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Geometry of Borromean halo nuclei

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We discuss the geometry of the highly quantal nuclear three-body systems composed of a core plus two loosely bound particles. These Borromean nuclei have no single bound two-body subsystem. Correlation plays a prominent role. From consideration of the $B(E1)$ value extracted from electromagnetic dissociation, in conjunction with HBT-type analysis of the two valence-halo particles correlation, we show that an estimate of the over-all geometry can be deduced. In particular we find that the opening angle between the two neutrons in ^{6}He and ^{11}Li are, respectively, $\theta_{nn} = 83^{\circ}{}^{+20}_{-10}$ and $66^{\circ}{}^{+22}_{-18}$. These angles are reduced by about 12% to $\theta_{nn} = 78^{\circ}{}^{+13}_{-18}$ and $58^{\circ}{}^{+10}_{-14}$ if the laser spectroscopy values of the rms charge radii are used to obtain the rms distance between the cores and the center of mass of the two neutrons. The opening angle in the case of ^{11}Li is more than 20% larger than recently reported by Nakamura *et al.* [Phys. Rev. Lett. **96**, 252502 (2006)]. The analysis is extended to ^{14}Be and the two-proton Borromean nucleus ^{17}Ne where complete data are still not available. Using available experimental data and recent theoretical calculations we find $\theta_{nn} = 64^{0}{}^{+9}_{-10}$ and $\theta_{pp} = 110^{\circ}$, respectively.

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Borromean nuclei are fragile three-body systems with all two-body subsystems being unbound. Typical examples are ^{6}He, ^{11}Li, and ^{14}Be which are two-neutron Borromean halo isotopes and ^{17}Ne which is a two-proton Borromean halo isotope of neon. The reason that the two-body subsystems are unbound while the three-body system is bound is entirely due to the effective (in-medium) two-nucleon correlations. How strong are these effective two-body correlations? Do they correspond to dinucleon systems, where spatial correlations are maximum, or to some kind of a Cooper correlation, where the two nucleons sit at opposite sides of the core?

From the experimental point of view, the answer to this question could be obtained from a concomitant measurement of the $B(E1)$ values and source size in a Hanbury Brown-Twiss (HBT) type correlation study [1]. We will argue here that this scheme should supply a mean of estimating the average value of the opening angle between the halo nucleons in Borromean nuclei.

In a recent publication, Nakamura *et al.* [2] studied the low lying dipole excitation in ^{11}Li. Their work has had a great impact in this field because new results, showing deviations from previous experimental analysis, have been reported [2]. They also deduced the opening angle between the two neutrons in the halo. By relating their measured $B(E1)$ to the rms value of the distance between the core, ^{9}Li and the center of mass of the two valence neutrons, viz.

$$B(E1) = \frac{3}{\pi}\left(\frac{Z_c}{A}\right)^2 e^2\langle r^2_{c-2n}\rangle \tag{1}$$

$$= \frac{3}{4\pi}\left(\frac{Z_c}{A}\right)^2 e^2\langle r_n^2 + r_{n'}^2 + 2r_n r_{n'}\cos\theta_{nn'}\rangle, \tag{2}$$

and using $r_n = r_{n'}$ obtained from the no-correlation value of $B(E1)$ ($\langle\theta_{nn}\rangle = \pi/2$) given in Ref. [3] using a dipole sum rule value, namely $B(E1) = 1.07\ e^2\text{fm}^2$ [2] obtained for $\langle\theta_{nn}\rangle$ the value

$$\langle\theta_{nn'}\rangle = 48^{\circ}{}^{+14}_{-18}.$$

Notice that the simple relation, Eq. (1), used by Nakamura *et al.* has a very simple interpretation in terms of θ_{NN}. When $\theta_{NN} = \pi$ one gets $B(E1) = 0$. This is because the two valence nucleons lie on opposite sides of the nucleus and the dipole operator vanishes identically due to their same charge-to-mass ratio. On the other hand, it $\theta_{NN} = 0$, i.e., when the valence nucleon wave functions agglomerate close to each other (dineutron), one gets a maximum value of $B(E1)$. Thus, assuming the validity of the three-body model for the Borromean nucleus, without the complications of effective charges, core-polarization, etc., the experimental values of $B(E1)$ are valuable telltales of the nuclear geometry.

