We also observe that the relativistic inertia effects contribute to the largest part of the corrections to the elastic cross section. For the relevant scattering angles it amounts to about a 10% increase in the elastic cross section. Magnetic and retardation effects increase the corrected cross sections by an additional 3–4 % difference.

The functions $h(\Theta)$ and $g(\Theta)$ in Eqs. (4.4) and (4.5) are always positive for the relevant angles of scattering. Therefore, we conclude that the relativistic corrections in the elastic Coulomb cross section are always positive if they are parametrized as a function of the kinetic energy [see Eq. (4.5a)]. On the other hand, if one fixes the initial velocity, instead of the kinetic energy, the corrections are

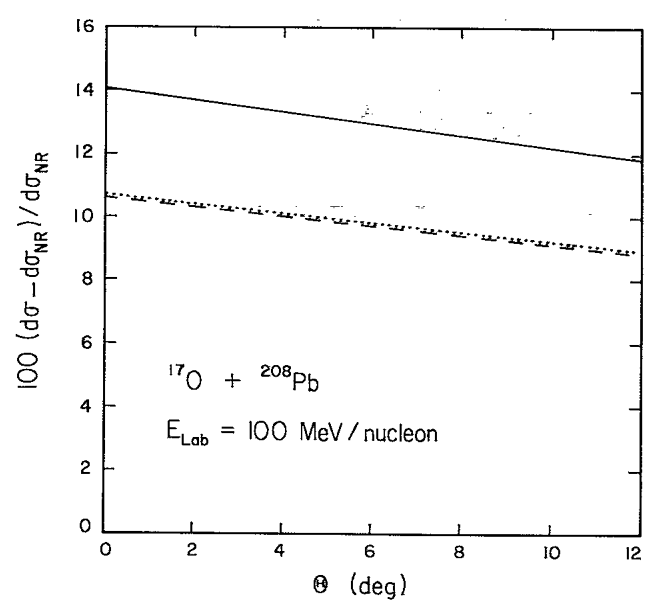

FIG. 6. Relative difference $\delta\sigma$ (in percent) between the Rutherford cross section and the elastic differential cross section calculated with relativistic corrections up to order c^{-2} (solid line) for the system $^{17}O+^{208}Pb$ with laboratory energy of 100 MeV per nucleon as a function of the center of mass scattering angle Θ (in deg). When the magnetic and retardation corrections are turned off, keeping only the corrections on the particle masses, the difference drops to values displayed by the dashed line. The dotted line represents the analytical approximation (4.5).

negative [see Eq. (4.4a)]. This result is in agreement with the predictions of Ref. 5.

As we pointed out at the beginning of this paper, good knowledge of the elastic cross section is necessary in order to obtain useful information on the inelastic one. Recently, inelastic scattering of ^{17}O on ^{208}Pb at the bombarding energy of 84 MeV per nucleon was measured.[3] The contribution of nuclear interaction to the process was separated and shown to be small in the measured angular region. In Fig. 7 we show their data (for excitation of giant dipole resonances in ^{17}O) divided by elastic Coulomb cross section calculated by means of the Rutherford formula (open circles) and by means of the solutions of the equations of motion corrected to order c^{-2} (filled circles) in the way explained above [i.e., via Eqs. (2.4), (3.1), and (3.2)]. The curves are just a guide to the eye. As implied by Eq. (1.1), this is just the semiclassical excitation probability for a giant dipole resonance state in ^{17}O at this colliding energy. They are very small and justify the use of first-order perturbation theory. The relativistic corrections in the elastic cross sections diminish the values of the excitation probabilities extracted from the experiments. It is worth noting that the excitation probability has a maximum for a given scattering angle, below grazing angle. In contrast to what was done in this paper, Ref. 4 deals with the relativistic corrections in the semiclassical calculations of the Coulomb excitation probabilities induced by an ion following a relativistically corrected trajectory. This is a different problem and involves the computation of the coupling of the field generated by one ion with the internal degrees of freedom of the other nucleus (excitation amplitudes). An explanation of the maximum of Fig. 7 could only be accomplished after proceeding with such a calculation and studying the interplay of the different parameters involved in this collision. This is beyond the scope of this paper but represents a straightforward theoretical prescription for future works. Also, the presently available experimental data are accurate within 10%, which is just the order of the relativistic corrections we have studied above. Therefore, more experimental information is needed to have a clearer understanding of the source of possible discrepancies with theoretical calculations.

