Color van der Waals Force Acting in Heavy-Ion Scattering at Low Energies

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The influence of the color van der Waals force in the elastic scattering of ^{208}Pb on ^{208}Pb at sub-barrier energies is studied. The conspicuous changes in the Mott oscillation found here are suggested as a possible experimental test.

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Several theoretical investigations have considered the possible existence of strong color van der Waals forces between hadrons.[1-6] Just as the usual electromagnetic van der Waals (VDW) force between neutral atoms arises from two-photon exchange, the color VDW force is suggested by several theorists to come about from two-gluon and multigluon exchanges between color-singlet hadrons. There are, of course, important differences between the QCD and the color VDW forces. The most notable of these differences are related to confinement (in the MIT bag model this force does not exist) and the nonlinear structure of the Yang-Mills gluon fields. Although the overwhelming majority of particle physicsts do not believe in the existence of the color VDW force, it is still important to set experimental limits on these forces. Estimates of the strength of the color VDW force have been made. We summarize these in the following:[4,5]

$$V^{C,6}_{\rm VDW}(r) = -\alpha_6 \frac{\hbar c}{r_0}\left(\frac{r_0}{r}\right)^6, \tag{1a}$$

$$V^{C,7}_{\rm VDW}(r) = -\alpha_7 \frac{\hbar c}{r_0}\left(\frac{r_0}{r}\right)^7, \tag{1b}$$

where $r_0 \approx 1$ fm, $\alpha_6 \cong 8$, and $\alpha_7 \cong 100$.

The phenomenological consequences of the existence of the color VDW interaction have been looked for in different systems and the results are not conclusive.[4] The purpose of the present paper is to suggest a physical system where the presumed color VDW is enhanced and consequently its effect on the system can be less ambiguously studied. Specifically, owing to the fact that in the collision between two heavy nuclei the effective interaction is obtained by double folding hadronic densities with the effective nucleon-nucleon interaction, any color VDW force between the constituent hadrons, such as (1a) or (1b), leads to a color nucleus-nucleus VDW force which at large separations goes as $A_1A_2\times$[(1a) or (1b)], where A_i is the atomic number of nucleus i. We therefore propose to look for the color VDW force in the low-energy scattering of ^{208}Pb+^{208}Pb. By low energy we mean low enough to avoid the action of the strong short-range nuclear interaction. At these energies, the Coulomb repulsion completely dominates the scattering and consequently the cross section is structureless and almost entirely Rutherford. Small perturbations such as the color VDW force have to be looked for in quantum interference effects which would arise from, e.g., the identity of the projectile and the target. Thus our choice of ^{208}Pb+^{208}Pb. We proceed now to describe our calculation.

We first remind the reader that other small effects, besides the VDW interaction, have to be taken into account. These include QED vacuum polarization $V_{\rm VP}$ (Uehling potential[7]), nuclear dipole and quadrupole polarizabilities V_D and V_Q, respectively,[8-10] electron screening $V_{\rm ES}$,[11] and relativistic corrections V_R arising from using the Darwin Hamiltonian.[10] These effects have been investigated experimentally by Lynch *et al.*[12] in the system ^{16}O+^{208}Pb. The sum of all these corrections and the VDW one is denoted by $\Delta V \equiv \sum_I V_I$. Thus the interaction potential felt by the two nuclei is

$$V_{A_1A_2}(r) = Z_1Z_2e^2/r + \Delta V(r)\,. \tag{2}$$

The Coulomb barrier of Pb+Pb is about 600 MeV. We take for the c.m. energy 500 MeV as a representative case. At this energy the classical distance of closest approach is 25 fm, at which the short-range nuclear interaction is negligible. So we are fully justified in taking (2) for an interaction.

