Electron screening and its effects on big-bang nucleosynthesis

Biao Wang* and C. A. Bertulani†
Department of Physics, Texas A&M University, Commerce, Texas 75429, USA

A. B. Balantekin‡
Physics Department, University of Wisconsin-Madison,1150 University Avenue, Madison, Wisconsin 53706, USA
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We study the effects of electron screening on nuclear reaction rates occurring during the big-bang nucleosynthesis epoch. The sensitivity of the predicted elemental abundances on electron screening is studied in detail. It is shown that electron screening does not produce noticeable results in the abundances unless the traditional Debye-Hückel model for the treatment of electron screening in stellar environments is enhanced by several orders of magnitude. This work rules out electron screening as a relevant ingredient to big-bang nucleosynthesis, confirming a previous study [see Itoh *et al.*,Astrophys. J. **488**, 507 (1997)] and ruling out exotic possibilities for the treatment of screening beyond the mean-field theoretical approach.

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During the big bang, the Universe evolved very rapidly, and only the lightest nuclides (e.g., D, ^{3}He, ^{4}He, and ^{7}Li) could be synthesized. The abundances of these nuclides are probes of the conditions of the Universe during the very early stages of its evolution, i.e., the first few minutes. The conditions during the big-bang nucleosynthesis (BBN) are believed to be well described in terms of standard models of cosmology and particle physics, which determine the values of, e.g., temperature, nucleon density, expansion rate, neutrino content, neutrino-antineutrino asymmetry, etc. Deviations from the BBN test the parameters of these models and constrain nonstandard physics or cosmology that may alter the conditions during BBN [1,2]. Sensitivity to several parameters and physics input in the BBN model have been investigated thoroughly in the past (see, e.g., [3–8]). In this paper, we consider whether the screening by electrons would have any impact on the BBN predictions. This paper reinforces the conclusions presented in Ref. [9], namely, that screening is not a relevant ingredient of the BBN.

Modeling the BBN and stellar evolution requires that one include the information on nuclear-reaction rates $\langle\sigma v\rangle$ in reaction network calculations, where σ is the nuclear-fusion cross section and v is the relative velocity between the participant nuclides. Whereas v is well described by a Maxwell-Boltzmann velocity distribution for a given temperature T, the cross section σ is taken from laboratory experiments on Earth, some of which are not as well known as desired [3–7]. The presence of atomic electrons in laboratory experiments also influences the measured values of the cross sections in a rather unexpected way (see, e.g., [10]). In the network calculations for the description of elemental synthesis in the BBN or in stellar evolution, one needs to account for the differences of the "bare" nuclear cross sections obtained in laboratory measurements $\sigma_b(E)$ to the corresponding quantities in stellar interiors $\sigma_s(E)$. One of these corrections is caused by stellar electron screening, as light nuclei in the stellar environments are almost completely ionized.

Using the Debye-Hückel model, Salpeter [11] showed that stellar electron screening enhances cross sections, reducing the Coulomb barrier that reacting ions must overcome, yielding an enhancement factor

$$f(E)=\frac{\sigma_s(E)}{\sigma_b(E)}. \tag{1}$$

The Debye-Hueckel model used by Salpeter yields a screened Coulomb potential, valid when $\langle V\rangle \ll kT$ (weak screening),

$$V(r)=\frac{e^2Z_i}{r}\exp\left(-\frac{r}{R_D}\right), \tag{2}$$

which depends on the ratio of the Coulomb potential at the Debye radius R_D to the temperature

$$f = \exp\left(\frac{Z_1\, Z_2\, e^2}{R_D kT}\right) = \exp\left(0.188\, Z_1\, Z_2\, \zeta\, \rho^{1/2}\, T_6^{-3/2}\right), \tag{3}$$

where

$$\zeta R_D=\left(\frac{kT}{4\pi e^2 n}\right)^{1/2} \tag{4}$$

is the Debye radius, n is the ion number density, ρ is a dimensionless quantity measured in units of g cm^3,

$$\zeta=\left(\sum_i X_i\frac{Z_i^2}{A_i}+\chi\sum_i X_i\frac{Z_i}{A_i}\right)^{1/2}, \tag{5}$$

where X_i is the mass fraction of nuclei of type i, and T_6 is the dimensionless temperature in units of 10^6 K. The factor χ corrects for the effects of electron degeneracy [11].

