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Multipole response of ^{11}Li

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We investigate the electric multipole response of ^{11}Li which could be tested in reactions with secondary beams at intermediate and high energies. We use simple arguments to show that, even in the most favorable cases, electric dipole excitations are by far more dominant. The contributions from higher order multipolarities will be less than the presently attainable experimental uncertainties.

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Experiments with secondary beams of ^{11}Li have evidenced large electromagnetic dissociation cross sections of the projectiles [1–3]. This is basically due to the weak binding of $^{9}\mathrm{Li} + (n + n)$ ($S_{2n} = 0.33 \pm 0.09$ MeV). The cross sections for the inelastic process $^{11}\mathrm{Li} + A \to X$ are of order of several barns at intermediate energy collisions [2, 3].

The electromagnetic excitation cross sections for nucleus-nucleus collisions can be written as [4]

$$\sigma = \sum_{\pi\lambda} \int \frac{1}{E_x}\, n_{\pi\lambda}(E_x)\, \sigma_\gamma^{\pi\lambda}(E_x)\, dE_x\ , \qquad (1)$$

where E_x is the excitation energy, $\pi\lambda$ (= $E1$, $M1$, ...) denotes the electromagnetic multipolarity, and $\sigma_\gamma^{\pi\lambda}$ is the photoabsorption cross section for the corresponding multipolarity and excitation energy. The functions $n_{\pi\lambda}$ (called the virtual photon numbers) have been extensively studied in Ref. [4]. They depend on the system, beam energy, and excitation energies involved. One expects that electric dipole excitations dominate the electromagnetic cross sections in high energy collisions, since it is quite natural that $E1$ photoabsorption cross sections are larger than $E2$ ones, unless selection rules inhibit the $E1$ transitions. In fact, calculations of the electric dipole response of ^{11}Li within several models have been performed and used as input to calculate the electromagnetic dissociation cross sections with reasonably good agreement with the experimental data [5]. It is the aim of this paper to determine the relative contribution of higher multipole resonances in the fragmentation of ^{11}Li projectiles. This is relevant for reactions with ^{11}Li since it has been advocated that soft multipole resonances, exhausting a large fraction of the sum rules, are present in this and other halo nuclei [6, 7]. In the following we use the formalism of Ref. [8] to calculate the equivalent photon numbers.

The electromagnetic response of ^{11}Li is calculated with the continuum random phase approximation (RPA) formalism [9, 10]. A Woods-Saxon mean field is used and the neutron central potential has been adjusted to bind the neutron $p_{1/2}$ level by 300 keV. This yields an rms radius of 5.4 fm for the valence neutrons and reproduces the single-particle density of the valence neutrons in ^{11}Li obtained in Ref. [11] by solving the Bethe-Goldstone equation explicitly including the correlation between the valence neutrons. While these correlations are important (^{11}Li is not bound without them), it has been seen [12, 5] that the EM response and resulting electromagnetic dissociation (EMD) cross sections are only slightly larger for the correlated model than the RPA model. In order to make a comparison, we have also calculated the multipole response of ^{9}Li. In this case, the neutron central potential is adjusted to reproduce the neutron separation energy of 4.06 MeV for the $p_{3/2}$ level.

We have examined the response to the isoscalar and isovector single-particle operators

$$F_{IS} = \sum_{i=p,n} r_i^\lambda Y_{\lambda,0},$$
$$F_{IV} = \sum_{i=p} r_i^\lambda Y_{\lambda,0} - \sum_{i=n} r_i^\lambda Y_{\lambda,0} \qquad (2)$$

for multipoles λ=1,2,3, as well as for the monopole, for which $F \sim r^2$ must be used. In Fig. 1, we show the RPA response for the isoscalar (IS) monopole, quadrupole, and octupole, as well as the "isovector" (IV) dipole for both ^{11}Li and ^{9}Li. At higher energies, the responses show very similar strength and structure, indicating that excitations of the ^{9}Li core within ^{11}Li look quite similar to excitations of the bare ^{9}Li. We also find, as previously observed [6, 7], there is a very large peak at threshold for the isoscalar and isovector response for all multipo-

