# Momentum fluctuations in the fragmentation of neutron-rich nuclei

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We generalize the Goldhaber momentum-dispersion formula to the case of a "two-fluid" nucleus, in which the nucleons of one fluid have a different mean square momentum from those of the other fluid. The model predicts that fragments originated from the core of neutron-rich nuclei have a transverse momentum width which is significantly larger than that of a weakly bound "halo" neutron, and that both are much smaller than the $\sim$ 100 MeV/$c$ widths observed in the fragmentation of less exotic nuclei.

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A statistical model with minimal correlations was derived by Goldhaber in 1979 [1] to explain the momentum distributions of projectile fragments originating in peripheral collisions with heavy ions. In these experiments [2] it was observed that the momentum distributions of the fragments are well described by Gaussians, whose widths $\sigma$ were found to follow a parabolic dependence on the mass of the fragment. In the statistical model this dependence is explained in terms of a single quantity, namely the rms momentum of the nucleons in the original projectile. In general, the Goldhaber formula is quite successful, but some discrepancies with experiments have been observed in the past. For example, in some cases it has been observed that the momentum dispersion $\sigma$ is about 30% smaller than predicted by the statistical model [3]. Experiments with radioactive nuclei have also shown some discrepancies with the statistical model [4–7]. In experiments with an $^{11}$Li projectile, a very narrow peak in the momentum distribution of $^9$Li fragments was found ($\sigma \sim 19$ MeV/$c$) which could not be explained in terms of the model. Experimentally it was also observed that the momentum distributions of neutrons are appreciably narrower than those for $^9$Li fragments. The momentum distributions in such a weakly bound nucleus are presumably determined by their nuclear matter size [4]. However, while the momentum distribution of $^9$Li suggests a halo size of 6–8 fm [6] in $^{11}$Li, the momentum distribution of neutrons seems to suggest a factor square root of 2 larger [8]. In this work we show that a possible explanation of these results and other forthcoming experimental results can be given with a statistical model for a system of two fluids of nucleons.

Let us first recall the original Goldhaber derivation. In the statistical model the momentum of the fragment with mass number $F$ is obtained by picking at random $F$ nucleons from the projectile. In the frame of the projectile the average value (dispersion) of the square of the fragment momentum is given by

$$\sigma^2 = \left\langle \left[\sum_i^F \mathbf{p}_i\right]^2 \right\rangle = F\langle p_i^2\rangle + F(F-1)\langle \mathbf{p}_i.\mathbf{p}_j\rangle \ , \quad (1)$$

*provided* all come from a common distribution.

The second term in the above equation can be estimated in terms of $\langle p_i^2\rangle$ by using the fact that the total momentum of the original nucleus is zero. This is in fact the only correlation used in the statistical model [1]. One obtains

$$\left\langle \left[\sum_i^A \mathbf{p}_i\right]^2 \right\rangle = A\langle p_i^2\rangle + A(A-1)\langle \mathbf{p}_i.\mathbf{p}_j\rangle = 0 \ , \quad (2)$$

where $i \neq j$.

These two equations yield for the momentum dispersion of a fragment $F$ the Goldhaber formula

$$\sigma^2 = \frac{F(A-F)}{A-1}\,\langle p_i^2\rangle \, . \quad (3)$$

The quantity $\langle p_i^2\rangle$ can be estimated by using shell-model wave functions, or the Fermi gas model for the nucleons. For a Fermi gas $\langle p_i^2\rangle = 3p_F^2/5$. Generally, only one Cartesian component of the momentum distribution is measured. We then replace $\langle p_i^2\rangle$ by $\sigma_0^2 = \langle p_i^2\rangle/3 = p_F^2/5$ [1].

This simple and elegant derivation, due to Goldhaber, nicely displays the parabolic dependence on the fragment mass $F$ as observed in many experimental data [2]. From this derivation one sees that the averaging procedure does not make any extra assumption about nucleon-nucleon correlations, other than that the total momentum of the initial distribution is zero. Other kinds of correlations presumably would affect the value of $\langle p_i^2\rangle$, but not of $\langle \mathbf{p}_i.\mathbf{p}_j\rangle$ when averaged over direction. However, Goldhaber's formula would not be well justified if, e.g., the spatial dependence of the fragmentation operator is so strong that a different treatment for nucleons at the nuclear surface and in the nuclear interior has to be considered separately [9]. In this regard we expect that the parallel momentum distributions are less sensitive to the fragmentation operator than the transverse distributions are. This is because transverse momentum distributions are more sensitive to the transverse geometry of the participating nuclei as well as to the Coulomb deflection of the projectile and fragments [10]. This was in fact observed experimentally [6]. When spatial dependence is relevant it is important to consider Pauli correlations among the nucleons in the statistical model [9], which would cause a reduction of the dispersion predicted by the Goldhaber model. We expect that other correlations, e.g., short-range correlations due to the nucleon-nucleon interaction, will also play an important role in such a case.