Corrections to the Salpeter formula are expected at some level. Nonadiabatic effects have been suggested as one source, e.g., a high Gamow energy allows reacting nuclei having velocities significantly higher than the typical ion velocity,

*biao_wang@leo.tamu-commerce.edu
†carlos_bertulani@tamu-commerce.edu
‡baha@physics.wisc.edu

so the response of slower plasma ions might be suppressed. Dynamic corrections were first discussed by [12] and later studied by [13]. Subsequent work showed that Salpeter's formula would be valid independent of the Gamow energy as a result of the nearly precise thermodynamic equilibrium of the solar plasma [14–16]. A number of contradictions were pointed out in later investigations claiming larger corrections, and a field-theoretic approach was shown to lead to the expectation of only small (∼4%) corrections to the standard formula for solar conditions [17].

Controversies about the magnitude of the screening effect have not entirely died out and have continued in some works [18–21]. These works are invariably based on molecular-dynamics simulations. Dynamic screening becomes important because the nuclei in a plasma are much slower than the electrons and are not able to rearrange themselves as quickly around faster moving ions. Since nuclear reactions require energies several times the average thermal energy, the ions that are able to engage in nuclear reactions in the stars are such faster moving ions, which therefore may not be accompanied by their full screening cloud. In fact, dynamic effects are important when particles react with large relative velocities. It appears that pairs of ions with greater relative velocities experience less screening than pairs of particles with lower relative velocities [18]. The correction for dynamical screening can be approximated by replacing R_D in Eq. (3) with a velocity-dependent quantity $R_D(v_p) = R_D\sqrt{1+\mu v_p^2/kT}$, where μ is the ion-pair reduced mass and v_p is their relative velocity [22]. This effect is relevant in stellar environments whenever nuclear reactions occur at energies that are greater than the thermal energy.

Experimentalists have exploited surrogate environments to test our understanding of plasma screening effects. For example, screening in $d(d,p)t$ has been studied for gaseous targets and for deuterated metals, insulators, and semiconductors [23,24]. It is believed that the quasifree valence electrons in metals create a screening environment quite similar to that found in stellar plasmas. The experiments in metals seem to have confirmed important predictions of the Debye model, such as the temperature dependence $U_e(T) \propto T^{-1/2}$ [24]. However, there are still controversies on the validity of the experimental analysis and the use of the Debye screening, or Salpeter formula, to describe the experiments [25–27].

A good measure of the screening effect is given by the screening parameter $\Gamma = Z_1Z_2e^2/\langle r\rangle kT$, where $\langle r\rangle = n^{-1/3}$. In the core of the Sun, densities are of the order of $\rho \sim 150$ g cm^3 with temperatures of $T \sim 1.5 \times 10^7$ K. For pp reactions in the Sun, we thus get $\Gamma \sim 1.06$, which validates the weak screening approximation; for p^7Be reactions, one gets $\Gamma_{p^7\mathrm{Be}} \sim 1.5$, which is one of the reasons to support modifications of the Salpeter formula. Also, in the Sun, the number of ions within a sphere of radius R_D (Debye sphere) is of the order of $N \sim 4$. As the Debye-Hueckel approximation is based on the mean-field approximation, i.e., for $N = n(4\pi R^3/3) \gg 1$, deviations from the Salpeter approximation are justifiable.

Based on the discussion above, there is a possibility that the screening enhancement factor [Eq. (1)] could appreciably differ from the Salpeter formula under several circumstances,

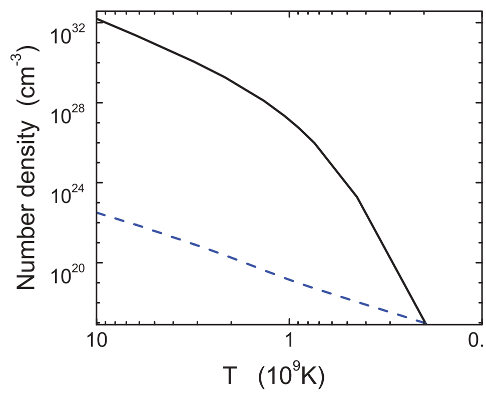

FIG. 1. (Color online) Number density of electrons (solid curve) and of baryons (dashed curve) during the early Universe as a function of the temperature in units of billion degrees Kelvin, T_9.

leading to non-negligible changes in the BBN and stellar evolution predictions. The purpose of this paper is to verify under which conditions this statement would be true.

In this paper, the BBN abundances were calculated with a modified version of the standard BBN code derived from Wagoner, Fowler, and Hoyle [2] and Kawano [28,29] (for a public code, see [30]).

The electron density during the early Universe varies strongly with the temperature as seen in Fig. 1, where T_9 is the temperature in units of 10^9 K (T_9). This can compared with the electron number density at the center of the Sun, $n_e^{\mathrm{Sun}} \sim 10^{26}$ cm^3. Figure 1 shows that, at typical temperatures $T_9 \sim 0.1 - 1$ during the BBN, the Universe had electron densities that are much larger that the electron density in the Sun. However, in contrast to the Sun, the baryon density in the early Universe is much smaller than the electron density. The large electron density is due to the e^+e^- production by the abundant photons during the BBN.

