

Proton vs neutron halo breakup

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In this paper we show how effective parameters such as effective binding energies can be defined for a proton in the combined nuclear-Coulomb potential, including also the target potential, in the case in which the proton is bound in a nucleus which is partner of a nuclear reaction. Using such effective parameters the proton behaves similar to a neutron. In this way some unexpected results obtained from dynamical calculations for reactions initiated by very weakly bound proton halo nuclei can be interpreted. Namely, the fact that stripping dominates the nuclear breakup cross section which in turn dominates over the Coulomb breakup even when the target is heavy at medium to high incident energies. Our interpretation helps also clarifying why the existence and characteristics of a proton halo extracted from different types of data have sometimes appeared contradictory.

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I. INTRODUCTION

This paper is concerned with the differences which might arise in reactions initiated by a neutron halo nucleus such as ^{11}Be and a proton halo nucleus such as ^{17}F or ^8B . Halo nuclei are a special case of radioactive beams for which the last nucleon is very weakly bound, with separation energies of the order of 0.5 MeV or less, and in a state of low angular momentum ($l=0,1$). They exhibit extreme properties such as very large total and breakup cross sections. Nuclear and Coulomb breakup of neutron halo nuclei have been studied in great detail both experimentally as well as theoretically and are now quite well understood processes [1]. On the other hand proton halo nuclei such as ^8B and ^{17}F are still under investigation. Their behavior as projectiles of nuclear reactions needs to be understood better, in particular, as ^8B is partner in (p,γ) radiative capture reactions of great astrophysical interest for the understanding of the neutrino flux from the sun (see, for example, the discussion and references of Ref. [2]). Also the existence of a proton halo has sometimes been questioned [3] and results from different experiments might seem to be contradictory [4]. For those nuclei Coulomb breakup reactions in the laboratory have been used to get indirect information on the radiative capture, since it has been shown that the Coulomb breakup cross section is proportional to the radiative capture cross section [5].

In the case of neutrons the Coulomb breakup cross section is largest for heavy targets and the interplay with nuclear breakup is well understood both experimentally as well as

theoretically, in particular, thanks to the measurements of angular distributions for both processes [6–9]. Then ^{208}Pb and ^{58}Ni have been used as targets with beams of ^8B or of ^{17}F at various energies [2,10–15]. Data on lighter targets such as ^9Be and ^{28}Si [3,17,18] also exist. At the same time a number of theoretical papers have appeared dealing with the problem of the accuracy necessary to interpret the data [19–23]. In particular, the problems of higher order effects in Coulomb breakup, of the inclusion of $E0$, $E1$, and $E2$ multipolarities in the Coulomb field, and of the relative magnitude of nuclear and Coulomb contributions and of their interference have been discussed at length.

A number of experimental papers [15,18] have shown that for a ^8B projectile it is the nuclear breakup and, in particular, the stripping (or absorption) [17,18] component that dominates the experimental cross section. In Refs. [17,18] a ^{28}Si target was used and different beam energies around 40A MeV were explored. The data of Table I of Ref. [18] show that stripping (110 ± 9 mb) is very close to the total diffraction (112 ± 12 mb) which contains both nuclear and Coulomb components. On the other hand at the same beam energy and on the same target the one-neutron breakup of ^{11}Be , measured in Ref. [24] and calculated in Ref. [25] gave a stripping cross section of 220 mb and a total diffraction of 300 mb of which 120 mb from Coulomb breakup. These results could be considered rather astonishing in view of the fact that the proton in ^8B has a separation energy of 0.14 MeV while the neutron separation energy in ^{11}Be is larger and equal to 0.5 MeV. On the other hand the data of Ref. [15] for the breakup of ^8B on ^{208}Pb at 142A MeV provided a one-proton removal cross section of 744 ± 9 mb of which about 300–450 mb were estimated to be due to nuclear breakup and 311 mb to Coulomb breakup. This is again a surprising result because for the system $^{11}\text{Be}+^{208}\text{Pb}$ at 120A MeV it was calculated in Ref. [9] that the cross

