

Energy dependence of breakup cross sections of the halo nucleus ${}^8\text{B}$ and effective interactions

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We study the energy dependence of the cross sections for nucleon removal of ${}^8\text{B}$ projectiles. It is shown that the Glauber model calculations with nucleon-nucleon t -matrix reproduce well the energy dependence of the breakup cross sections of ${}^8\text{B}$. A distorted wave Born approximation (DWBA) model for the breakup cross section is also proposed and results are compared with those of the Glauber model. We show that to obtain an agreement between the DWBA calculations, the Glauber formalism, and the experimental data, it is necessary to modify the energy behavior of the effective interaction. In particular, the breakup potential has a quite different energy dependence than the strong absorption potential. [S0556-2813(98)04001-1]

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I. INTRODUCTION

The study of breakup reactions with halo nuclei is one of the main tools for understanding their structure. The measurements of the width of the momentum distributions of fragments, the magnitude of the total reaction cross sections, and single- and double-nucleon removal cross sections have been of major usefulness to unravel their internal properties (for a review, see, e.g., [1]). These measurements have been carried out at relatively high energies, in the beam energy range of 30–1200 MeV/nucleon.

The Glauber formalism is the major theoretical approach in use to analyze these measurements. This formalism is well established and yields very reasonable results for the reactions involving stable nuclei at high energies. In particular, a direct connection of the quantum-mechanical breakup amplitudes and semiclassical calculations can be done in the Glauber formalism in a very intuitive way [2].

In perturbation theory the transition amplitude is given by

$$T_{fi} = \langle \phi_f | \Psi^- | U | \phi_i | \Psi^+ \rangle, \quad (1)$$

where $\langle \phi_i |$ ($|\phi_f\rangle$) denotes the initial (final) internal wave function of the nuclei, Ψ^- (Ψ^+) is the incoming (outgoing) scattering wave of the center of mass, and U is the interaction potential. The Glauber formalism uses eikonal wave functions for the scattering waves. The product $\Psi^- \cdot \Psi^+$ is then simply a plane wave displaced by a (eikonal) phase which is directly proportional to the integral of the absorptive potential along the beam direction, the z axis.

The use of eikonal wave functions is a crucial step in the Glauber formalism. Indeed, as shown by Glauber in his ex-

cellent lecture notes [2], the z integration can be done by parts, and the potential U in Eq. (1) will only appear in the exponential phase. This procedure is valid if the excitation potential U is the same as the absorptive potential appearing in the eikonal phases. This is the case for example in the calculation of total reaction cross sections. However, it is not a general premise. For example, in inelastic excitations of surface vibrations, the absorptive potential, U_{abs} , causing the phase shifts in the elastic channel is not the same as the interaction potential U_{ex} which leads to surface vibrations, although (in some models) they can be related by derivatives. But, even in such situations, the energy dependence of both potentials are roughly the same. However, while the excitation potential is related to a few reaction channels, the absorptive potential carries information of *all* channels which may lead to the absorption of the scattering waves. Thus, one expects that a difference in the energy dependence of the interaction and the absorptive should be manifest in some sensitive cases.

A good place to look for a deviation from the Glauber theory is the breakup reactions involving halo nuclei. This is because the energies involved in the breakup are basically the separation energies of the valence nucleons, while the core nucleons which are also relevant for the absorptive part of the potential have much larger separation energies. Also, the spatial distribution of the valence and core nucleons are very different so that they influence differently on the absorptive (for which all nucleons participate) and the excitation (for which only the valence nucleons participate) potential.

In Sec. II we illustrate the connection of the Glauber formalism and the nucleon-nucleon cross sections. We apply the theory to the calculation of proton removal cross sections from ${}^8\text{B}$ projectiles at several bombarding energies. In Sec. III we develop a distorted wave Born approximation (DWBA) formalism for the stripping reactions. In Sec. IV

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we show how to relate the DWBA calculation with the Glauber formalism, using effective interactions. Our conclusions are presented in Sec. V.

