Neutron removal in peripheral relativistic heavy-ion collisions

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We investigate the relativistic Coulomb fragmentation of ^{197}Au by heavy ions, leading to one-, two-, and three-neutron removal. To resolve the ambiguity connected with the choice of a specific minimum impact parameter in a semiclassical calculation, a microscopic approach is developed based on nucleon-nucleon collisions ("soft-spheres" model). This approach is compared with experimental data for ^{197}Au at 1 GeV/nucleon and with a calculation using the "sharp-cutoff" approximation. We find that the harmonic-oscillator model predicting a Poisson distribution of the excitation probabilities of multiphonon states gives a good agreement with one-neutron removal cross sections but is unable to reach an equally good agreement with three-neutron removal cross sections.

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Experiments with relativistic heavy-ion beams have accumulated evidence for the population of two-phonon giant-dipole resonances (GDR) both directly via the observation of neutron [1,2] or γ decay [3,4] and indirectly via the measurement of neutron-emission cross sections [5]. Theoretical descriptions of these processes were based on a semiclassical approach [6]. In the present communication we review some specific aspects of semiclassical Coulomb-excitation calculations: First, we discuss the differences between perturbation theory and a harmonic oscillator model in calculating two-phonon-GDR excitation cross sections. We then show how one can avoid the ambiguity connected with the choice of a specific lower integration limit in the semiclassical calculation when integrating over impact parameter. This is done by performing a Glauber-type transparency calculation. To check the validity of our calculations, we compare our results to measured $1n$- to $3n$-removal cross sections from ^{197}Au bombarded with several projectiles at relativistic energies [5]. The key question that we try to answer is whether our improved calculations are able to solve the puzzle contained in the fact that $1n$-removal cross sections could be reproduced reasonably well, whereas $3n$-removal cross sections (which are dominated by 2-phonon GDR excitation) were underestimated. This amounted to a deficit in the 2-phonon GDR of roughly a factor of 2 [5]. Similar deficits were observed also in the exclusive experiments (see Ref. [7] for a detailed discussion of this subject).

Two different approaches have been used in the literature to describe the excitation probabilities and cross sections in relativistic Coulomb excitation, namely, first- and second-order perturbation theory [8–10], and a harmonic oscillator model [6,11]. As we will show in more detail below, the two models represent two extreme assumptions: in perturbation theory, the multiphonon excitations are assumed to be completely independent of each other, whereas in the harmonic oscillator model they are assumed to be coupled up to infinite order. Our choice of one of these extremes will be guided by comparison with experimental data for ^{197}Au.

If first-order perturbation theory is valid, one can show that the excitation of a GDR state with energy E yields for the differential excitation probability the expression [6]

$$\mathcal{P}(E,b) \equiv \frac{dP(E,b)}{dE} = \frac{1}{E}\, N(E,b)\sigma_\gamma(E)\;, \qquad (1)$$

where b is the impact parameter, $\sigma_\gamma(E)$ is the photonuclear cross section for a photon with energy E, and $N(E,b)$ is the number density of equivalent photons with energy E, as given in Ref. [6].

To calculate the probability of exciting a double-phonon state, i.e., a state composed of two GDR states, one can use second order perturbation theory. Apart from a small interference term, the excitation probability of a state with energy E is a simple product of the probability to excite an intermediate state with energy E' and the probability to go from this state to the final state, summed over all intermediate states. A drawback of this method is that for small impact parameters b, for which the probability is large, the loss of probability for one-phonon excitation due to the two-phonon (and higher order) excitation is not accounted for, i.e., unitarity is violated. This problem can be eliminated by incorporating higher order corrections, but a proper treatment of this procedure depends strongly on the model assumed for the nuclear states. As a conclusion from the above considerations, we expect perturbation theory to *overestimate* single-phonon excitation probabilities.