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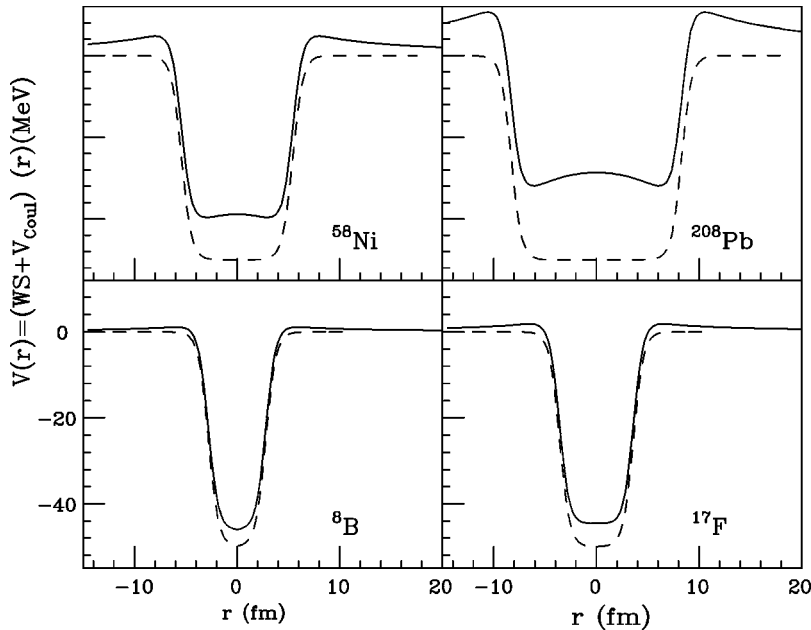


FIG. 1. Nuclear (dashed) and nuclear-Coulomb (solid) potentials for ${}^8\text{B}$, ${}^{17}\text{F}$, ${}^{58}\text{Ni}$, and ${}^{208}\text{Pb}$.

sections would be 321 mb for nuclear breakup and 1050 mb for Coulomb breakup, the model of Ref. [9] being very reliable as it agrees with exclusive data [6]. Similarly, the recent data from GSI [16] at the relativistic beam energy of 936A MeV give for the one-proton removal cross section of ${}^8\text{B}$ on ${}^{208}\text{Pb}$ and ${}^{12}\text{C}$, 662 ± 60 mb and 94 ± 8 mb, respectively, while at a similar energy (790A MeV) the one neutron removal from ${}^{11}\text{Be}$ on the same targets was 960 ± 60 mb and 169 ± 4 , respectively [26].

On the other hand, very recently a new theoretical work has appeared where the authors treat the nuclear breakup of ${}^{17}\text{F}$ to first order [27], contrary to what it has been established in the literature, namely, that halo breakup should be treated to all orders in the neutron-target interaction. An *a posteriori* justification of the approach of Bertulani and Danielewicz [27] is that the calculated nuclear breakup is larger by several orders of magnitude than the Coulomb breakup. In fact the approach of Ref. [27] can be justified with the results of another theoretical work by Esbensen and Bertsch [22] on the proton halo nucleus ${}^8\text{B}$, where it was shown that starting from about 40A MeV in the reaction ${}^8\text{B} + {}^{208}\text{Pb}$, dynamical calculations and first order perturbation theory with or without far field approximation yield nearly the same Coulomb breakup cross sections for distances of closest approach for the core-target trajectory of 20 fm or larger. Also in Ref. [21] the same authors found that for ${}^{17}\text{F}$ nuclear diffraction and Coulomb breakup have very similar probabilities to occur and the values are also close to those for nuclear stripping. An earlier calculation by Esbensen and Hencken [20] showed that nuclear one-proton removal cross sections for a ${}^8\text{B}$ projectile would be larger than Coulomb cross sections up to target mass $A_T=100$. Similar conclusions were reached by Dasso, Lenzi, and Vitturi [28].

In order to get some insight into the peculiarities of the proton halo reactions, in particular, in comparison to neutron halos, we introduce here an effective treatment of proton single particle states which simplifies their treatment and

makes them behaving as neutrons. Related approaches have recently been introduced by other authors [29]. We do not propose our method as opposed to dynamical calculations, but we are simply concerned with the understanding of the underlying physics and the interpretation of numerical results from more sophisticated methods such as direct solutions of the Schrödinger equation or coupled channels.

We show in the following how to treat proton transfer and breakup in a way that is similar to neutron transfer and breakup by using an effective potential in which the weakly bound protons behave as “normally” bound neutrons and then we come to some simple conclusions. The basic idea is that breakup is a kind of “transfer to the continuum” and as such its main features come from matching conditions and Q value effects [30].