Let us now consider a nucleus composed of two "fluids" of nucleons with mass numbers $A_1$ and $A_2$ ($A = A_1+A_2$). An alpha particle inside a larger nucleus, or a $^9$Li core in $^{11}$Li, could be an example. Since the two fluids have to interact in order to keep the nucleus bound, we have to introduce another parameter $K$ which is the relative momentum between the two fluids. Momentum conservation implies

$$\sum_i^{A_1} \mathbf{p}_i^{(1)} = \mathbf{K} = -\sum_j^{A_2} \mathbf{p}_j^{(2)} , \tag{4}$$

so that $2\mathbf{K}$ is the relative momentum of the fluids.

Using this relation we obtain

$$\langle \mathbf{p}_i^{(1)} . \mathbf{p}_j^{(2)} \rangle = -\frac{\langle K^2 \rangle}{A_1 A_2} \tag{5}$$

and

$$\langle \mathbf{p}_i . \mathbf{p}_j \rangle_N = \frac{\langle K^2 \rangle}{A_N (A_N - 1)} - \frac{\langle p^2 \rangle_N}{A_N - 1} , \quad i \neq j,$$

where $N = 1, 2$ and $\langle \rangle_N$ means average over nucleons in only one of the two fluids. Equation (5) is an average over nucleons in different fluids. We also express the nucleon momenta with respect to the center of mass of their respective fluids. The relationship between their averages is

$$\langle p^2 \rangle_N = \langle \tilde{p}^2 \rangle_N + \langle K^2 \rangle / A_N^2 . \tag{6}$$

If we pick $F$ nucleons from the projectile to make a fragment, the mean square momentum involves the cross products given by the above equations. Assuming that the fragment can be composed of nucleons with equal probability from either of the two fluids, we can use Eq. (1) for the dispersion of the momentum distribution. In this case the quantities $\langle p_i^2 \rangle \equiv \langle p^2 \rangle$ and $\langle \mathbf{p}_i . \mathbf{p}_j \rangle$ are to be averaged over the two distributions. We find

$$\begin{aligned} \sigma^2 \equiv \left\langle \left[ \sum_i^F \mathbf{p}_i \right]^2 \right\rangle &= F \left[ \frac{A_1}{A} \langle p^2 \rangle_1 + \frac{A_2}{A} \langle p^2 \rangle_2 \right] \\ &\quad + F(F-1) \left[ \frac{2A_1A_2}{A(A-1)} \langle \mathbf{p}_i^{(1)} . \mathbf{p}_j^{(2)} \rangle + \frac{A_1(A_1-1)}{A(A-1)} \langle \mathbf{p}_i . \mathbf{p}_j \rangle_1 + \frac{A_2(A_2-1)}{A(A-1)} \langle \mathbf{p}_i . \mathbf{p}_j \rangle_2 \right] \\ &= \frac{F(A-F)}{A-1} \left[ \frac{A_1}{A} \langle \tilde{p}^2 \rangle_1 + \frac{A_2}{A} \langle \tilde{p}^2 \rangle_2 + \frac{\langle K^2 \rangle}{A_1 A_2} \right] , \end{aligned} \tag{7}$$

where the subindices mean averages over the respective fluids. For $A_1 = A$ ($\langle K^2 \rangle = 0$) this equation reduces to Eq. (3).

Useful limits of Eq. (7) above can be obtained. For example, when $\langle K^2 \rangle$ and $\langle \tilde{p}^2 \rangle_1 \ll \langle \tilde{p}^2 \rangle_2$ one gets (unless $A_1 \gg A_2$)

$$\sigma^2 \simeq \frac{A_2}{A} \frac{F(A-F)}{A-1} \langle \tilde{p}^2 \rangle_2 . \tag{8}$$

In such a situation we come to the important conclusion that the momentum dispersion of the fragments is reduced by a factor $A_2/A$ relative to that obtained by means of Eq. (3). This suggests that if the momentum variance in a particular experiment is found to be appreciably less than that expected from Eq. (3), as in the experiment of Ref. [3], it might be an indication that the projectile should be considered as a multifluid system. In this case, Eq. (7) (together with Pauli correlations [9], if the spatial dependence of the fragmentation operator is important) might be more appropriate to describe the momentum distributions.