II. GLAUBER MODEL AND NUCLEON-NUCLEON SCATTERING AMPLITUDES

In the Glauber theory, after the z integration, the remaining integrals in Eq. (1) can be easily related to the concept of impact parameter and to absorption and survival probabilities. For example, simple manipulations show that the nucleon removal cross sections in high-energy collisions are described in the Glauber theory by

$$\sigma = 2\pi \int db b [1 - \exp(-2 \operatorname{Im}\chi_v)] \exp(-2 \operatorname{Im}\chi_c), \quad (2)$$

where Im stands for the imaginary part, $v(c)$ denotes the valence (core) particles, and χ are the eikonal phases given by [2]

$$\chi_i(b) = -\frac{1}{\hbar v} \int_{-\infty}^{\infty} dz' U(r'), \quad (3)$$

where v is the projectile velocity, $r' = \sqrt{b^2 + z'^2}$, and U is the optical potential for the system composed of the particle i ($=v, c$) and the target. The term inside the brackets in Eq. (2) can be interpreted as the probability that the valence nucleons will be removed in a collision with impact parameter b , while the exponential term outside brackets is the probability that the core nucleons will survive. This product integrated over all impact parameters gives the cross section for (valence) nucleon removal.

A great simplification introduced by Glauber was to relate the optical potentials to the nucleon-nucleon cross sections. This can be done easily by noticing that the optical theorem for the forward nucleon-nucleon amplitude yields [2,3]

$$\begin{aligned} t_{NN}(\mathbf{q}=0) &= -\frac{4\pi\hbar^2}{2\mu} f(\theta=0^\circ) \\ &= -i \frac{\hbar v}{2} \sigma_{NN}(E) [1 - i\alpha(E)], \end{aligned} \quad (4)$$

where σ_{NN} is the nucleon-nucleon cross section and α is the real-to-imaginary ratio of the nucleon-nucleon scattering amplitude. With the assumptions that only very forward angles are involved, and that the nucleon-nucleon interaction is of very short range (i.e., a δ -function interaction), one can construct optical potentials for the nuclear scattering in terms of the folding integrals [2,3]

$$U_i(R) = t_{NN}(\mathbf{q}=0) \int \rho_i(\mathbf{r}) \rho_A(\mathbf{R}-\mathbf{r}) d^3r, \quad (5)$$

where ρ_i and ρ_A are the ground-state densities of the projectile i , and the target A , respectively, and \mathbf{R} is the coordinate separating the center of mass of the two nuclei.

As an application of this model, let us consider the proton removal cross sections of ^8B projectiles in reactions with carbon targets. For the valence nucleon we get the density

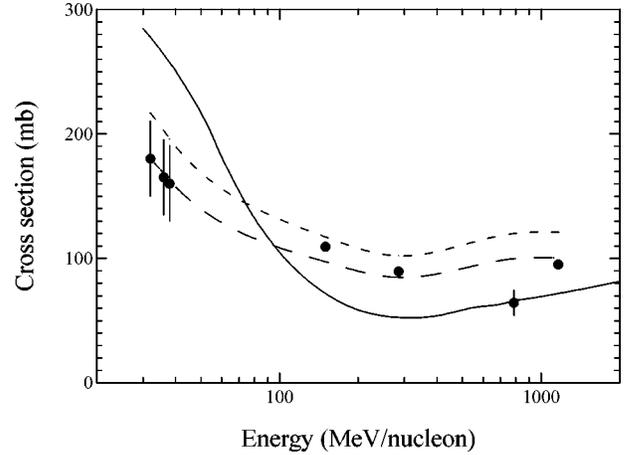


FIG. 1. Proton removal cross sections of ^8B projectiles on carbon targets as a function of the incident beam energy. Data points are from Ref. [4]. Solid line is a calculation based on a model by Hansen [5]. Short-dashed line is a calculation based on the Glauber model, Eq. (2). Dashed line is simply the short-dashed line downshifted by a factor of 0.83.