II. PROTON VERSUS NEUTRON: EFFECTIVE POTENTIAL

We begin this section by noticing the differences in the treatment of a neutron halo breakup and a proton halo breakup. In Ref. [9] the neutron breakup was studied to all orders in the nuclear and Coulomb fields. The nuclear potential responsible for the neutron transition to the continuum was taken to be the neutron-target optical potential. On the other hand it was shown that Coulomb breakup originates from an effective repulsive force acting on the neutron and due to the core-target Coulomb potential. If we were to extend the same model to proton breakup we should add the two Coulomb interactions of the proton itself with its core in the initial state and with the target in the final state. Now, because of the slow variation of the Coulomb field, we can use the adiabatic approximation or frozen halo [31], for these two Coulomb interactions which make the proton breakup different from the neutron breakup. The detailed derivation of the formalism is given in the Appendix.

The effect of the proton-core and proton-target Coulomb potentials can be understood qualitatively by discussing Fig.

TABLE I. Barrier radii from Fig. 1 and initial binding energies.

	${}^8\text{B}$	J^π	${}^{17}\text{F}$	J^π	${}^{58}\text{Ni}$	${}^{208}\text{Pb}$
$R_{i,f}$ (fm)	6.0		6.5		8.0	10.5
ε_i (MeV)	-0.14	$1p_{3/2}$	-0.6	$1d_{5/2}$		
ε_i^* (MeV)			-0.1	$2s_{1/2}$		

1 which shows the potentials felt by a neutron (dashed line) and a proton (full line) in ${}^8\text{B}$, ${}^{17}\text{F}$, ${}^{58}\text{Ni}$, and ${}^{208}\text{Pb}$. Supposing the two particles have the same binding energy $\varepsilon_i < 0$ the proton wave function inside the potential of Fig. 1 is like a neutron wave function with binding energy $\varepsilon_i - Z_p e^2 / R_i$ up to the radius R_i . The proton potential is like the neutron potential pushed up by $Z_p e^2 / R_i$, where R_i is the barrier radius. For any given nucleus this radius is rather larger than the nuclear or Coulomb radius values usually quoted in the literature. But from Fig. 1 one can see that it is the value corresponding to the barrier peak. We give these values in Table I, together with the experimental binding energies of the halo state in ${}^8\text{B}$ and of two states in ${}^{17}\text{F}$.

But as it is shown in the Appendix, in a scattering process there is also an effect due to the Coulomb potential of the projectile. It can be understood by looking at Fig. 2 which shows the nuclear-Coulomb potentials for ${}^8\text{B} + {}^{58}\text{Ni}$ (top) and ${}^{17}\text{F} + {}^{208}\text{Pb}$ (bottom) at several distances. Short and long dashed lines are the separate projectile and target potentials, respectively. Full line is the projectile-target combined potential. The effect of the target potential on the projectile potential is actually twofold.

(a) The center of the projectile potential shifts up by an amount $Z_T e^2 / d$, where d is the distance of closest approach between the two nuclei.

(b) The height of the barrier on the side near the target goes up by an amount $Z_T e^2 / |d - R_i|$ relative to the center. While on the other side it goes up by $Z_T e^2 / |d + R_i|$.

This suggests that the true binding energy ε_i could be replaced by

$$\varepsilon_i \rightarrow \tilde{\varepsilon}_i = \varepsilon_i - \Delta_i, \quad (1)$$

where

$$\Delta_i = \frac{Z_p e^2}{R_i} + Z_T e^2 \left[\frac{1}{2} \left(\frac{1}{|d + R_i|} + \frac{1}{|d - R_i|} \right) - \frac{1}{d} \right]. \quad (2)$$

$(Z_T e^2 / 2)(1/|d + R_i| + 1/|d - R_i|)$ is the average effect of the target Coulomb potential at the points $r = \pm R_i$ on the left and right sides of the projectile.

In the reactions we are discussing the initial states are always bound. According to Eq. (1) they will be shifted down by a Δ_i . Therefore the phase space for breakup states will be reduced and thus breakup probabilities for protons will be smaller than for neutrons having the same binding energy. Furthermore there will be an important target dependence.

Then we conclude that some features of proton breakup could be understood by analogy with neutron breakup by using effective parameters in the following way.

(a) Use effective $\tilde{\gamma}_i$ calculated from

$$\frac{\hbar^2 \tilde{\gamma}_i^2}{2m} = |\tilde{\varepsilon}_i|. \quad (3)$$

(b) Calculate the normalization constants \tilde{C}_i of asymptotic wave functions [32] as for neutron wave functions with binding energies $\tilde{\varepsilon}_i$.