A rather well-studied case of fragmentation of a neutron-rich nucleus is that provided recently with $^{11}$Li projectiles. The data can be understood in a quite simple form, without resorting to our more involved formulas, but they do provide a very useful example of how the formulas would function in a more complex situation.

In the fragmentation of $^{11}$Li projectiles very narrow components were observed for the momentum distributions of $^9$Li fragments and of single neutrons [4–7]. These widths (e.g., $\sim 19$ MeV/$c$ for $^9$Li fragments [6]) cannot be explained by Eq. (8), unless an unrealistic value of the mean square momentum in $^9$Li is assumed. To understand this we recall that Eqs. (7) and (8) are only adequate if enough energy is given to the projectile so that a fragment could be formed with nucleons equally likely from fluid $A_1$ or $A_2$. But, in some situations [4] it has been observed that the energy transferred to the projectile is not enough to remove nucleons from the tightly bound core, say fluid $A_2$. The statistical average then has to be carried out using a different procedure, as we explain below.

Let us consider the simple case of fragmentation of $^{11}$Li projectiles. In the case of $^9$Li fragments from $^{11}$Li projectiles, we can identify the fragment itself as one of the constituent fluids of the nucleus. Then, obviously, its momentum distribution has a variance determined by the momentum $K$. As a trivial consequence of Eq. (4) we get

$$\sigma_9^2 = \langle K^2 \rangle . \tag{9}$$

We observe that the nuclear potential binding the halo neutrons in $^{11}$Li has a range not greater than 3–4 fm. However, due to their low binding energy these neutrons extend to a very large distance from the core, with an empirical rms radius for the halo matter distribution of about 6 fm [6]. Thus, the halo is a manifestation of a quantum tunneling of the valence neutrons which extend to a region where their momenta are imaginary. Their wave functions in this region depend primarily on their binding energy. Therefore, we suggest it is more appropriate to equate $p_{\rm rel}^2/2\mu_{12}$ with the separation energy $\langle B\rangle$ of the two neutrons from the core. Doing so, we find

$$\langle B\rangle = \frac{(2K)^2}{2\mu_{12}} = \frac{2\,|\langle K^2\rangle|}{m_N}\,\frac{A}{A_1A_2}. \qquad (10)$$

Using the momentum width of 19 MeV/$c$ for $^9$Li fragments [6], we find $\langle B\rangle = 0.47$ MeV, which is in fact very close to the separation energy of two neutrons in $^{11}$Li, i.e., $B = 0.34 \pm 0.05$ MeV.

As for the momentum width of a single halo neutron from $^{11}$Li, we can use Eq. (6) to obtain

$$\sigma_n^2 = \langle \tilde{p}^2\rangle_n + \frac{1}{4}\,\langle K^2\rangle\,, \qquad (11)$$

where $\langle \tilde{p}^2\rangle_n$ is the average square momentum of the neutrons in the halo. Since the radius of the halo is empirically about 6 fm, each neutron is confined within $\Delta x \simeq 12$ fm, so $\Delta p_x \geq \frac{1}{2}(\frac{1}{12\ \rm fm}) = 8$ MeV/$c$. Using this value for $\sqrt{\langle \tilde{p}^2\rangle_n}$, and Eq. (9) to calculate the second term in Eq. (11) we obtain $\sigma_n = 12.4$ MeV/$c$, in satisfactory agreement with the experimental values [5,11].

The expressions obtained above for the case of $^{11}$Li fragments are a particular limit of a more general situation which can arise in reactions with neutron-rich nuclei. In fact, if one knows what fraction of nucleons in a given fragment come from one or the other nuclear fluid then, instead of Eq. (7), it is straightforward to show that

$$\begin{aligned}\sigma^2 &= F_1\,\langle p^2\rangle_1 + F_2\langle p^2\rangle_2 + F_1(F_1-1)\,\langle \mathbf{p}_i.\mathbf{p}_j\rangle_1 + F_2(F_2-1)\,\langle \mathbf{p}_i.\mathbf{p}_j\rangle_2 + 2F_1F_2\,\langle \mathbf{p}_i^{(1)}.\mathbf{p}_j^{(2)}\rangle \\ &= \frac{F_1(A_1-F_1)}{A_1-1}\,\langle \tilde{p}^2\rangle_1 + \frac{F_2(A_2-F_2)}{A_2-1}\,\langle \tilde{p}^2\rangle_2 \\ &\quad + \left[\frac{F_1(F_1-1)}{A_1(A_1-1)} + \frac{F_2(F_2-1)}{A_2(A_2-1)} - \frac{2F_1F_2}{A_1A_2} + \frac{F_1(A_1-F_1)}{A_1^2(A_1-1)} + \frac{F_2(A_2-F_2)}{A_2^2(A_2-1)}\right]\langle K^2\rangle\,, \qquad (12)\end{aligned}$$

where $F = F_1 + F_2$, and $F_N$ comes from fluid $N$.