distribution from a Woods-Saxon+spin-orbit potential model for a proton in the $p_{3/2}$ orbital (the parameters are given in Sec. IV). For the core (^7Be) density we use the ground-state density parametrized as $\rho(r) = \rho_0 [1 + cr^2/a^2] \exp(-r^2/a^2)$, with $a = 1.77$ fm and $c = 0.327$ fm. The result of the calculation is shown by the short-dashed line of Fig. 1. The experimental data are from Ref. [4]. Although the magnitude of the cross section is a little overestimated, we see that the energy dependence follows very closely that of the experimental data. The dashed curve is the calculation renormalized to the lowest energy experimental data point. In fact, the reasonably good agreement between the energy dependence deduced from the Glauber theory and the experimental data on the total nuclear cross sections, and nucleon removal cross sections, is well established, both for stable and unstable nuclei.

Also shown in Fig. 1 (solid curve) is the model developed by Hansen [5]. In his model, the nucleon removal cross section is forced to have the same energy dependence as the total nucleon-nucleon cross section. The total reaction cross section has a slightly different energy dependence than the valence nucleon removal cross section. This can be best seen from the Glauber theory. The calculation of the total reaction cross section amounts in replacing the integrand in Eq. (2) by $b[1 - \exp(-2 \operatorname{Im}\chi_{aA})]$, where now χ_{aA} is the eikonal phase for the collision of the projectile a and the target A . In fact, we see that the Hansen's model predicts a rather different energy dependence of the proton removal cross section. The data favor the calculation following Eq. (2).

From a general point of view, the energy dependence of the total and the nucleon removal cross sections are directly related to the underlying optical potentials for the reaction. From the above discussion we can see that these optical potentials should have a similar energy dependence as the nucleon-nucleon cross section. To study this idea further, let us formulate a DWBA model for the breakup cross section. The use of an effective nucleon-nucleon interaction, the

M3Y interaction, will serve as guide to understand the link between the optical potentials and the nucleon-nucleon cross sections.

III. DWBA BREAKUP AMPLITUDES

Let us consider the general case of the stripping of the projectile a incident on target A :

$$a + A \rightarrow b(a-x) + B(A+x). \quad (6)$$

The Hamiltonian for the system is

$$H = H_a + H_A + T_{aA} + V_{aA} = H_b + H_B + T_{bB} + V_{bB}, \quad (7)$$

The transition matrix element for this reaction is given by

$$T = \langle \Psi^-(\mathbf{k}_b, \mathbf{r}_{bB}) \phi_b(\mathbf{s}_b) \phi_B(\xi, \mathbf{s}_x, \mathbf{r}_{xA}) | U_{bu} | \Psi_a^+(\mathbf{k}_a, \mathbf{r}_{aA}) \phi_a(\mathbf{s}_b, \mathbf{s}_x, \mathbf{r}_{bx}) \phi_A(\xi) \rangle, \quad (8)$$

where $\phi_a, \phi_b, \phi_A, \phi_B$ are eigenstates of H_a, H_b, H_A, H_B , respectively, i.e., $H_A \phi_A = \epsilon_A \phi_A$, $H_a \phi_a = \epsilon_a \phi_a$, etc., Ψ^\pm are distorted waves of the particles a and b , i.e., $(T_{aA} + U_{aA})\Psi_a^\pm = (E - \epsilon_a - \epsilon_A)\Psi_a^\pm$. The internal coordinates of a , b , and x , respectively, are denoted by \mathbf{s}_i ($i = a, b, x$), \mathbf{r}_{ij} are the relative coordinates of particles i and j , and ξ is the internal coordinate of particle A .