The approach here corresponds to an adiabatic approximation for the effect of the Coulomb force of one nucleus on the other and it was introduced for the first time in Ref. [33] where it was also shown that it is equivalent to the sudden approximation which was instead discussed in Ref. [34]. We used it already in Ref. [35] to discuss the proton transfer to the continuum reaction ${}^{197}\text{Au}({}^{20}\text{Ne}, {}^{19}\text{F}){}^{198}\text{Hg}$ [36]. The approximation of using effective parameters for protons so that they could be treated similar to neutrons has a long story in direct reaction theories, see, for example, Ref. [37]. However it is direct noticing that the definitions, Eqs. (1), used here represent a generalization and improvement with respect to

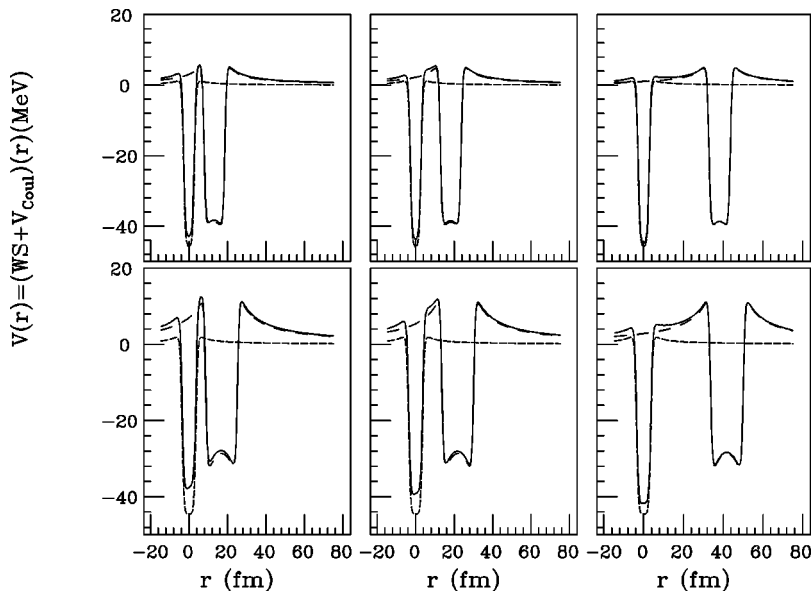


FIG. 2. Nuclear-Coulomb potentials for ${}^8\text{B} + {}^{58}\text{Ni}$ (top) and ${}^{17}\text{F} + {}^{208}\text{Pb}$ (bottom) at distances between the centers equal to $d = 1.4(A_p^{1/3} + A_T^{1/3}) \text{ fm} + s$, with $s = 5, 15, \text{ and } 30 \text{ fm}$. Short and long dashed lines are the projectile and target potentials, respectively. Full line is the projectile-target combined potential.

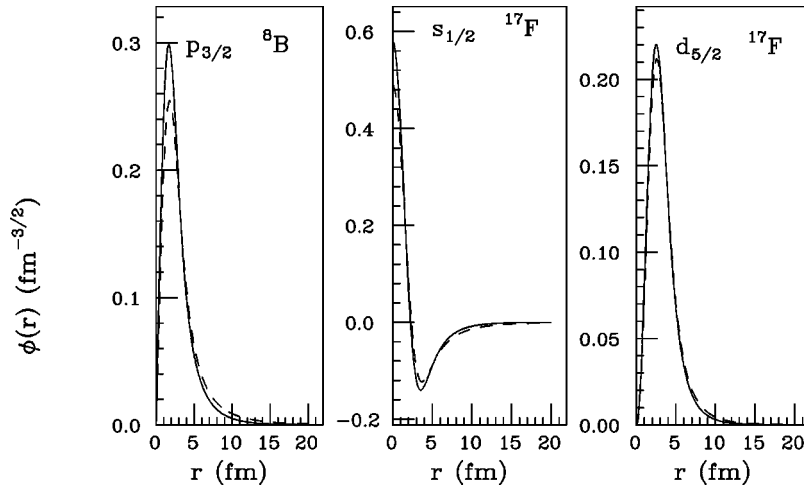


FIG. 3. Proton (dashed) and neutron (solid) wave functions for ^8B , ^{17}F as indicated. Neutron wave functions obtained for effective energies as in Table II, in the case of the ^{58}Ni target.

those used in Refs. [33,34,37]. This is because we not only take into account the effect of the Coulomb barrier in the projectile and target potentials, but also consider the “polarization” effect that the target Coulomb potential has on the projectile and vice versa. As Fig. 2 (bottom part) shows, in the case of a light projectile and a heavy target, the long range effect of the Coulomb potential gives a considerable shift upwards of the projectile potential. The importance of such effects and the meaning of the approach discussed here will be clearer later on when we will discuss Fig. 3 for the wave functions.