At 500 MeV, the Sommerfeld parameter $\eta = Z_1Z_2e^2/\hbar v$ is about 480 which is large enough to guarantee that the semiclassical description of the scattering is quite adequate.[8-10] With the aid of the stationary-phase approximation, we obtain for the elastic-scattering amplitude

$$f(\theta) = \frac{1}{k\sqrt{\sin\theta}}\left(\frac{\lambda_i}{|\Theta'(\lambda_i)|}\right)^{1/2} \exp[2i\delta(\lambda_i) - i\lambda_i\theta]\left(\frac{d\sigma_{\rm cl}}{d\Omega}\right)^{1/2} \exp[2i\delta(\lambda_i) - i\lambda_i\theta]\,, \tag{3}$$

where

$$\frac{d\sigma_{\rm cl}}{d\Omega} \equiv \frac{1}{k\sin\theta}\frac{\lambda_i}{|\Theta'(\lambda_i)|}$$

is the classical cross section, $\delta(\lambda_i)$ is the total phase shift, $\lambda_i = l_i + \frac{1}{2}$, with l_i being the orbital angular momentum of the stationary phase defined by

$$2\frac{d}{d\lambda}\delta(\lambda)\bigg|_{\lambda_i} \equiv \Theta(\lambda_i) = \theta ,$$

and Θ is the classical deflection function. In Eq. (3), $\Theta' = d\Theta/d\lambda|_{\lambda_i}$.

The phase $\delta(\lambda_i)$ is written as

$$\delta(\lambda_i) = \sigma(\lambda_i) + \Delta\delta \equiv \sigma(\lambda_i) + \sum_j \Delta\delta_j(\lambda_i) , \qquad (4)$$

where σ is the Coulomb phase and $\Delta\delta_j(\lambda_i)$ is the change in the phase due to the perturbing potential V_j. The classical cross section $d\sigma_{\rm cl}/d\Omega$ can be calculated to first order in $\Delta\Theta \equiv 2d(\Delta\delta)/d\lambda$ as[8-10]

$$\frac{d\sigma_{\rm cl}}{d\Omega} = \frac{d\sigma_{\rm Ruth}}{d\Omega}\left[1 + \frac{1}{2}\Delta\theta\tan\frac{\theta}{2} + \frac{3}{2}\Delta\theta\cot\frac{\theta}{2} - \frac{d}{d\theta}\Delta\theta\right], \qquad (5)$$

where $d\sigma_{\rm Ruth}/d\Omega \equiv a^2/(4\sin^4\frac{1}{2}\theta)$ is the Rutherford cross section and $a \equiv Z_1Z_2e^2/2E_{\rm c.m.}$. With the above preliminaries, we can finally write the symmetrized cross section in the following form:

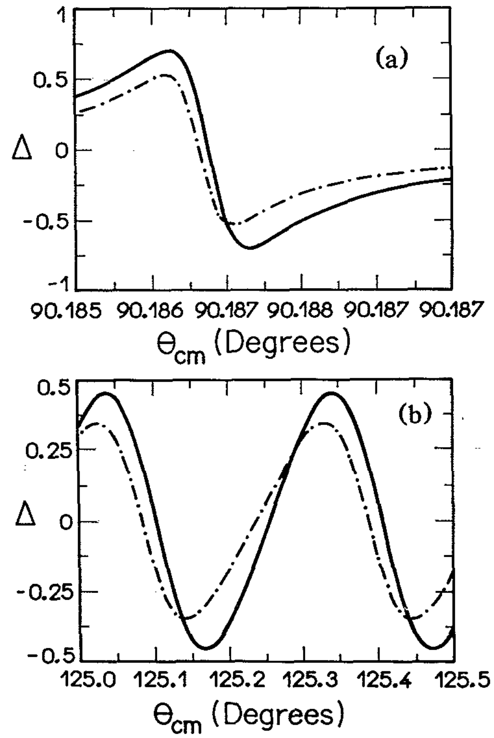

FIG. 1. Calculated $(\sigma - \sigma_{\rm Mott})/(\sigma + \sigma_{\rm Mott})(\theta) \equiv \Delta$ for $^{208}Pb + ^{208}Pb$ at $E_{\rm c.m.} = 500$ MeV. The solid curve includes all effects including $V^{C7}_{\rm VDW}(r)$. The dash-dotted curve corresponds to no color van der Waals force. (a) The 90° region and (b) the 125° region.