We use the coordinate relationship

$$\mathbf{r}_{bB} = \mathbf{r}_{bx} + \left(\frac{m_A}{m_B}\right) \mathbf{r}_{xA}, \quad \mathbf{r}_{aA} = \mathbf{r}_{xA} + \left(\frac{m_b}{m_a}\right) \mathbf{r}_{bx}, \quad (9)$$

and we integrate over the internal coordinate, ξ , of A

$$\psi_x(\mathbf{s}_x, \mathbf{r}_{xA}) = \int d\xi \phi_B^*(\xi, \mathbf{s}_x, \mathbf{r}_{xA}) \phi_A(\xi), \quad (10)$$

and over the internal coordinates of b and x ,

$$\int d\mathbf{s}_b d\mathbf{s}_x \phi_b^*(\mathbf{s}_b) \psi_x(\mathbf{s}_x, \mathbf{r}_{xA}) \phi_a(\mathbf{s}_b, \mathbf{s}_x, \mathbf{r}_{bx}) = C_{bx} \phi_a(\mathbf{r}_{bx}) \Psi_x(\mathbf{r}_{xA}), \quad (11)$$

where $|C_{bx}|^2$ is the spectroscopic factor.

We get for the transition-matrix element

$$T = C_{bx} \int d^3 r_{bx} d^3 r_{xA} \Psi_b^{-*} \left(\mathbf{k}_b, \mathbf{r}_{bx} + \frac{m_A}{m_B} \mathbf{r}_{xA} \right) \times \Psi_x(\mathbf{r}_{xA}) U_{bu}(\mathbf{r}_{xA}, \mathbf{r}_{bx}) \phi_a(\mathbf{r}_{bx}) \Psi_a^+ \left(\mathbf{k}_a, \mathbf{r}_{xA} + \frac{m_b}{m_a} \mathbf{r}_{bx} \right). \quad (12)$$

The potential U_{ex} for the breakup channel is given by

$$U_{bu}(\mathbf{r}_{xA}, \mathbf{r}_{bx}) = U_{bA}(\mathbf{r}_{bA}) + U_{xA}(\mathbf{r}_{xA}) - U_{aA}(\mathbf{r}_{aA}) = U_{bA}(\mathbf{r}_{xA} + \mathbf{r}_{bx}) + U_{xA}(\mathbf{r}_{xA}) - U_{aA} \left(\mathbf{r}_{xA} + \frac{m_b}{m_a} \mathbf{r}_{bx} \right). \quad (13)$$

If we now integrate over \mathbf{r}_{bx} , we can use the fact that the bound-state wave function is peaked at small \mathbf{r}_{bx} values, so that

$$\int d^3 r_{bx} \Psi_b^{-*} \left(\mathbf{k}_b, \mathbf{r}_{bx} + \frac{m_A}{m_B} \mathbf{r}_{xA} \right) U_{bu}(\mathbf{r}_{xA}, \mathbf{r}_{bx}) \times \phi_a(\mathbf{r}_{bx}) \Psi_a^+ \left(\mathbf{k}_a, \mathbf{r}_{xA} + \frac{m_b}{m_a} \mathbf{r}_{bx} \right) \approx \Psi_b^{-*} \left(\mathbf{k}_b, \frac{m_A}{m_B} \mathbf{r}_{xA} \right) \Psi_a^+(\mathbf{k}_a, \mathbf{r}_{xA}) \times \int d^3 r_{bx} U_{bu}(\mathbf{r}_{xA}, \mathbf{r}_{bx}) \phi_a(\mathbf{r}_{bx}). \quad (14)$$

We now define a ‘‘transition,’’ or ‘‘excitation,’’ potential as

$$U_{ex}(\mathbf{r}_{xA}) = \int d^3 r_{bx} \phi_a(\mathbf{r}_{bx}) U_{bu}(\mathbf{r}_{xA}, \mathbf{r}_{bx}), \quad (15)$$

so that

$$T = C_{bx} \int d^3 r_{xA} \Psi_b^{-*} \left(\mathbf{k}_b, \frac{m_A}{m_B} \mathbf{r}_{xA} \right) \times \Psi_x(\mathbf{r}_{xA}) U_{ex}(\mathbf{r}_{xA}) \Psi_a^+(\mathbf{k}_a, \mathbf{r}_{xA}). \quad (16)$$

The above equation is our main result. It gives the t matrix in terms of the scattering waves of particle a , b , and x , and a ‘‘transition’’ potential U_{ex} . This potential contains the information on the structure of particle a .