From Fig. 2 one sees clearly that the effect of the barrier is very important even at distances as large as 30 fm. In order to quantize the effects discussed above we give in Table I the barrier radii, called R_i and R_f , for two nuclei usually used as projectiles ^8B , ^{17}F , and for ^{58}Ni , ^{208}Pb , which have been used as targets. In this case R_f has been taken as the barrier radius which can be deduced from Fig. 1. In Table II we give the effective energy shift Δ_i , the effective binding energies for the possible projectile-target combinations discussed in this paper. For completeness we add the effective length parameters $\tilde{\gamma}_i$ and asymptotic normalization constants \tilde{C}_i of the initial asymptotic wave functions. It is indeed the tail of the wave function which determines the main characteristics of the breakup mechanism (Ref. [38], and references therein).

We illustrate the last point by Fig. 3 where the dashed lines represent the proton single particle wave functions corresponding to the three initial states of Table I, calculated in

TABLE II. Effective parameters.

	$^8\text{B}+^{58}\text{Ni}$	$^8\text{B}+^{208}\text{Pb}$	$^{17}\text{F}+^{58}\text{Ni}$	$^{17}\text{F}+^{208}\text{Pb}$
Δ_i (MeV)	-1.85	-2.29	-2.7	-3.2
$\tilde{\epsilon}_i$ (MeV)	-1.99	-2.43	-3.3	-3.8
$\tilde{\gamma}_i$ (fm^{-1})	0.29	0.34	0.39	0.42
\tilde{C}_i ($\text{fm}^{-1/2}$)	0.69	0.79	0.75	0.89
$\tilde{\epsilon}_i^*$ (MeV)			-2.8	-3.3
$\tilde{\gamma}_i^*$ (fm^{-1})			0.36	0.39
\tilde{C}_i^* ($\text{fm}^{-1/2}$)			3.06	3.5

a Woods-Saxon plus spin-orbit [38] plus Coulomb potential with parameters: $r_0=1.27$ fm, $a=0.65$ fm, $V_{so}=7$ MeV, $r_c=1.3$ fm. The Woods-Saxon depth is fitted to give the correct binding energy. The solid lines represent the neutron wave functions calculated with the effective binding energies in the case of the ^{58}Ni target. One sees clearly that in each case the true proton wave function is very close to the “effective energy” neutron wave function. We remind the reader that at small distances the breakup is strongly reduced due to the core-target absorption into more complicated reaction channels.

From the values shown one clearly sees that the proton halo behaves in a breakup reaction with a heavy target as a neutron state bound with a “normal” energy of several MeV, for which it is very well known that the nuclear breakup is comparable to the Coulomb breakup and on the other hand that the stripping is dominant on diffraction [39–41].

To give an idea of the orders of magnitude involved, we have calculated total breakup cross sections for two reactions: $^{11}\text{Be} \rightarrow ^{10}\text{Be}+n$ and $^{17}\text{F}(1/2^+) \rightarrow ^{16}\text{O}+p$, both at 40A MeV on a ^{208}Pb target. Nuclear and Coulomb breakup of ^{11}Be have been studied in many experiments on heavy targets and absolute breakup cross sections are very well known [6,7]. For Coulomb breakup we used first order perturbation theory and for nuclear breakup we used the transfer to the continuum model [30,35]. Our aim here is only to give some order of magnitude estimates. For the breakup of ^{17}F we used a neutron wave function and the “effective parameters” of Table II. The values obtained are given in Table III. In the case of ^{11}Be we have used a spectroscopic factor $C^2S=0.77$ for the initial state, while for ^{17}F we have used a unit spectroscopic factor. One sees that for the proton “halo” state in ^{17}F there is a strong reduction of about a factor of 7 for Coulomb breakup and of a factor of 4 for diffraction because both require the neutron to be in a final free particle

TABLE III. Cross sections in mb.

	σ_C	σ_S	σ_D
$^{11}\text{Be}+^{208}\text{Pb}$	2724	312	240
$^{17}\text{F}+^{208}\text{Pb}$	382	131	53

state, which is obviously less probable the stronger the “effective binding” of the nucleon in the initial state. For stripping instead the reduction is just a factor of 3. It is interesting to note that the reduction in the proton removal cross sections from ^{17}F as compared to ^{11}Be calculated here and in Ref. [27] would be stronger than the reduction already seen in the data for ^8B discussed in the Introduction. This is because ^{17}F has a larger Z_p than ^8B and therefore as shown in Fig. 2 and Table II its effective separation energies are larger.