$$\frac{d\sigma}{d\Omega} = |f(\theta) + f(\pi-\theta)|^2 = \frac{d\sigma_{\rm cl}}{d\Omega}(\theta) + \frac{d\sigma_{\rm cl}}{d\Omega}(\pi-\theta) + 2\left[\frac{d\sigma_{\rm cl}}{d\Omega}(\theta)\frac{d\sigma_{\rm cl}}{d\Omega}(\pi-\theta)\right]^{1/2} \times\cos\left[2\eta\ln\cot\frac{\theta}{2} + 2[\Delta\delta(\lambda_i(\theta)) - \Delta\delta(\lambda_i(\pi-\theta))]\right]. \qquad (6)$$

In obtaining the phase in (6) the formula[13]

$$2\sigma(\lambda_i(\theta)) - 2\sigma(\lambda_i(\pi-\theta)) = 2\eta\ln\cot\frac{\theta}{2}$$

has been used (this formula is valid as long as $\eta^{-1} \ll 1$).

The numerical results presented below are obtained by calculating $\Delta\sigma$ using first-order perturbation theory, which should be quite adequate considering the smallness of the perturbations involved. The quantity

$$\Delta \equiv \frac{d\sigma/d\Omega - d\sigma_{\rm Mott}/d\Omega}{d\sigma/d\Omega + d\sigma_{\rm Mott}/d\Omega}$$

is then evaluated with $d\sigma_{\rm Mott}/d\Omega$ being the Mott cross section. In Fig. 1 we summarize our results for $E_{\rm c.m.} = 500$ MeV which is slightly below the Coulomb barrier, in both the 90° and the 125° angle regions.

For $V^{C6}_{\rm VDW}(r)$, Eq. (1a), with $\alpha_6 = 8.0$, the effect is extremely small (not shown in Fig. 1). However, when $V^{C7}_{\rm VDW}(r)$, Eq. (1b), is used with $\alpha_7 = 100.0$, the effect on the Mott cross section is quite conspicuous, as can be seen in Fig. 1(a), where the shift in the position of the minima ξ due to the color VDW force is about $\xi = 5 \times 10^{-4}$ deg in the 90° region and becomes much larger in the 125° region, attaining a value of $\xi \simeq 1 \times 10^{-2}$ deg. Therefore, an angle precision of 0.0005° is required in the 90° region and about 0.1° in the 125° region, in order to confirm our finding. It is clear from our calculation that larger values of ξ can be obtained with increasing (or decreasing) angles.

A clearer picture of the change in the oscillatory structure of the cross section due to the color VDW force is exhibited for completeness in Fig. 2. In Fig. 3 we show the individual contributions of the different effects mentioned earlier for comparison.

Recently, Vetterli *et al.*[14] have studied QED vacuum-polarization effects in $^{12}C + ^{12}C$ at $E_{\rm lab} = 4$ MeV with a precision which is slightly smaller than the one required in our test calculation in the 90° region.

At this point, it is important to assess the importance of the possible uncertainties of other small contributions as compared to the size of the color-VDW-force effect.

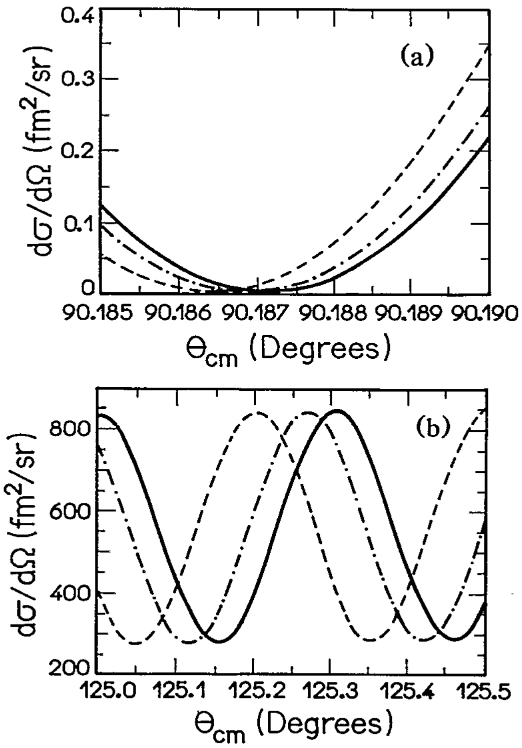

FIG. 2. Same as Fig. 1 but for $d\sigma/d\Omega$. (a) The 90° region and (b) the 125° region. Also shown is $d\sigma_{\mathrm{Mott}}/d\Omega$ (dashed curve).