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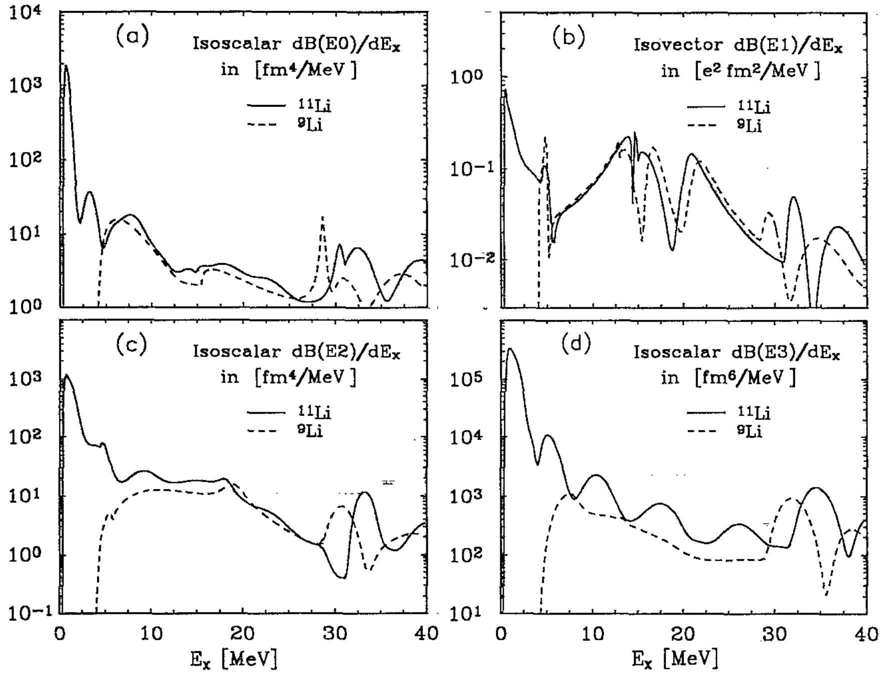

FIG. 1. RPA response of ^{11}Li (solid) and ^{9}Li (dashed) for the (a) isoscalar monopole, (b) "isovector" dipole, (c) isoscalar quadrupole, and the (d) isoscalar octupole.

larities in the ^{11}Li responses. The low energy response (<4–5 MeV) is due entirely to the promotion of a valence neutron from the $p_{1/2}$ level into the continuum. This excitation process decouples from the higher energy excitations. Hence, with no configuration mixing the IS and IV responses coincide with the free response (with no residual interaction) in this energy region.

The energy-weighted sum rule (EWSR) for isoscalar transitions ($\lambda > 1$) is [13]

$$\begin{aligned} S_1 &= \frac{1}{2}\langle 0|[F,[H,F]]|0\rangle = \frac{\hbar^2}{2m}\frac{(2\lambda+1)\lambda}{4\pi}A\langle r^{2\lambda-2}\rangle \\ &= \frac{\hbar^2}{2m}\frac{(2\lambda+1)\lambda}{4\pi}[N_{\rm core}\langle r^{2\lambda-2}\rangle_{n,\rm core} \\ &\qquad +Z_{\rm core}\langle r^{2\lambda-2}\rangle_{p,\rm core} \\ &\qquad +N_{\rm val}\langle r^{2\lambda-2}\rangle_{n,\rm val}]. \end{aligned} \tag{3}$$

For the breathing mode ($\lambda = 0$), the expectation values are $\langle r^2\rangle$ as in the quadrupole case. We clearly see how the valence neutrons can contribute significantly to the EWSR when their rms radius becomes large.