It is easy to show that Eqs. (9) and (11) are consequences of Eq. (12). The first one follows by using $F_1 = 0$, $F_2 = A_2$. The second one follows from $F_1 = 1$, $F_2 = 0$, and $A_1 = 2$. Other applications of Eq. (12) could, e.g., be the case of fragmentation of neutron-rich nuclei via the excitation of a collective dipole vibration. Such process is expected to occur by means of Coulomb excitation in high energy collisions. In such collisions the Coulomb field of the target gives a fast kick to the protons in the projectile. Due to neutron halos, or thick neutron skins, this collective excitation can lie at low energies. For this kind of excitation (soft dipole modes), we can identify $A_1$ ($A_2$) as the number of neutrons (protons) in the projectile. $F_1$ ($F_2$) will be the neutron (proton) number in the fragment.

The above derivation can be extended to three or more nuclear fluids, but this would be at the cost of more unknowns for their intrinsic momenta. This procedure is beyond the scope of this article and might not be reasonable in view of other processes that can affect the momentum distributions. It is clear however that, if we want to insist in using statistical fragmentation models to describe the momentum distributions in reactions with neutron-rich nuclei, we have to consider two or more interacting fluids, which yield different weights to the observed momentum variances. We hope that the discussion and formulas described in this work may be useful for this purpose.

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[1] A.S. Goldhaber, Phys. Lett. **53B**, 306 (1974).
[2] D.E. Greiner, P.J. Lindstrom, H.H. Heckman, B. Cork, and F.S. Bieser, Phys. Rev. Lett. **35**, 152 (1975).
[3] Y.P. Viyogi, T.J.M. Symons, P. Doll, D.E. Greiner, H.H. Heckman, D.L. Hendrie, P.J. Lindstrom, J. Mahoney, D.K. Scott, K. Van Bibber, G.D. Westfall, H. Wieman, H.J. Crawford, C. McParland, and C.K. Gelbke, Phys. Rev. Lett. **42**, 33 (1979).
[4] T. Kobayashi, O. Yamakawa, K. Omata, K. Sugimoto, T. Shimoda, N. Takahashi, and I. Tanihata, Phys. Rev. Lett. **25**, 2599 (1988).
[5] R. Anne, S.E. Arnell, R. Bimbot, H. Emling, D. Guillemaud-Mueller, P.G. Hansen, L. Johannsen, B. Jonson, M. Lewitowicz, S. Mattsson, A.C. Mueller, R. Neugart, G. Nyman, F. Pougheon, A. Richter, K. Riisager, M.G. Saint-Laurent, G. Schrieder, O. Sorlin, and K. Wilhelmsen, Phys. Lett. B **250**, 19 (1990).
[6] N. Orr, N. Anantaraman, S.M. Austin, C.A. Bertulani, K. Hanold, J.H. Kelley, D.J. Morrissey, B.M. Sherrill, G.A. Souliotis, M. Thoenessen, J.S. Winfield, and J. Winger, Phys. Rev. Lett. **69**, 2050 (1992).
[7] D. Sackett, K. Ieki, A. Galonsky, C.A. Bertulani, H. Esbensen, J.J. Kruse, W.G. Lynch, D.J. Morrissey, N.A. Orr, B.M. Sherrill, H. Schultz, A. Sustich, J.A. Winger, F. Deák, Á. Horváth, Á. Kiss, Z. Seres, J.J. Kolata, R.E. Warner, and D.L. Humphrey, Phys. Rev. C **48**, 118 (1993).
[8] P.G. Hansen, Nature **361**, 501 (1993).
[9] G.F. Bertsch, Phys. Rev. Lett. **46**, 472 (1981)
[10] C.A. Bertulani and K.W. McVoy, Phys. Rev. C **46**, 2638 (1992).
[11] K. Riisager, R. Anne, S.E. Arnell, R. Bimbot, H. Emling, D. Guillemaud-Mueller, P.G. Hansen, L. Johannsen, B. Jonson, A. Latimier, M. Lewitowicz, S. Mattsson, A.C. Mueller, R. Neugart, G. Nyman, F. Pougheon, A. Richard, A. Richter, M.G. Saint-Laurent, G. Schrieder, O. Sorlin, and K. Wilhelmsen, Nucl. Phys. **A540**, 365 (1992).