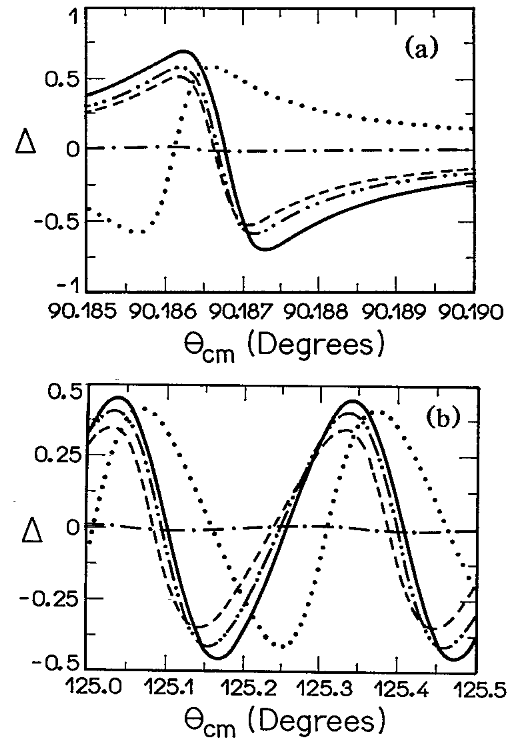

FIG. 3. Calculated Δ_i, where i refers to a given effect: dipole polarizability (dashed curve), quadrupole polarizability (dash-dotted curve), relativistic corrections (dash-double-dotted curve), and vacuum polarization (dotted curve); the solid curve represents Δ of Fig. 1 (taking into account all effects including the color VDW force). (a) The 90° region and (b) the 125° region.

We proceed now to discuss this in detail. Possible uncertainties in the polarization potentials can be assessed by looking at, for example, the dipole case. The next higher-order contribution to $V_{\mathrm{dip}}(r) = -V^0_{\mathrm{dip}}/r^4$ goes as $+(V^0_{\mathrm{dip}}/r^4)^2$ and thus it would diminish even more the already negligible octopole polarization potential, which goes as $-V^0_{\mathrm{oct}}/r^8$ (not shown in Fig. 3 as it is much smaller than the quadrupole contribution, which is already extremely small, as seen in the figure).

As far as the QED vacuum-polarization potential effects are concerned, we are assured that effects higher than the dominant α^2 $[(\frac{1}{137})^2]$ contribution contribute by as little as 1×10^{-2} (Ref. 14) to the polarization potential and as little as 0.01% to the proposed effect exemplified by $\xi(90°)$ and $\xi(125°)$ (other angular regions could be easily explored as well).

Finally, a word about our semiclassical treatment of the elastic scattering. It is well known that the condition for the applicability of semiclassical concepts in scattering conditions can be generally expressed as[15]

$$\frac{1}{2\eta}+\left(\frac{\Delta E}{E}\right)^2\cos^2\frac{\theta}{2}\ll\frac{(\theta/2)^2}{\sin^2(\theta/2)}\,, \tag{7}$$

where η is the Sommerfeld parameter, and ΔE is the uncertainty in the energy of the incident beam. As far as Fig. 1 is concerned, the above condition translates to $(2\eta)^{-1}+\frac{1}{2}(\Delta E/E)^2\ll 1$ at $\theta\sim 90°$ and slightly different for $\theta\sim 125°$. It is obvious that condition (7) is completely satisfied in our case; $\eta\sim 500$ and $\Delta E/E\sim 10^{-4}$ for $E_{\mathrm{c.m.}}\sim 500$ MeV in a machine like GANIL.

Before ending, we should mention that we have not explicitly considered the charge form factors of the nuclei even though the momentum transfer in the experiment that we propose is large. Our discussion, however, is completely justified since in the semiclassical analysis that we give, what is relevant is the distance of closest approach, and at this distance the form-factor effect is negligible.

In summary, we have studied in this paper the influence of the color van der Waals force on the low-energy elastic scattering of identical heavy nuclei. With a precision in angle measurement which can be easily attained with a slight improvement of the procedure used in Ref. 14, one should be able to set an upper limit on the strength of the r^{-7} color VDW force.

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