It appears also clear that under such conditions Coulomb and nuclear breakup could not need to be calculated to all orders. Also the effect of the effective parameters introduced here has to be studied in more detail and results should be compared to full dynamical calculations.

III. CONCLUSIONS

In this paper we have tried to draw the attention to the physical origins of the differences in the behavior in a reaction of a proton halo nucleus as compared to a neutron halo. We have shown that if the target is heavy, but also if the projectile is heavier, as in the case of ^{17}F vs ^8B there is an effective barrier which makes the proton “effectively” bound by several MeV, so that some typical halo features might change in breakup reactions. In particular, nuclear breakup and its stripping component could be of comparable magnitude as Coulomb breakup. This could explain the apparent discrepancy in the interpretation in terms of halo structure between data from different types of experiments. Also first order calculations are not completely unjustified. Therefore approaches of the type used in Ref. [27] although not very accurate would give reasonable order of magnitude predictions for weakly bound protons interacting with a heavy target but not for interactions with light targets or in the case of neutron breakup.

It is known that Coulomb breakup on a heavy target can be useful to simulate the (p, γ) reactions of astrophysical interest. However, exclusive measurements need to be done to separate Coulomb from nuclear breakup. Measuring proton angular distributions as done in Ref. [6] for neutron would help disentangling the dominant reaction mechanism, but also separating the large core-target impact parameter contributions as done in Refs. [2,7] is very useful.

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APPENDIX: COULOMB POTENTIALS

We consider the breakup of a proton halo nucleus such as ^{17}F consisting of a proton bound to a ^{16}O core in a collision with a target nucleus. The system of the halo nucleus and the target is described by Jacobi coordinates (\mathbf{R}, \mathbf{r}) where \mathbf{R} is

the position of the center of mass of the halo nucleus relative to the target nucleus and \mathbf{r} is the position of the neutron relative to the halo core, and the coordinate \mathbf{R} is assumed to move on a classical path. This allows target recoil to be included in a consistent way. The Hamiltonian of the system is

$$H = T_R + T_r + V_{pc}(\mathbf{r}) + V_2(\mathbf{R}, \mathbf{r}), \quad (\text{A1})$$

where T_R and T_r are the kinetic energy operators associated with the coordinates \mathbf{R} and \mathbf{r} and V_{pc} is the potential describing the interaction of the proton with the core, and it contains nuclear and Coulomb parts. The potential V_2 describes the interaction between the projectile and the target. It is a sum of two parts depending on the relative coordinates of the proton and the target and of the core and the target

$$V_2(\mathbf{R}, \mathbf{r}) = V_{pt}(\beta_2 \mathbf{r} + \mathbf{R}) + V_{ct}(\mathbf{R} - \beta_1 \mathbf{r}). \quad (\text{A2})$$

Here $\beta_1 = m_p/m_P, \beta_2 = m_c/m_P = 1 - \beta_1$, where m_p is the proton mass, m_c is the mass of the projectile core, and $m_P = m_p + m_c$ is the projectile mass. Both V_{pt} and V_{ct} are represented by complex optical potentials. The imaginary part of V_{pt} describes absorption of the proton by the target to form a compound nucleus. It gives rise to the stripping part of the halo breakup. The imaginary part of V_{ct} describes reactions of the halo core with the target. The potentials V_{pt} and V_{ct} also includes the Coulomb interaction between the proton and the target and the halo core and the target. This part of V_{ct} is responsible for Coulomb breakup.

The mass ratio β_1 is small for a halo nucleus with a heavy core. For example, $\beta_1 \approx 0.06$ and $\beta_2 \approx 0.94$ in the case of ^{17}F . This property is used here to approximate the proton-target and core-target potentials by

$$V_{pt}(\beta_2 \mathbf{r} + \mathbf{R}) \approx V_{pt}(\mathbf{r} + \mathbf{R}), \quad (\text{A3})$$

$$V_{ct}(\mathbf{R} - \beta_1 \mathbf{r}) \approx V_{ct}(\mathbf{R}) + \mathbf{V}_{eff}(\mathbf{r}, \mathbf{R}). \quad (\text{A4})$$

The halo breakup is caused by the direct proton-target interaction V_{pt} or by a recoil effect due to the core-target interaction. Coulomb breakup of a one-proton halo nucleus is mainly a recoil effect due the Coulomb component V_{ct} of the core-target interaction and is contained in $\mathbf{V}_{eff}(\mathbf{r}, \mathbf{R})$. It is proportional to the mass ratio β_1 .