For the electromagnetic response, we should not use the isoscalar or isovector transition operator, but rather just the proton response to the external field,

$$E\lambda = e\sum_{i=p} r_i^\lambda Y_{\lambda 0}. \tag{4}$$

For ^{11}Li, there is no low energy response to operators of this type since about $1\hbar\omega$ is needed to create a proton ph pair (the p states are bound still). It is well known that the electric dipole operator

$$E1 = e\sum_{i=p} r_i\cos\theta_i = e\sum_{i=p} z_i \tag{5}$$

is not the correct operator to use since it induces spurious center-of-mass motion. Including one-body corrections to remove this c.m. motion yields the intrinsic dipole operator

$$D = e\sum_{i=p}(N/A)z_i - e\sum_{i=n}(Z/A)z_i, \tag{6}$$

which is equivalent to using the $E1$ operator with an effective charge of $(N/A)e$ for protons and $-(Z/A)e$ for neutrons. With this operator, we find the low energy soft dipole mode which has been greatly discussed. This is in fact the operator that has been used for the "isovector" dipole response of Fig. 1. It is not the true isovector dipole operator, but is instead the effective charge corrected electric dipole operator. Since the low energy response is due entirely to (valence) neutrons, the response to the operator D is $\sim (Z/A)^2 = (3/11)^2$ of the IS, true IV, or free response.

For higher multipoles, there are also one-body corrections for spurious c.m. motion that yield different effective charges [14] with the result that the corrected electric multipole operator is

$$\begin{aligned} E\lambda^{\rm corr} &= e\sum_{i=p}\left[\left(1-\frac{1}{A}\right)^\lambda + (-1)^\lambda\frac{(Z-1)}{A^\lambda}\right] r_i^\lambda Y_{\lambda 0} \\ &\quad + e\sum_{i=n} Z\left(-\frac{1}{A^\lambda}\right) r_i^\lambda Y_{\lambda 0}. \end{aligned} \tag{7}$$

Since the corrections are of order $1/A$, they have usually been neglected. However, for light nuclei, especially with greatly different proton and neutron responses, they are important. The neutrons carry an effective charge of $-eZ/A^\lambda$. Again, for all multipoles of ^{11}Li, the low energy response is due to (valence) neutrons only, and, hence, the electric response is $\sim (Z/A^\lambda)^2$ of the IS, IV, or free response. For quadrupole (λ=2), we are reduced by $[3/(11)^2]^2 = 6.15\times 10^{-4}$ from the free response while for octupole (λ=3) the reduction is $[3/(11)^3]^2 = 5.08\times 10^{-6}$.

There is no correction to the electric monopole, so there is no low energy $E0$ response. There are also no $E0$ virtual photons generated by the projectile, so the monopole response is not of interest to the EMD process. In Fig. 2, we show for ^{11}Li the (effective charge corrected) $E2$ and $E3$ responses, together with the isoscalar quadrupole and octupole responses for comparison. Recall that the "isovector" dipole response of Fig. 1 is already the (effective-charge-corrected) $E1$ response. Note that the low energy response (< 3 MeV) is unchanged in shape, but just reduced by the square of the effective neutron charge.

The response functions are related to the photonuclear cross sections by means of the relations

$$\sigma_\gamma^{\pi\lambda}(E_x) = \frac{(2\pi)^3(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\left(\frac{E_x}{\hbar c}\right)^{2\lambda-1}\frac{dB(E\lambda;E_x)}{dE_x}\,. \tag{8}$$

Inserting the response functions calculated above into Eqs. (1) and (8), the total electromagnetic dissociation cross section can be calculated. This is shown in Fig. 3 where the solid line corresponds to the excitation of isovector dipole states, the dashed line corresponds to the excitation of (effective-charge-corrected) quadrupole states, and the dotted line corresponds to the excitation of (effective-charge-corrected) octupole states in ^{11}Li. We observe that the excitation of isovector dipole states is by far the dominant mode of excitation at all energies. At very high energies $\sigma_C^{E2}/\sigma_C^{E1} \sim 10^{-4}$ and $\sigma_C^{E3}/\sigma_C^{E1} \sim 10^{-7}$.