A similar procedure can be employed for the other Borromean nuclei when data are available. However, the method of Nakamura *et al.* relies on the use of the no-correlation value of r_n, and thus is heavily model-dependent. Namely, from Ref. [3], one has with $\theta_{nn'} = \pi/2$ (no nn correlation),

$$B(E1) = \frac{3}{4\pi}\left(\frac{Z_c}{A}\right)^2 e^2\langle r_n^2 + r_{n'}^2\rangle = \frac{3}{2\pi}\left(\frac{Z_c}{A}\right)^2 e^2\langle r_n^2\rangle. \tag{3}$$

The above equation supplies a value for $\langle r_n^2\rangle$ if the dipole sum rule (DSR) value of $B(E1)$, $B(E1)_{\text{DSR}}$, is used

$$\langle r_n^2\rangle = \frac{2\pi}{3e^2}\left(\frac{A}{Z_c}\right)^2 B(E1)_{\text{DSR}}. \tag{4}$$

For ^{11}Li $B(E1)_{\text{DSR}} = 1.07\ e^2\text{fm}^2$ [3]. Nakamura *et al.* [2] then used the above value of $\langle r_n^2\rangle$ in Eq. (3), with their experimental value of $B(E1)$, $B(E1)_{\text{Exp}}$, after setting $r_n = r_{n'}$:

$$B(E1) = \frac{3}{4\pi}\left(\frac{Z_c}{A}\right)^2 e^2\langle r_n^2 + r_{n'}^2 + 2r_n r_{n'}\cos\theta_{nn}\rangle$$

$$= \frac{3}{4\pi}\left(\frac{Z_c}{A}\right)^2 e^2[\langle r_n^2 + r_{n'}^2\rangle + \langle 2r_n r_{n'}\cos\theta_{nn}\rangle]$$

$$\simeq \frac{3}{2\pi}\left(\frac{Z_c}{A}\right)^2 e^2\langle r_n^2\rangle[1 + \langle\cos\theta_{nn}\rangle], \quad (5)$$

where the average of the product $\langle r_n r_{n'}\cos\theta_{nn}\rangle$ is approximated by the product of averages

$$\langle r_n r_{n'}\cos\theta_{nn}\rangle \simeq \langle r_n^2\rangle\langle\cos\theta_{nn}\rangle. \quad (6)$$

With Eq. (5), and with the further assumption $\langle r_n^2 + r_{n'}^2\rangle = 2\langle r_n^2\rangle$ and $\langle r_n r_{n'}\rangle = \langle r_n^2\rangle$, we get the Nakamura prescription for determining $\langle\cos\theta_{nn}\rangle$, i.e.,

$$B(E1)_{\rm Exp} = B(E1)_{\rm DSR}[1 + \langle\cos\theta_{nn}\rangle], \quad (7)$$

which gives the value for $\langle\theta_{nn}\rangle = \cos^{-1}\langle\cos\theta_{nn}\rangle$ quoted above.

The above procedure is strongly model-dependent as it relies on only *one* set of experimental observables, $B(E1)$, obtained from Coulomb excitation measurements. Clearly, to reduce the model dependence one needs *more* sets of experimental observables. It is thus very important to seek other observables in order to determine, in a less model dependent way, $\langle\theta_{nn}\rangle$, for Borromean nuclei. In this article we will focus on this endeavor. We avoid the use of Eq. (2) altogether.

In the work of Marques *et al*. [4,5], the two neutron correlation function is measured. This function is defined as

$$C(\mathbf{p}_1, \mathbf{p}_2) = \frac{P_2(\mathbf{p}_1, \mathbf{p}_2)}{P_1(\mathbf{p}_1)P_1(\mathbf{p}_2)}, \quad (8)$$