The theory in this paper is based on a time-dependent approach which can be derived from an eikonal approximation. The projectile motion relative to the target is described by a time-dependent classical trajectory $\mathbf{R}(t) = \mathbf{d} + vt\hat{\mathbf{z}}$ with constant velocity v and impact parameter \mathbf{d} ($\hat{\mathbf{z}}$ is a unit vector parallel to the z axis). As discussed in Ref. [9] the main effect of $V_{ct}(\mathbf{R})$ is to give an absorption for small core-target impact parameters and thus it reduces the core survival probability.

Then with the approximations (A3) and (A4) to the potentials the wave function $\phi(\mathbf{r}, \mathbf{d}, t)$ describing the dynamics of the halo proton satisfies the time-dependent equation

$$i\hbar \frac{\partial \phi(\mathbf{r}, \mathbf{d}, t)}{\partial t} = \{H_r + V_{pt}[\mathbf{r} + \mathbf{R}(t)] + \mathbf{V}_{eff}(\mathbf{r}, \mathbf{R}(t))\} \times \phi(\mathbf{r}, \mathbf{d}, t), \quad (\text{A5})$$

where $H_r = T_r + V_{pc}(\mathbf{r})$ is the Hamiltonian for the halo nucleus. In the present paper we neglect the nuclear part of final state interactions between the proton and the halo core, but include the Coulomb proton-core interaction $V_{pc}(\mathbf{r}) = Z_h Z_C e^2 / |\mathbf{r}|$ and the final state interactions between the proton and the target. This approximation should be satisfactory unless there are resonances in the proton-core final state interaction which are strongly excited during the reaction. The proton-core potential does not act dynamically and it cannot cause breakup. It gives the maximum contribution at the top of the proton-core barrier where $|\mathbf{r}| = R_i$. Therefore we take it constant as $V_{pc} = Z_h Z_C e^2 / R_i$.

When the nuclear proton-core final state interactions are neglected we can define a potential

$$\begin{aligned} \bar{V}_2(\mathbf{r}, t) &= V_{pt}^N[\mathbf{r} + \mathbf{R}(t)] + V_{pt}^C[\mathbf{r} + \mathbf{R}(t)] + \mathbf{V}_{eff}(\mathbf{r}, \mathbf{R}(t)) \\ &= V_{pt}^N(\mathbf{r} + \mathbf{R}) + V_{Coul}, \end{aligned} \quad (\text{A6})$$

and V_{pt}^N and V_{pt}^C are the nuclear and Coulomb parts of the proton-target interaction.

Thus in the case of a proton breakup and for a heavy target, besides the proton-target nuclear potential it is necessary to include in the total Hamiltonian the proton-core, proton-target, and core-target Coulomb potentials. The proton-target potential and the core-target potential are included dynamically but the effect on the center of mass has to be subtracted. We have then

$$V_{Coul} = Z_T e^2 \left(\frac{Z_h}{|\mathbf{R} + \beta_2 \mathbf{r}|} + \frac{Z_C}{|\mathbf{R} - \beta_1 \mathbf{r}|} - \frac{Z_C}{|\mathbf{R}|} - \frac{Z_h}{|\mathbf{R}|} \right), \quad (\text{A7})$$

where charges and masses are the following: core (A_C, Z_C), halo (A_h, Z_h), target (A_T, Z_T). We used also two ratios: $\beta_1 = A_h / A_p$ and $\beta_2 = A_c / A_p \approx 1$, with $A_p = A_C + A_h$.

Now we approximate V_{Coul} with something simpler. One approach is to make the dipole expansion. This is quite good for the core-target recoil term because β_1 is small, but is less good for the halo-target term because $\beta_2 \approx 1$. It would be good for large separations when $R \gg r$. Another possibility is to make a dipole approximation rather than a dipole expansion.

It means making an approximation to V_{Coul} which is reasonable over the region of the projectile by writing

$$V_{Coul}(\mathbf{r}, \mathbf{R}) \approx V_{C0} + C^{(D)} \frac{\mathbf{r} \cdot \mathbf{R}}{|\mathbf{R}|^3} \quad (\text{A8})$$

and choosing V_{C0} and $C^{(D)}$ so that, when \mathbf{r} and \mathbf{R} are aligned, the approximation fits the exact expression, Eq. (A7), at $r = \pm R_i$, at the barrier tops on the left and right of the projectile (see Fig. 2).