The baryonic density is best seen in Fig. 2. It varies as

$$\rho_b \simeq hT_9^3, \tag{6}$$

where h is the baryon density parameter [31]. It can be calculated by using Eq. (3.11) of Ref. [31] and the baryon-to-photon ratio $\eta = 6.19 \times 10^{-10}$ at the BBN epoch (from WMAP data [32]). Around $T_9 \sim 2$, there is a change of the value of h from $h \sim 2.1 \times 10^{-5}$ to $h \sim 5.7 \times 10^{-5}$. Equation (6) along with the two values of h are shown as dashed lines in Fig. 2.

In Ref. [9], a theory was developed to show how the abundant e^+e^- pairs during the BBN, for temperatures below the neutrino decoupling ($T_9 \sim 0.7$ MeV), result in modifications in the Salpeter formula. In fact, only a very small fraction of the electrons present in the medium is needed to neutralize the charge of the protons. The majority of the electrons are accompanied by the respective positrons created via $\gamma\gamma \to e^+e^-$, so the total charge of the Universe is zero. At the decoupling temperature, the neutron density is only about 1/6 of the total baryon density. Thus, the ion charge density (proton density) at this epoch is approximately equal to the baryon density. With this assumption, the enhancement factor

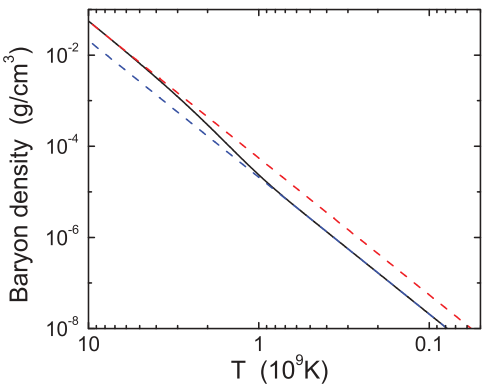

FIG. 2. (Color online) Baryon density (solid curve) during the early Universe as a function of the temperature in units of billion degrees Kelvin, T_9. The dashed curves are obtained from Eq. (6) with $h \sim 2.1 \times 10^{-5}$ and $h \sim 5.7 \times 10^{-5}$, respectively.

in Eq. (3) becomes independent of the temperature:

$$f^{\mathrm{BBN}} = \exp(4.49 \times 10^{-8} \zeta Z_1 Z_2) \sim 1 + 4.49 \times 10^{-8} \zeta Z_1 Z_2 \quad \text{for } T_9 \lesssim 1 \qquad (7)$$

and

$$f^{\mathrm{BBN}} = \exp(2.71 \times 10^{-8} \zeta Z_1 Z_2) \sim 1 + 2.71 \times 10^{-8} \zeta Z_1 Z_2 \quad \text{for } T_9 \gtrsim 2, \qquad (8)$$

which yield small screening corrections for all known nuclear reactions.

It is also worthwhile to calculate the Debye radius as a function of the temperature. This is shown in Fig. 3, where we plot Eq. (4) with the ion density equal to the proton density. The accompanying dashed lines correspond to the approximation of Eq. (6), with $h \sim 2.1 \times 10^{-5}$ and $h \sim 5.7 \times 10^{-5}$. This leads to two straight lines in a logarithmic plot of

$$R_D = R_D^{(0)} T_9^{-1}, \qquad (9)$$

with $R_D^{(0)} \sim 6.1 \times 10^{-5}$ cm and $R_D^{(0)} \sim 3.7 \times 10^{-5}$ cm, respectively. In Fig. 3, we also show the inter-ion distance by the lower dashed line. It is clear that the number of ions inside the Debye sphere is at least of the order of 10^3, which would justify the mean-field approximation for the ions. In contrast to protons, electrons and positrons are mostly relativistic and their chaotic motion will probably average out the effect of screening around the ions. However, because the number density of electrons is large, an appreciable fraction of them still carry velocities comparable to those of the ions. The effects of such electrons on the modification of the Debye-Hueckel scenario might be worthwhile to investigate theoretically.

Based on the above arguments, the screening by electrons during the BBN is likely a negligible effect. However, one needs to verify arguments that sensitive quantities such as the Li/H ratio predicted by BBN might be impacted. This ratio is very small, but is one of the major problems for the big-bang predictions. In fact, there are discrepancies between the BBN theory and observation for the lithium isotopes ^{6}Li and ^{7}Li. These discrepancies are substantiated by recent observations of metal-poor halo stars [33] and the high precision measurement of the baryon-to-photon ratio η of the Universe by WMAP [32,34,35]. In view of the relevance of this topic to BBN and its predictions, it is worthwhile to check the influence of the electron screening on the elemental abundances.