where $P_1(\mathbf{p}_i)$ is the one-neutron momentum distribution and $P_2(\mathbf{p}_1, \mathbf{p}_2)$ is the two-neutron momentum distribution. The indices 1 and 2 attached to the momenta refer to first and second emitted neutrons. The authors of Refs. [4,5] compared the measured $C(\mathbf{p}_1, \mathbf{p}_2)$ with Eq. (8) with an analytical expression for P_2 extracted from Ref. [6] to account for the case of direct two-neutron independent emission from a Gaussian source. From such analysis approximate, model-dependent, values of $\langle r_{nn}^2\rangle$ were determined for ^{6}He, ^{11}Li, and ^{14}Be. We should stress that the above HBT analysis was based on the use of a simple model of the emission of the two neutrons from a supposed random source. It is not yet clear how large are the coherent effects, and how much these effects would affect the final results of the analysis. Furthermore, the HBT probes the average n-n configuration of the continuum states and not the ground state, as the nucleus is excited above the threshold before the emission occurs. Bearing all of the above in mind, one would only hope to use the HBT results to get, at most, an estimate of the average value of r_{nn}. For a recent review containing, among other things, an account of the difficulties encountered in the correlation measurement and the extraction of r_{nn}, see Ref. [7].

In what follows we will use the HBT study results for the distance between the two neutrons, given by Marques *et al*. [4,5] and show that the opening angle between the two neutrons in ^{11}Li is 25% larger than the above. We also calculate the opening angle for ^{6}He, where full measurement is available [both the $B(E1)$, laser spectroscopy and the HBT analysis] and also supply the value of this angle for ^{14}Be as well as for ^{17}Ne using available data and model calculations. We find using the laser spectroscopy data on the rms values of the charge radii [20–22], $\langle\theta_{nn}\rangle = 58^{\circ}{}^{+10}_{-14}$, $78^{\circ}{}^{+13}_{-18}$, and $64^{\circ}{}^{+9}_{-10}$ for ^{11}Li, ^{6}He, and ^{14}Be, respectively.

For the two-proton Borromean nucleus, ^{17}Ne we use the general cluster formula for the dipole strength function [8]

$$B(E1) = \frac{3}{\pi}Z_{\rm eff}^2\, e^2\langle r_{c-2p}^2\rangle$$
$$= \frac{3}{4\pi}Z_{\rm eff}^2\, e^2\langle r_p^2 + r_{p'}^2 + 2r_p r_{p'}\cos\theta_{pp}\rangle, \quad (9)$$

with $Z_{\rm eff} = (Z_v A_c - Z_c A_v)/A = 2N_c/A$, and obtain

$$\langle\theta_{pp}\rangle = 110^{\circ} \text{ for } ^{17}\text{Ne}.$$

We supply the details of our calculation in what follows.

The experimental analysis of Refs. [4] and [5] can be summarized by giving the average distances between the valence nucleons obtained through two-particle correlations that supplies the size of the source. The obtained values are $\langle r_{nn}\rangle = 6.6$ fm, 5.9 fm, and 5.6 fm, for ^{11}Li, ^{6}He, and ^{14}Be, respectively. From the calculation of [9], one extracts $\langle r_{pp}\rangle = 4.45$ fm.

From the measured $B(E1)$ for ^{6}He [16] and for ^{11}Li [2] and the calculated ones for ^{14}Be [17] and for ^{17}Ne [9], and using Eq. (1), the rms value of y, which is identified as $r_{c\text{-}NN}$, the average distance between the c.m. of the core and the c.m. of the two nucleons, is determined to be 3.36(39), 5.01(32), 4.50, and 1.55 fm, respectively. More accurate values of this latter quantity can be obtained [22] from measurements of the rms charge radii [20,21]. They supply for $r_{c\text{-}NN}$ for ^{6}He [21] and ^{11}Li [20] employing the analysis of [22] the values 3.71(07) and 5.97(22), respectively. Moreover, the rms value of x is the quantity measured in the HBT studies. From these two experimentally determined and theoretically calculated quantities, the opening angle is approximately obtained without resorting to any further model dependence (besides the model dependence of the measured r_{NN}) except for the assumption $r_N = r_{N'}$.