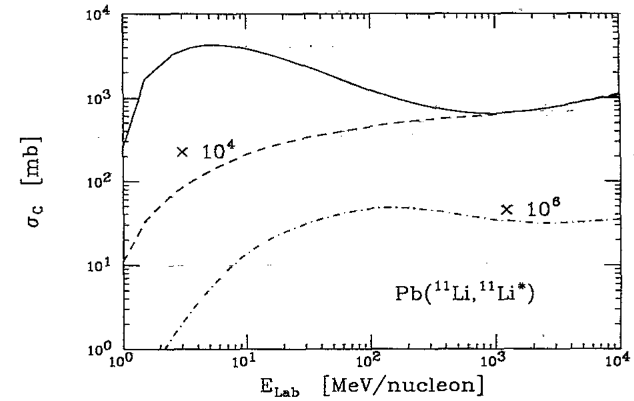

FIG. 3. Total Coulomb excitation cross section of ^{11}Li projectiles incident on a lead target as a function of the beam energy. The solid (dashed) line represents the contribution of isovector dipole (effective-charge-corrected quadrupole) excitations. The dash-dotted line represents the contribution of effective-charge-corrected octupole excitations. The quadrupole and the octupole results have been multiplied by 10^4 and 10^6, respectively.

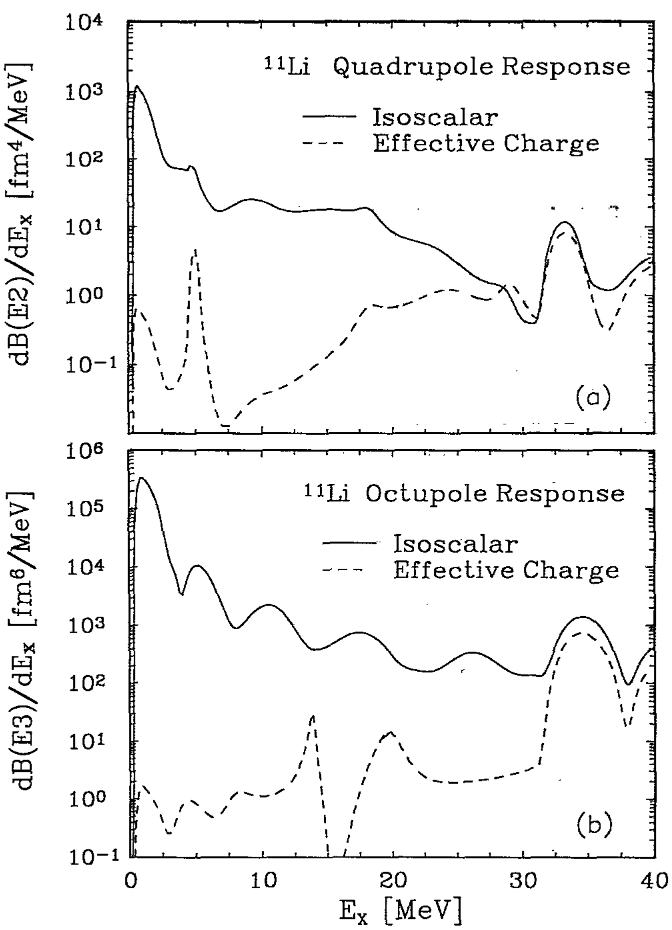

FIG. 2. Effective-charge-corrected electric response (dashed) and isoscalar response (solid) in ^{11}Li for (a) quadrupole, and (b) octupole.

It has been shown [11] that the correlations between the valence neutrons (which are not included in the RPA class of correlations) are very important to describe the properties of the ^{11}Li nucleus. The net effect of the correlations is also to increase the dipole response at low energies. Since the dipole virtual photon spectrum is larger at smaller energies, this effect enhances the total $E1$ cross section. It has been claimed [6, 7] that the increase in low energy response seen in the dipole case will also be present in other multipole responses. The question is whether such effects are sizeable and contribute appreciably to the total electromagnetic dissociation cross sections, or if this can be verified in other kinds of experiments. Let us consider the $E2$ and $E3$ response for example. The calculation of these responses with neutron-neutron correlations is quite difficult. But, we can show that it is not necessary to perform the calculations. Our arguments are based on the simple cluster model. This extreme correlated model considers the valence neutrons in ^{11}Li as a bound system. The $E1$ response function obtained from this model is peaked at low energies, $E_0 = 8S_{2n}/5$ and has a width of $\Delta E_x \sim 2S_{2n}$. That is, the response function is only characterized by the separation energy of ^{11}Li; the smaller the energy is, the narrower the peak is and the more it is positioned at lower energies. The response in the cluster model is the upper limit for the response function at lower energies. In general, the cluster model overestimates the total electromagnetic dissociation cross sections by a factor 1.5–2.