If we are only interested in particle b , assuming that the particle x is not observed, we can use the closure relation

$$\sum_{\mathbf{k}_x} \Psi_x^{(\mathbf{k}_x)*}(\mathbf{r}_{xA}) \Psi_x^{(\mathbf{k}_x)}(\mathbf{r}'_{xA}) = \delta(\mathbf{r}_{xA} - \mathbf{r}'_{xA}), \quad (17)$$

to obtain

$$\sum_{\mathbf{k}_x} |T|^2 = |C_{bx}|^2 \int d^3 r_{xA} \left| \Psi_b^{-*} \left(\mathbf{k}_b, \frac{m_A}{m_B} \mathbf{r}_{xA} \right) \right|^2 \times |U_{ex}(\mathbf{r}_{xA})|^2 |\Psi_a^+(\mathbf{k}_a, \mathbf{r}_{xA})|^2, \quad (18)$$

The calculation becomes very transparent if we use eikonal functions for the distorted waves:

$$\Psi_b^- = \exp \left[i \frac{m_A}{m_B} \mathbf{k}_b \cdot \mathbf{r}_{xA} + i \chi_b \left(\frac{m_A}{m_B} \mathbf{r}_{xA} \right) \right], \quad \Psi_a^+ = \exp [i \mathbf{k}_a \cdot \mathbf{r}_{xA} + i \chi_a(\mathbf{r}_{xA})], \quad (19)$$

with the eikonal phases given by

$$\chi_b \left(\frac{m_A}{m_B} \mathbf{r}_{xA} \right) = - \frac{1}{\hbar v_b} \int_{z_{xA}}^{\infty} U_{bA} \left(\frac{m_A}{m_B} r'_{xA} \right) dz'_{xA}, \quad \chi_a(\mathbf{r}_{xA}) = - \frac{1}{\hbar v_a} \int_{-\infty}^{z_{xA}} U_{aA}(r'_{xA}) dz'_{xA}, \quad (20)$$

where $r'_{xA} = \sqrt{b_{xA}^2 + z'_{xA}{}^2}$. Note that, since only the moduli of the Ψ 's enter into Eq. (18), we get

$$\sum_{\mathbf{k}_x} |T|^2 = |C_{bx}|^2 \int d^3 r_{xA} S_a(r_{xA}) S_b(r_{xA}) |U_{ex}(r_{xA})|^2, \quad (21)$$

where

$$S_b(r_{xA}) = \exp \left[\frac{2}{\hbar v_b} \int_{z_{xA}}^{\infty} \text{Im} U_{bA} \left(\frac{m_A}{m_B} r'_{xA} \right) dz'_{xA} \right],$$

$$S_a(r_{xA}) = \exp \left[\frac{2}{\hbar v_a} \int_{-\infty}^{z_{xA}} \text{Im} U_{aA}(r'_{xA}) dz'_{xA} \right], \quad (22)$$

Finally, the cross section for removal of particle x from a is given by

$$\sigma = \frac{m_a m_b k_b}{\pi \hbar^4 k_a} \frac{\sum_{\mathbf{k}_x, \text{spins}} |T|^2}{(2J_A + 1)(2J_a + 1)}, \quad (23)$$

and we shall assume that $v_b \approx v_a$, valid for high-energy collisions and small binding energies of the incident projectile.

It is worth mention that we call Eqs. (21)–(23) the DWBA approximation, and we compare it to the Glauber model described in Sec. II. Eikonal wave functions are well known as proper solutions of the Schrödinger equation for the scattering of high-energy particles. They simplify enormously the numerical calculations, replacing the sum over partial waves by a much simpler integral. One should not confuse the Glauber model with the use of eikonal approximations. The Glauber model describes the high-energy scattering of composite particles in terms of cross sections for its constituents (see Sec. II). Glauber has also shown how the multiple scattering of the constituents (which we will not discuss here) affects the cross sections of the composite objects in the high-energy limit. It is noted that several authors have recently (see, e.g., [6,7]) investigated the inclusive fragmentation cross sections of halo nuclei in the Glauber model for ^{11}Li projectiles.