Given $r_{c\text{-}NN}$ and r_{NN}, can one determine the opening angle θ_{NN}? From Fig. 1 it is easy to write

$$\cos\theta_{NN}/2 = \frac{y}{\sqrt{y^2 + x^2/4}}. \quad (10)$$

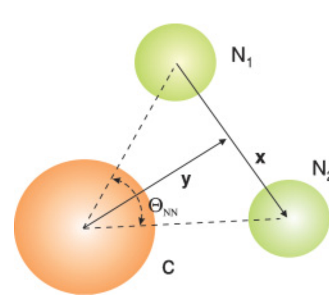

FIG. 1. (Color online) Jacobian coordinates (x and y) for a Borromean nucleus of a core (C) and two nucleons (N_1 and N_2). The average values of the coordinates in the three-body ground state $\sqrt{\langle x^2\rangle} \equiv r_{NN}$ and $\sqrt{\langle y^2\rangle} \equiv r_{C\text{-}2N}$.

The rms value of the cosine above clearly does not correspond to the cosine of the average value of the angle, $\overline{\theta}_{NN}$. This latter can be estimated from

$$\cos \overline{\theta}_{NN}/2 = \frac{r_{c\text{-}NN}}{\sqrt{r^2_{C\text{-}NN} + r^2_{NN}/4}}. \tag{11}$$

The calculation of the rms value of the cosine in Eq. (10) can be performed using the Gaussian model for the source. For our purposes in this paper we use instead Eq. (11) to get the already reported estimates of $\overline{\theta}_{NN}$.

How does our current analysis of the geometry of the ground state of Borromean nuclei bear on the values of the rms matter radii tabulated in [10]? To answer this, we use the formula for the rms radius of a two-cluster nucleus, where the two halo nucleons are treated as an extended entity of radius $r_{NN}/2$,

$$R^2_{\text{rms}} = \left(\frac{A_c}{A}\right) R^2_c + \left(\frac{2}{A}\right)\left(\frac{r_{NN}}{2}\right)^2 + \left(\frac{2A_c}{A^2}\right)(r_{c\text{-}NN})^2. \tag{12}$$

We have used the radii of the cores, $R_{^4\text{He}} = 1.57(4)$, $R_{^9\text{Li}} = 2.32(1)$, $R_{^{12}\text{Be}} = 2.59(6)$, and $R_{^{15}\text{O}} = 2.44(4)$ fm, all taken from [10]. With the values of r_{NN} cited above and $r_{c\text{-}NN}$ from the measured $B(E1)$'s we find $R_{\text{rms}} = 2.67(36)$ fm, 3.17(27) fm, 3.10 fm, and 2.70 fm, for the Borromean nuclei ^{6}He, ^{11}Li, ^{14}Be, and ^{17}Ne, respectively. These values are to be compared to the tabulated ones given in [10], namely, 2.48(3)fm, 3.12(16) fm, 3.16(38) fm, and 2.75(7) fm, respectively. Our results are summarized in Table I. We did not indicate the error bars in the radius of ^{17}Ne since no data are available.

If we use the values of $r_{c\text{-}NN}$ extracted from the rms charge radii of ^{6}He and ^{11}Li (see above) we get for the rms matter radii the values 2.78 and 3.4 fm, respectively. These values are larger than those of [10] but closer to the ones obtained by improved Glauber calculation of the reaction cross sections. For example [26] obtained the value 3.5(6) fm for ^{11}Li.

We should reiterate here a point already mentioned in the paper: the HBT probes the average *n-n* configuration of the continuum states and not the ground state, as the nucleus is excited above the threshold before the emission occurs. It is therefore expected that the values of r_{NN} corresponding to the ground state would be smaller than the ones quoted in the text and the table. This will result in smaller opening angles, perhaps within the range the errors already indicated in the table.

It is worth mentioning here that the opening angles we have obtained for ^{6}He and ^{11}Li are consistent with the the recent three-body pairing calculation of Hagino and Sagawa [18].

Notwithstanding the large size of the error bars in the measured r_{NN} and the small difference (2°) between $\overline{\theta}_{NN}$ for ^{11}Li and ^{14}Be, this implies that there is a gradual increase in the intensity of spatial correlations between the two halo neutrons. The case of ^{17}Ne is quite different; owing to the Coulomb repulsion between the two protons the above trend ceases to operate. This may be traced to the scattering lengths of the two nucleon pairs. For the *nn* case one has the so far accepted value of $a_{nn} = -18.6$ (4) fm [11,12]. Though charge symmetry says that the nuclear (hadronic) value of a_{pp} should be equal to that of a_{nn}, the presence of electromagnetic repulsion and other effects render a_{pp} almost one third of a_{nn}. Precisely [11,13], one has $a_{pp} = -7.8063$ (26) fm. It would be quite interesting to check the above by performing both $B(E1)$ measurement and HBT correlation analysis for the ^{17}Ne two-proton Borromean nucleus. Such an endeavor is currently in the planning stage at the GSI [14]. Due to the long-range Coulomb interaction, the HBT analysis has to be carried out with care for charged particles [19].