The excitation probabilities are sensitive to small variations of the classical trajectories. The probabilities are larger if the *adiabaticity parameter* $\xi = \omega a_0 / v$ is smaller than 1. When ξ is greater than unity the excitation probability diminishes with ξ proportionally to $\exp(-\pi\xi)$. In the case of excitation of giant resonances and in grazing collisions at intermediate energies, $\xi \sim 1$. Therefore, small variations in the distance of closest approach, $2a_0$, will appreciably vary the excitation probabilities. Following this reasoning, a variation of 5% in the closest approach disance, as an outcome of relativistic corrections in the elastic scattering, will result in about a 15% variation in the excitation probabilities.[4]

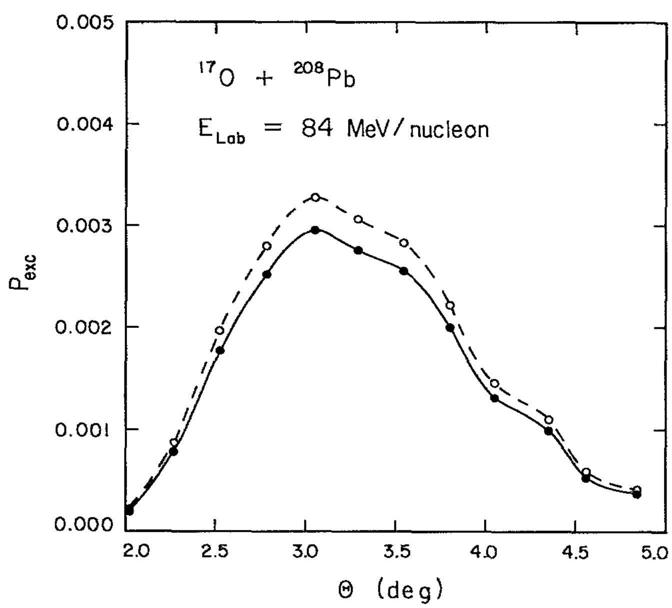

FIG. 7. Excitation probabilities P_{exc} of giant resonance states in ^{17}O as a function of the center of mass scattering angle Θ in the collision ^{17}O+^{208}Pb at a laboratory energy of 84 MeV per nucleon. The open circles were obtained by dividing the experimentally obtained inelastic differential cross sections (see Ref. 3) by the Rutherford elastic differential cross sections. The filled circles were obtained dividing the same data by the Coulomb elastic differential cross sections with relativistic corrections up to order c^{-2}. The curves are a guide to the eye.

V. CONCLUSIONS

We have studied relativistic corrections to the scattering of two heavy ions interacting electromagnetically. It was shown that in intermediate energy collisions, around some hundreds of MeV's per nucleon, the c^{-2} corrections are large and account for most of the relativistic effects. For symmetric systems it is possible to extend the corrections up to c^{-4} order and the convergences is seen to be very fast.

For elastic scattering of a light particle by a heavy particle the effects of retardation become small. In this case a simple and reliable approximate formula was obtained for the elastic cross section, which takes into account only kinematic corrections.

The above study is useful for an analysis of the experimental data on inelastic scattering of heavy ions in intermediate energy collisions. The extension of these calculations to all orders of $1/c$ is an intriguing problem, and an appropriate tool could be the use of the action-at-a-distance electrodynamics of Fokker, Wheeler, and Feynman.[9] Certainly, more studies in this direction are needed, and heavy-ion physics at intermediate energies seem to be a promising tool to investigate such effects.

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