The BBN is sensitive to certain parameters, including the baryon-to-photon ratio, number of neutrino families, and the neutron decay lifetime. We use the values $\eta = 6.19 \times 10^{-10}$, $N_\nu = 3$, and $\tau_n = 878.5$ s for the baryon-photon ratio, number of neutrino families, and neutron-day lifetime, respectively. We have included all reactions of the standard BBN model using the values of the cross sections as published in Ref. [3]. The reaction rates were modified to include screening factors calculated with Eq. (7).

In order to test under which circumstances the screening by electrons would make an appreciable impact on the BBN predictions, we have artificially modified the Salpeter formula for f^{BBN} by rescaling it with a fudge factor w, i.e.,

$$f' = \exp\left(w \frac{Z_1 Z_2 e^2}{R_D k T}\right), \quad \text{or} \quad \ln f' = w \ln f, \qquad (10)$$

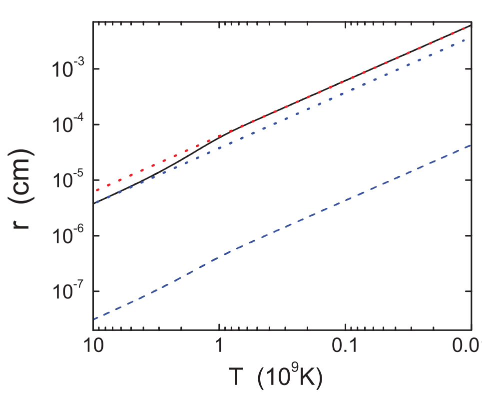

FIG. 3. (Color online) Debye radius during the BBN as a function of the temperature in units of billion degrees Kelvin (solid line). The dotted lines are the approximation given by Eq. (9) with $R_D^{(0)} \sim 6.1 \times 10^{-5}$ cm and $R_D^{(0)} \sim 3.7 \times 10^{-5}$ cm, respectively. The inter-ion distance is shown by the isolated dashed line.

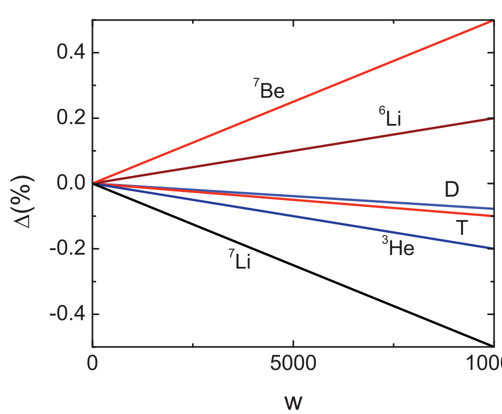

FIG. 4. (Color online) Variation (in percent) of the abundances of several light nuclei as a function of a multiplicative factor w artificially enhancing Salpeter's formulation of screening. w is defined in Eq. (10).

with w varying between 1 and 10^4. We quantify the dependence of BBN on the electron screening by the quantity

$$\Delta = \frac{Y' - Y}{Y}, \tag{11}$$

where Y denotes the abundance Y of an element produced during the BBN. The primed quantities Y' denote the abundances modified by the screening effect.

In Fig. 4, we show the value of Δ (in percent) as a function of the screening enhancement factor w in Eq. (7). For $w \sim 1$, any of the abundances are modified by less than 10^{-5} in the case wherein the screening factor is calculated according to Eq. (7). If the w is increased, the abundances of ^{6}Li and ^{7}Be increase, while those of D, T, ^{3}He, and ^{7}Li decrease as w increases. This shows that the elemental abundance ratios have a different sensitivity to the electron screening. These calculations also make it evident that the relative abundances in the BBN would be somewhat modified only if the electron screening would be enormously enhanced (by a factor w larger than 10^4) compared to the prediction of Salpeter's model.

In conclusion, using a standard numerical computation of the BBN, we have shown that electron screening can not be a source of measurable changes in the elemental abundance. This is verified by artificially increasing the screening obtained by traditional models [11]. We back our numerical results with very simple and transparent estimates. This is also substantiated by the mean-field calculations of screening as a result of the more abundant free e^+e^- pairs published in Ref. [9]. They conclude that screening due to free pairs might yield a 0.1% change on the BBN abundances. However, even if mean-field models for electron screening were not reliable under certain conditions, which we have discussed thoroughly in the text, it is extremely unlikely that electron screening might have any influence on the predictions of the standard big-bang nucleosynthesis.

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