TABLE I. The average distance between the two nucleons in the halo and the core-2N average distance shown in the first and second columns, respectively. The values of $r_{c\text{-}2N}$ and the rms radii for ^{6}He and ^{11}Li are obtained both from the $B(E1)$'s values, [16] and [2], and from [20,21] with the help of [22]. The core radii were taken from [10]. The RMS radii for the other two nuclei are tabulated according to Eq. (12)and [10]. Also indicated within parentheses are the compiled values of the Ref. [10]. The $B(E1)$ values were collected from the indicated references. The last column exhibits the values of the opening angle, $\bar{\theta}_{NN}$, calculated from Eq. (11). See text for details.

	r_{NN} (fm)	$r_{c\text{-}2N}$ (fm)	R_{rms} (fm)	$B(E1)$ (e^2fm^2)	$\bar{\theta}_{NN}$
^{6}He	5.9±1.2 [4]	3.36 (39) [16]	2.67 (2.48)	1.20 (20) [16]	$83^{\circ+20}_{-10}$
		3.71(07) [21]	2.78		$78^{\circ+13}_{-18}$
^{11}Li	6.6±1.5 [4]	5.01 (32) [2]	3.17 (3.12)	1.42 (18) [2]	$66^{\circ+22}_{-18}$
		5.97(22) [20]	3.4		$58^{\circ+10}_{-14}$
^{14}Be	5.60±1.0 [5]	4.50 [17]	3.10 (3.16)	1.69* [17]	$64^{\circ+9}_{-10}$
^{17}Ne	4.45 [9]	1.55 [9]	2.70 (2.75)	1.56* [9]	110°

It is tempting to compare our finding for the opening angle between the two halo protons in ^{17}Ne with the opening angle between the two hydrogen atoms in the water molecule H_2O. This latter angle is quite well known and its value is $\theta_{HH} = 104.45^\circ$, almost equal the nuclear counterpart, θ_{pp}. In H_2O, $r_{O-2H} = 78.15$ pm and $r_{HH} = 247.33$ pm (picometer). Though the physics is different, we believe that several universal properties may be common in these quantum three-body systems [23], one of which is the Efimov effect; the limit of infinite s-wave scattering length of at least one of the two-body subsystems. This allows for the existence of infinite number of three-body bound states close to the two-body threshold even in the absence of two-body bound states. Such states have been experimentally observed as giant recombination resonances that deplete the Bose-Einstein condensate in cold Cs gases [24]. In our present case we are finding a similarity in the three-body geometry of H_2O and ^{17}Ne (p_2O) which lures us to call ^{17}Ne the nuclear "water" molecule.

In conclusion we have supplied an estimate of the geometry of the Borromean nuclei, ^{6}He, ^{11}Li, ^{14}Be, and ^{17}Ne using available values of $B(E1)$ and the average distance between the valence nucleons supplied by two-particle correlation HBT-type analysis. We have found that the opening angle between the valence nucleons seems to evolve in a decreasing fashion as the mass of the system increases in the case of two-neutron Borromean nuclei. This conclusion is however not definite as it is hampered by the error bars in the measured values of r_{NN} [4,5]. In the case of the two-proton Borromean halo nucleus ^{17}Ne, the opening angle was found to be 110°, large enough to suggest that the *pp* subsystem in this nucleus is close to be a Cooper pair [15], in contrast to the *nn* subsystems in the two-neutron Borromean nuclei referenced above, where the corresponding *nn* opening angles were found to be much smaller. After completing a first version of this paper, we became aware of a similar work completed quite recently by Hagino and Sagawa [25]. They deduced opening angles for ^{6}He, ^{11}Li close to ours.

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