A general expression for the electric multipole response in the cluster model is given by

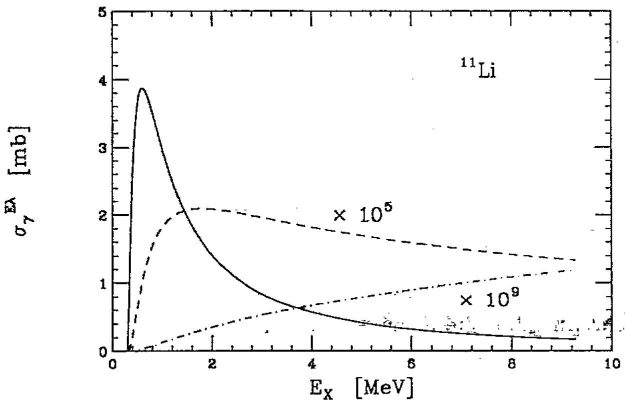

FIG. 4. Photoabsorption cross section of ^{11}Li in the cluster model. The solid (dashed) [dash-dotted] line corresponds to the dipole (quadrupole) [octupole] multipolarity. The quadrupole (octupole) results were multiplied by a factor of 10^5 (10^9).

$$\frac{dB(E\lambda;\,E_x)}{dE_x}=\frac{2^{\lambda-1}}{\pi^2}\,(\lambda!)^2\,(2\lambda+1)\,\left(\frac{\hbar^2}{\mu}\right)^{\lambda}$$
$$\times Z^2e^2\,\left(\frac{N_{\rm val}^{\lambda}}{A^{\lambda}}\right)^2\,\frac{\sqrt{S}\,(E_x-S)^{\lambda+1/2}}{E_x^{2\lambda+2}}\,, \qquad (9)$$

where S is the separation energy of the valence neutrons and μ is the reduced mass of the system ^{9}Li $+\,2n$. The maxima of the cluster response function occurs at

$$E_0=\frac{2\lambda+2}{\lambda+3/2}\,S\,. \qquad (10)$$

The width of this response function is given by $\Delta E_x = fE_0$, with f=1–2. The cluster response functions peak at approximately the same energies as the RPA ones. However, they lie above the low energy peaks of the RPA response functions and decrease steadily and much faster than those.

A comparison of the photonuclear cross section of ^{11}Li obtained with Eqs. (8) and (9) is shown in Fig. 4 for the $E1$, $E2$, and $E3$ multipolarities. The $E2$ and $E3$ photonuclear cross sections are smaller than the $E1$ by factors of order of 10^5 and 10^9, respectively. They have a longer tail than the $E1$ photonuclear cross section due to the phase factor $E_x^{2\lambda-1}$ in Eq. (8). These results show that a clusterlike correlation between the valence neutrons is not so effective in producing an enhancement of the low energy part of the photonuclear cross sections for higher multipolarities, as it does for the dipole case. So, one expects that such correlations in ^{11}Li will contribute very little to its total electromagnetic excitation cross section.

One could expect that the nuclear interaction with the target could access other relevant states, not necessarily of electric dipole nature, in reactions with ^{11}Li. But, due to the weak binding of this nucleus, and due to the uncertainties involved in extracting this information from the experiments with a distorted wave Born approximation analysis, we do not believe that this could be accomplished. Another possibility would be through electron scattering experiments. But these are far from our present experimental situation.

In summary, we have shown that low-energy excitations in ^{9}Li are not as large as in ^{11}Li. It is the weakly bound valence neutrons in ^{11}Li which enhance its electric response function at low energies. Therefore, the excitation of the core (^{9}Li-nucleus) will not influence the magnitude of the Coulomb dissociation cross sections obtained in secondary beam reactions. Also, we have presented simple arguments showing that only the $E1$ type excitations of the "halo" nucleons need to be considered in these experiments. The study of higher order soft multipole resonances is basically impossible with the actual accuracy attainable in the experiments.

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