IV. WAVE FUNCTIONS AND EFFECTIVE INTERACTIONS

The ground-state wave function of ^8B , in a given magnetic substate, M , is taken as

$$\phi_a^{(M)}(\mathbf{r}_{bx}) = \sum_{m, M_A} \langle jm I_x M_x | JM \rangle \phi_{jm}(\mathbf{r}) |I_x M_x\rangle, \quad (24)$$

where $|I_x M_x\rangle$ is the wave function of the ^7Be ($I_x^\pi = 3/2^-$), and $\phi_{j,m}$ is the single-particle wave function of the proton $j^\pi = 3/2^-$, coupled to a total angular momentum $J^\pi = 2^+$. Thus, the potential U_{ex} in Eqs. (15)–(18) and (21) depends on the initial orientation of ^8B and the target, which means that Eq. (23) carries an average over the magnetic substates of these nuclei.

Using the properties of the Clebsch-Gordan coefficients, and the orthogonality of the core wave functions, we get for the spin-averaged potential (and a spin zero target)

$$U_{ex} \equiv \frac{1}{(2J_A + 1)(2J_a + 1)} \sum_{\text{spins}} |U_{ex}(r_{xA})|^2$$

$$= \frac{2}{5} \left| \int d^3 r_{bx} \frac{R_j(r_{bx})}{r_{bx}} Y_{1,0}(\hat{\mathbf{r}}_{bx}) U_{bu}(\mathbf{r}_{xA}, \mathbf{r}_{bx}) \right|^2, \quad (25)$$

where $R_j(r)/r$ is the radial part of the single-particle wave function ϕ_{jm} . The cross section is

$$\sigma = \frac{2m_b^2}{\pi \hbar^4} |C_{bx}|^2 \int db b S_{ab}(b) \mathcal{F}_{ex}(b), \quad (26)$$

where $S_{ab} \equiv S_a S_b$ (we neglect the small dependence of S_{ab} on z) and

$$\mathcal{F}_{ex}(b) = \int_{-\infty}^{\infty} dz U_{ex}(\sqrt{b^2 + z^2}). \quad (27)$$

Now we need to determine the optical potentials to proceed with the calculation. Usually these optical potentials are obtained from elastic scattering experiments. But, for unstable nuclei the situation is quite different. One generally has to construct these optical potentials theoretically from effective nucleon-nucleon interactions. Among these, one of the most popular is the M3Y interaction, which has been shown to work quite reasonably for elastic and inelastic scattering of heavy ions at low and intermediate energy collisions [8,9].

In its simplest form the M3Y interaction is given by two direct terms with different ranges, and an exchange term represented by a δ interaction:

$$t(s) = A \frac{e^{-\beta_1 s}}{\beta_1 s} + B \frac{e^{-\beta_2 s}}{\beta_2 s} + C \delta(\mathbf{s}), \quad (28)$$

where $A = 7999 \text{ MeV}$, $B = -2134 \text{ MeV}$, $C = -276 \text{ MeV fm}^3$, $\beta_1 = 4 \text{ fm}^{-1}$, and $\beta_2 = 2.5 \text{ fm}^{-1}$. The real part of the optical potential is obtained from a folding of this interaction with the ground-state densities of the nuclei:

$$U_{ij}(\mathbf{R}) = \int d^3 r_1 d^3 r_2 \rho_A(\mathbf{r}_1) \rho_j(\mathbf{r}_2) t(s), \quad (29)$$

with $\mathbf{s} = \mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1$. The imaginary part of the optical potential is usually parametrized to be $\text{Im} U = \lambda U_{M3Y}$, with $\lambda = 0.6 - 0.8$.

The M3Y interaction (28) has been modified to account for the energy dependence on the beam energy. However, for the energy range of 5–50 MeV/nucleon, only a small energy dependence was introduced [9] as a variation of the exchange term.