A simple and transparent result is obtained under the assumption of a harmonic vibrator model. In this case, the excitation probability of multiphonon states is given in terms of a Poisson distribution of the probabilities obtained in first order perturbation theory [12]. In this approximation the excitation of the one-phonon state is modified to yield

$$\mathcal{P}^{(1)}(E,b) = \mathcal{P}(E,b)\exp\Big\{-\mathcal{P}(b)\Big\}\;, \qquad (2)$$

and the excitation probability of the double-phonon state is given by

$$\mathcal{P}^{(2)}(E,b) = \frac{1}{2!} \int dE' \, \mathcal{P}(E-E',b) \, \mathcal{P}(E',b) \times \exp\Big\{ -\mathcal{P}(b) \Big\} , \quad (3)$$

where the integral is over the energies of all intermediate states. The exponential on the right hand side takes care of the flux of probability to higher order excitations. Of course, the harmonic vibrator model is only a rough approximation to the nuclear states. To obtain Eq. (2) it is implicitly assumed that all states contribute equally to the unitarity condition. In other words, even high-lying multiphonon states are considered to take out flux from the probability to excite the one-phonon state. Thus, while the second order perturbation theory is expected to overestimate the excitation probabilities, the harmonic vibrator model is likely to *underestimate* them.

A choice between the two approaches is hampered by the fact, however, that the usual way of semiclassical calculations involves a more or less free parameter, namely, the minimum impact parameter, the choice of which can yield agreement with experimental data for either model. The total cross section for relativistic Coulomb excitation is obtained by integrating the excitation probabilities over impact parameter, starting from a minimum value. It is assumed that below this minimum value the interaction is exclusively nuclear, whereas above pure Coulomb interactions occur ("sharp-cutoff" approximation). It has been found that with this approximation the Coulomb cross sections are very sensitive to the parametrization of the minimum impact parameter [1,3,5,8]. One commonly used parametrization at relativistic energies is that of Benesh *et al.* [13], fit to Glauber-type calculations of total reaction cross sections, which we refer to hereafter as "BCV." In Ref. [13] a detailed study has been made concerning the parametrization procedure of the minimum impact parameter. It was also found that the nuclear contribution to the neutron removal channels in peripheral collisions has a negligible interference with the Coulomb excitation mechanism. This is a very useful result since the Coulomb and nuclear parts of the cross sections may be treated separately. Another parametrization is that of Kox *et al.* [14], which reproduced well measured total reaction cross sections of light and medium-mass systems. We have used this parametrization previously [5] and found reasonable agreement with the measured data for $1n$ cross sections. It should be noted, however, that the Kox parametrization of total interaction cross sections has been derived mainly from experiments with light projectiles and that its application to heavy systems involves an extrapolation into a region where no data points are available.

It is well known that at relativistic energies and grazing impact parameters nuclei are partly transparent to each other and that it is much better to replace the sharp-cutoff approximation by a smooth transition from purely nuclear collisions at $b \ll b_{\min}$ to pure Coulomb collisions at $b \gg b_{\min}$ [11]. Such a "soft-spheres" model can be derived from Glauber theory and can be incorporated in our semiclassical calculation by replacing $N(E,b)$ in Eq. (1) by

$$N(E,b)\exp\Big\{ -\sigma_{NN} \int dz \int d^3r \; \rho_p(\mathbf{r}) \; \rho_t(\mathbf{R}-\mathbf{r}) \Big\} , \quad (4)$$

where $\mathbf{R} \equiv (\mathbf{b}, z)$, with z being the coordinate along and $\mathbf{b}$ perpendicular to the beam direction. The quantity σ_{NN} is the nucleon-nucleon cross section, and $\rho_{p,t}$ are the ground state nuclear densities of projectile and target, respectively. The parametrization of the nuclear densities has been taken from the droplet model [15] in accordance with Shen *et al.* [16]. Since we are dealing with nucleus-nucleus collisions at energies of the order of one GeV/nucleon, we adopt a value of $\sigma_{NN} = 40$ mb in our calculations.

In the following we will apply the soft-spheres approximation to the case of ^{197}Au where inclusive $1n$- to $3n$-removal cross sections have been measured by Aumann *et al.* [5]. Apart from the exponential function on the right hand side of Eq. (4) which accounts for nuclear transparency in near-grazing collisions, the calculation is identical to the one described in [5].