Thus we put

$$V_{C0} = \frac{1}{2} [V_{Coul}(R_i, R) + V_{Coul}(-R_i, R)]. \quad (\text{A9})$$

In other words V_{C0} is the average of V_{Coul} at the points $r = \pm R_i$, on the left and right sides of the projectile (see Fig. 2). In this way the maximum contribution of the proton-target Coulomb potential $Z_h Z_T e^2 / |\mathbf{R} + \beta_2 \mathbf{r}|$ is taken into account to all orders.

The constant $C^{(D)}$ is chosen so that

$$C^{(D)} \frac{R_i}{R^2} = \frac{1}{2} [V_{Coul}(R_i, R) - V_{Coul}(-R_i, R)]. \quad (\text{A10})$$

In the limit when R is very large the dipole expansion is good and we have $V_{C0} \approx 0, C^{(D)} \approx \beta_1 Z_T Z_C e^2 - \beta_2 Z_T Z_h e^2$. For smaller values of R the approximate form of V_{Coul} fits the exact Coulomb potential near the left and right barriers of the projectile. The constants V_{C0} and $C^{(D)}$ are dependent on R but in the calculations one would use the values at the point of closest approach in the path of relative motion.

This approximation would have several consequences.

(1) There would be an effective binding energy for the initial state

$$\tilde{\varepsilon}_i = \varepsilon_i - \frac{Z_C e^2}{R_i} - V_{C0}. \quad (\text{A11})$$

This is Eq. (1) of the text.

(2) Because the approximate form of V_{Coul} is a dipole field the breakup can be calculated by the methods used for neutron breakup. The only differences would be the effective strengths $C^{(D)}$ and the effective binding energy in the initial state $\tilde{\varepsilon}_i$.

(3) The main conclusion of the present discussion is that the effective binding modifies the halo character of proton breakup reactions.

Then the initial condition that as $t \rightarrow -\infty$ the wave function tends to the initial halo nucleus wave function reads

$$\phi(\mathbf{r}, \mathbf{d}, t) \rightarrow \phi_{lm}(\mathbf{r}, t) = \phi_{lm}(\mathbf{r}) \exp(-i\tilde{\varepsilon}_i t / \hbar) \quad (\text{A12})$$

provided the separation energy is given by Eq. (A11).

Now

$$\tilde{\mathbf{V}}_{eff}(\mathbf{r}, t) = C^{(D)} \frac{\mathbf{r} \cdot \mathbf{R}}{|\mathbf{R}|^3} \quad (\text{A13})$$

and the final proton wave function $\phi_f(t)$ satisfies the equation

$$i\hbar \frac{\partial \phi_f(t)}{\partial t} = [T_r + \tilde{V}_2(\mathbf{r}, t)] \phi_f(t), \quad (\text{A14})$$

with

$$\tilde{V}_2(\mathbf{r}, t) = V_{pt}^N[\mathbf{r} + \mathbf{R}(t)] + \tilde{\mathbf{V}}_{eff}(\mathbf{r}, \mathbf{R}(t)),$$

V_{pt}^N would just be the nuclear part of the proton-target final state interaction and the boundary condition that $\phi_f(t) \sim \exp(i\mathbf{k} \cdot \mathbf{r} - \varepsilon_f t / \hbar)$ when t is large.

The final step is to make an eikonal approximation for $\phi_f(t)$,

$$\phi_f(t) = \exp(i\mathbf{k} \cdot \mathbf{r} - i\varepsilon_f t/\hbar) \exp\left(-\frac{1}{i\hbar} \int_t^\infty \tilde{V}_2(\mathbf{r}, t') dt'\right), \quad (\text{A15})$$

such that we obtain for the amplitude

$$\begin{aligned} \tilde{g}_{lm}(\mathbf{k}, \mathbf{d}) = & \frac{1}{i\hbar} \int d^3\mathbf{r} \int dt e^{-i\mathbf{k}\cdot\mathbf{r} + i\tilde{\omega}t} \exp\left[\frac{1}{i\hbar} \int_t^\infty \tilde{V}_2(\mathbf{r}, t') dt'\right] \\ & \times \tilde{V}_2(\mathbf{r}, t) \phi_{lm}(\mathbf{r}), \end{aligned} \quad (\text{A16})$$

with a new $\tilde{\omega} = (\varepsilon_f - \tilde{\varepsilon}_i)/\hbar$. Equation (A16) is formally the same as the neutron breakup amplitude of Ref. [9] with the difference that the effect of the halo charge has been included in the effective energy $\tilde{\varepsilon}_i$.

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