To study the breakup of ^8B projectiles, we will use the form given by Eq. (28) for the M3Y interaction with the ^7Be and ^8B densities as in Sec. I, and a proton Gaussian density of radius equal to 0.7 fm. The radial wave function of the proton, $R_{3/2}$, was obtained in a Woods-Saxon+spin-orbit potential, i.e., $V(r) = V_0 [1 - F_{so}(\mathbf{l} \cdot \mathbf{s})(r_0/r) d/dr] f(r)$, with $f(r) = [1 + \exp((r-R)/a)]^{-1}$ with parameters $V_0 = -44.66 \text{ MeV}$, $a = 0.56 \text{ fm}$, $r_0 = 1.25 \text{ fm}$, $R = 2.391 \text{ fm}$, F_{so}

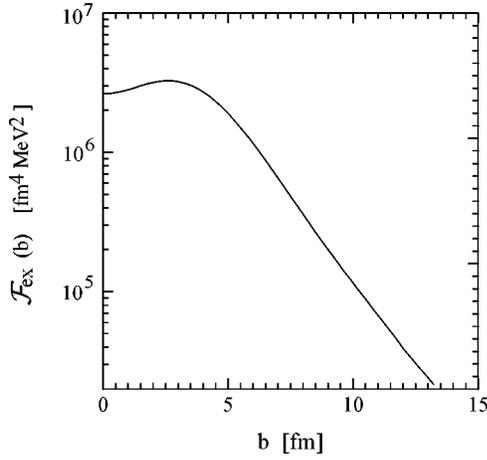


FIG. 2. Excitation function \mathcal{F}_{ex} [see definition in text; Eq. (27)], as a function of the impact parameter.

$=0.351$ fm, which reproduces the binding energy, $\epsilon = 0.138$ MeV, of ${}^8\text{B}$. The spectroscopic factor is taken as unity.

In Fig. 2 we plot the function $\mathcal{F}_{\text{ex}}(b)$, which contains the information not only of the ground-state wave function of the ${}^8\text{B}$, but also on the effective interaction. In Fig. 3 we plot the profile function $S_{ab}(b)$, which depends only on the effective interaction. We have calculated it for the energies $E/A = 30, 150, 300, 800,$ and 1200 MeV, respectively. The magnitude of the cross section is proportional in the area below $S_{ab}(b) \times \mathcal{F}_{\text{ex}}(b)$. Since \mathcal{F}_{ex} does not depend on the beam energy, the energy dependence is solely due to S_{ab} . Since the M3Y interaction does not depend on the energy, the energy dependence is a consequence of the $\hbar v$ factors in the denominators appearing in Eq. (22). This causes the nuclear transparency, described by the factor S_{ab} , to increase for small b as the beam energy increases. As a consequence, the cross sections *increase* with energy. A comparison with the experimental data in Fig. 4 (dotted curve) shows the departure of the calculated cross sections from the experimental data at large energies.

It is clear that we have to modify the effective interaction in Eq. (29) so as to incorporate the energy dependence. A simple way to do that is to make Eq. (29) have the same

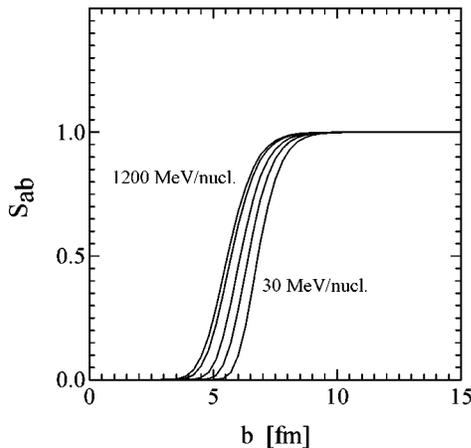


FIG. 3. Nuclear transparency function for the proton removal of ${}^8\text{B}$ projectiles at several energies incident on carbon targets.