As input to our calculations we will use the experimental photo-neutron emission cross sections from Ref. [17]. A Lorentzian fit to the (γ, xn) data is used to parametrize the GDR in ^{197}Au. The parameters are an excitation energy of 13.72 MeV, a width of 4.61 MeV, and a strength of 128% of the dipole sum rule [17]. The Lorentz parameters for the isoscalar and isovector GQR are identical to

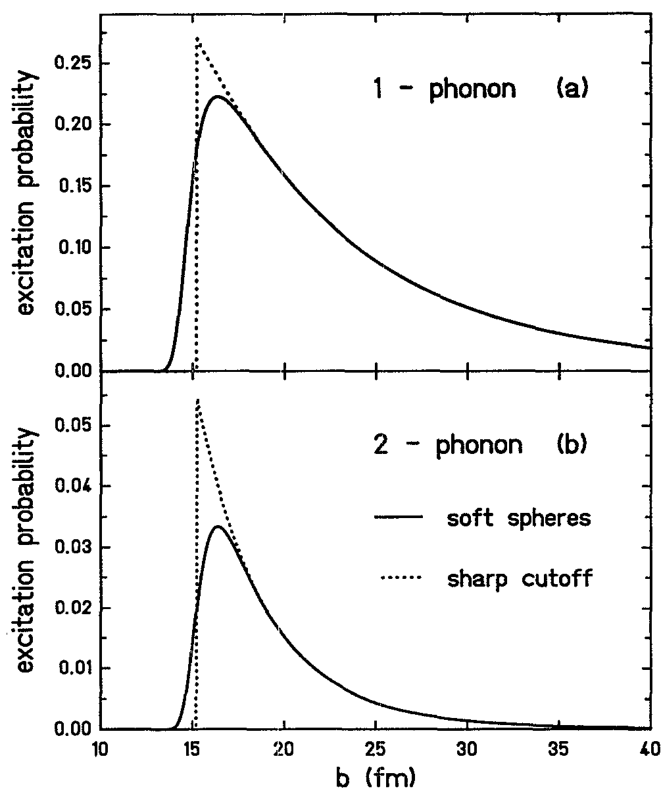

FIG. 1. Excitation probabilities of one-phonon (a) and two-phonon (b) GDR states in ^{197}Au due to relativistic Coulomb excitation by a gold projectile at 1 GeV/nucleon, as a function of the impact parameter. The solid (dashed) curves are obtained using a "soft-spheres" ("sharp-cutoff") model as described in the text.

those listed in Table V of Ref. [5]. With these parameters we calculate the excitation cross sections $d\sigma(E)/dE$ for one- and two-phonon dipole and quadrupole excitations. The respective neutron emission cross sections are given by $\sigma_{xn} = \int \frac{d\sigma(E)}{dE} f_{xn}(E)dE$, where $f_{xn}(E)$ is the probability to evaporate x neutrons at excitation energy E. For excitation energies below 27 MeV, $f_{xn}(E)$ is taken from the experimental (γ, xn) data [17], and for higher energies from a statistical decay calculation with the code HIVAP [18]. Since the three-neutron emission threshold in gold is above the energy of the GDR state, this channel is fed mainly by the two-phonon excitation mechanism, while the $1n$ cross section is dominated by the excitation of the GDR.

In Fig. 1 we plot the one- and two-phonon excitation probabilities for gold-gold collisions at 1 GeV/nucleon as a function of the impact parameter using the harmonic-oscillator model [Eqs. (2) and (3)]. The solid curve is the result of the soft-spheres model. We observe that this model gives an excitation probability which is a smoothly increasing function of b up to a maximum value, after which it decreases exactly as the sharp-cutoff approximation (dashed curve), obtained with the BCV parametrization. We expect that the BCV parametrization of $b_{\rm min}$ should yield similar results as the soft-spheres calculation since it was derived in fitting the complementary process, the nuclear interaction, calculated also with Glauber theory.

In Fig. 2 we examine how well we can reproduce the electromagnetic $1n$- and $3n$-removal cross sections with our model. Again, the solid curve denotes the soft-spheres calculation using the harmonic oscillator model. This calculation is in good agreement with the $1n$ cross sections. The dotted curve, which is from a sharp-cutoff calculation with the BCV parametrization of $b_{\rm min}$, deviates only insignificantly from the soft-spheres result, as expected. This remarkable agreement tells us that for practical purposes we can avoid the extra numerical complication connected with the use of Eq. (4) and corroborates the use of the BCV parametrization of $b_{\rm min}$ in sharp-cutoff calculations in the earlier work [11,19]. It also indicates that, contrary to our previous choice [5], the use of $b_{\rm min}$ given by Kox *et al.* [14] is physically less justified.