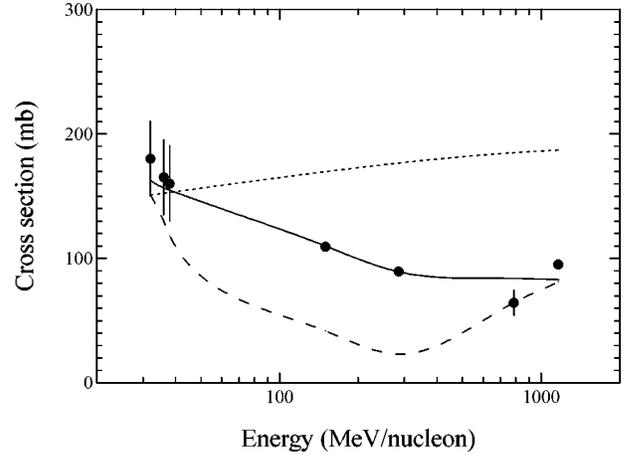


FIG. 4. Proton removal cross sections of ${}^8\text{B}$ projectiles incident on carbon targets as a function of the incident energy. Dotted line is the result of a DWBA calculation with the folding potentials with M3Y interaction. For the dashed line the effective interaction was taken with the same energy dependence as the nucleon-nucleon scattering amplitude. The solid line is the result obtained with a fit for the energy dependence of the breakup potential, different from that of the absorption potential. For details, see the text.

energy dependence as in Eqs. (4) and (5). It should be noticed that the potential U_{ex} in Eq. (16) is not the same as the potentials appearing in the phase of the scattering waves, in the sense that it does not have neither the same magnitude, nor the same spatial dependence. If we take, although it is not necessary, the same energy dependence as in Eq. (4) we can obtain the effective interaction as

$$t(E, s) = -i \frac{\hbar v}{2t_0} \sigma_{NN}(E) [1 - i\alpha(E)] t(s), \quad (30)$$

where $t_0 = 421$ MeV is the volume integral of the M3Y interaction. Note that Eq. (30) gives the same removal cross section as the M3Y interaction for $E = 30$ MeV. Inserting this result [Eq. (30)] in Eq. (29), we can determine the imaginary part of the optical potential automatically.

We repeat the calculation for the proton removal cross sections of ${}^8\text{B}$ using the effective interaction (30) in Eq. (29) and the calculated cross sections by Eq. (26) are shown in Fig. 4 (dashed line). We see that the energy dependence of the cross section changes drastically and follows more closely the trend of the experimental data.

As mentioned above we do not need to assume that the absorption potential and the excitation potential have the same energy dependence. It is reasonable to assume that the absorption potential follows the receipt of Eq. (29), with t given by Eq. (30), since this has the same energy dependence occurring in all calculations based on the Glauber formalism for total reaction cross sections, which are known to agree reasonably with the experimental data. Thus, we change the excitation potential to adjust its energy dependence to the data points. We find out that a simple energy dependence of the form $t(E, s) \propto E^{-0.25} t(s)$, for $E < 200$ MeV, and $U_{\text{ex}} \propto \text{const } t(s)$, for $E \geq 200$ MeV, reproduces the trend of the experimental data, as we show in Fig. 4 by the solid line, with a normalization factor which best fits the data.

V. CONCLUSIONS

In summary, it is found that the energy dependence of the experimental removal cross sections of ${}^8\text{B}$ can be obtained by Glauber model calculations with the nucleon-nucleon t matrix. We have studied also the relation between the Glauber model and the DWBA formalism. The DWBA approach to nucleon removal cross sections in general agrees with Glauber calculations if the optical potential in the elastic channel has the same energy dependence as the breakup potential. For halo nuclei this is not necessarily true, as we have shown for the breakup of the ${}^8\text{B}$ nucleus. This finding might have important consequences, not only for the breakup

of halo nuclei but also for their excitations to bound states. More studies with halo nuclei are needed in order to clarify the role of effective interactions in the construction of optical potentials, and of their connection to nucleon-nucleon scattering amplitudes.

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