The dashed curve in Fig. 2 denotes a soft-spheres calculation with perturbation theory used to calculate the excitation probabilities. As expected from the previous discussion, the $1n$ cross sections are much higher than with the harmonic oscillator approach and deviate considerably from the data. Since our model avoids an arbitrary choice of $b_{\rm min}$ as in Ref. [8], we conclude that the harmonic oscillator model of multiple giant resonances is appropriate for the case of large-Z systems and that the

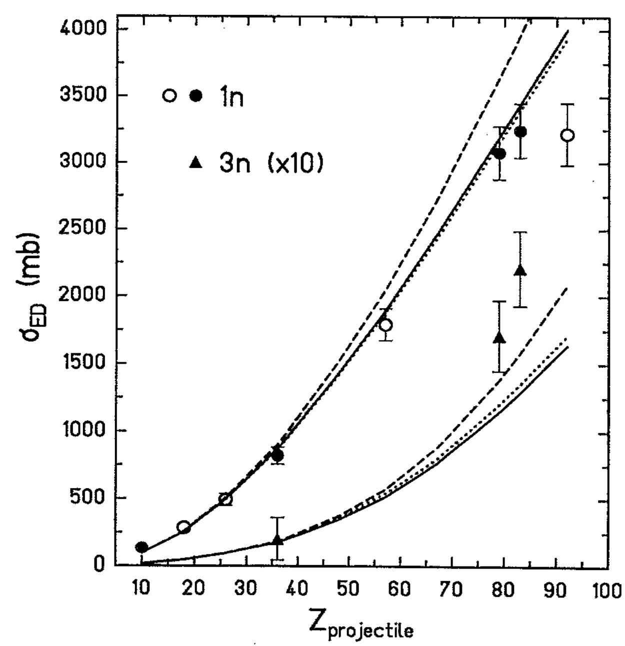

FIG. 2. Electromagnetic $1n$- and $3n$-removal cross sections (scaled to 1 GeV/nucleon) for ^{197}Au obtained by subtracting the nuclear cross section from the measured cross section (from Refs. [5,19]) in comparison with theoretical calculations from this work (solid curve: "soft-spheres" calculation with the harmonic-oscillator model; dashed curve: same for perturbation theory; dotted curve: "sharp-cutoff" calculation with the harmonic-oscillator model using the BCV parametrization of $b_{\rm min}$ [13]).

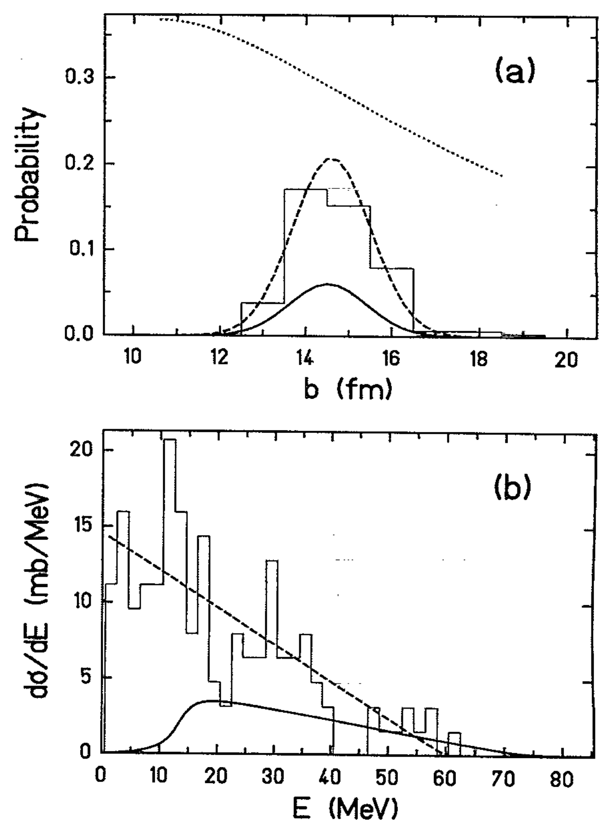

FIG. 3. (a) Impact-parameter distribution from an intranuclear-cascade calculation [5] for the formation of ^{196}Au without (dashed curve) and with Coulomb excitation (solid curve). The latter is obtained by multiplying the former distribution with the Coulomb excitation probability (dotted curve). (b) Excitation-energy distribution obtained by folding the nuclear excitation-energy spectrum of ^{196}Au from the cascade calculation (histogram, approximated by the dashed line) with the Lorentz curve representing the GDR excitation of ^{196}Au.

violation of the unitarity condition in the perturbation theory approach leads to discrepancies with the experimental data far beyond the error bars.

The lower set of curves in Fig. 2 shows the results for the $3n$ channel using the same models as in the upper part of the figure. We note that, as expected, perturbation theory yields higher cross sections, which in this case are closer to the measured data than those calculated with the harmonic oscillator model. Since we have chosen, however, to use the $1n$ cross sections as the test case, where the statistical accuracy is better and the nuclear contribution can be neglected completely, we are left with the conclusion that it is not possible to reproduce $1n$ and $3n$ cross sections (i.e., one- and two-phonon excitation) with the same model and that the lack of two-phonon excitation probability observed previously [1,3,5] is not connected with an improper choice of $b_{\min}$. Including the three-phonon excitation probabilities would not explain these discrepancies, since they are very small.

Up to now it was tacitly assumed that in a peripheral nuclear collision either a nuclear interaction or, in the case of transparency, a Coulomb interaction may take place. It is conceivable, however, that in the same collision both processes occur. For an estimate of the contribution of such processes to the $1n$ to $3n$ channels studied in the present work, we have modified our intranuclear-cascade calculations of the nuclear processes [5] to take into account also possible electromagnetic excitations. We note that the only channel that needs to be studied is the one-neutron knock-out in the intranuclear-cascade step of the collision. The ^{196}Au prefragment formed in this process then feeds the $2n$ and $3n$ channels by evaporation. The inclusion of Coulomb processes proceeds in our estimate in the same way as in the soft-spheres calculation of the total nuclear interaction probability: The impact-parameter distribution of ^{196}Au formation from the cascade calculation (upper part in Fig. 3) has to be multiplied by the probability of Coulomb excitation [dotted curve in Fig. 3(a)]. As a result, about 30% of the ^{196}Au prefragments are Coulomb excited and are thus shifted towards higher excitation energies which are obtained by folding the nuclear-excitation energy distribution taken from the cascade calculation with the Lorentz curve of GDR excitation (lower part of Fig. 3).

The net effect of the inclusion of nuclear-plus-Coulomb processes is small: on the one hand, the nuclear part of the $1n$ to $3n$ channels is depleted by 30%, since the corresponding ^{196}Au prefragments have been shifted to a different excitation energy distribution. On the other hand, the $2n$ and $3n$ channels are fed by evaporation from this very distribution, yielding a net increase, e.g., in the $3n$ channel of about 10 mb — a value that is less than the accuracy of the cross sections of Ref. [5] and also much less than the deficit of about 100 mb found in the theoretical cross section for the $3n$ channel as compared to the experimental one.

We conclude that an obvious modification of the semiclassical theory of relativistic Coulomb excitation, namely, the transition from a "sharp-cutoff" to a "soft-spheres" model, resolves ambiguities connected with the choice of a specific expression for the minimum impact parameter necessary in previous calculations. Once this previously free parameter is fixed, one can make a decision which of two rather extreme assumptions, namely, complete independence or complete coupling of the multiphonon excitations, is more appropriate. Our calculations show that the harmonic oscillator model describes $1n$-removal cross sections for the interactions of relativistic heavy ions with ^{197}Au with good accuracy. The basic discrepancy, however, that we and others have noted earlier, which lies in a good description of one-phonon excitation and a large deficit in the calculated two-phonon excitation, persists. In a simple estimate we have shown that this deficit cannot be attributed to a neglect of nuclear-plus-Coulomb interactions. It is possible that a coupled-channels calculation could be able to remove this discrepancy. More generally, a truly microscopic description of multiphonon excitations would be desirable, that circumvents the problems connected with the inadequacies of the presently used harmonic oscillator and perturbation-theory models.

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