# Core longitudinal momentum distributions in the stripping reactions of two-neutron halo nuclei

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**Abstract.** The core longitudinal momentum distributions in two-neutron stripping reactions for halo nuclei of $^{20}$C and $^{22}$C are computed and compared to the experimental data obtained by detecting the core nucleus. The three-body wave function from the zero-range renormalized model is used as input in our calculations. We approximate the wave function of the projectile with a three-body structure, namely neutron-neutron-core, to an effective two-body one by integrating over neutron-neutron relative distance, such that the one has a core-dineutron wave function. The eikonal approximation is used in the description of the fragment-target interactions where the São Paulo optical potential is used for modeling the core-target and a Woods-Saxon potential is used for the dineutron-target interaction.

## 1. Introduction

Thanks to technological developments particle beams of unstable neutron rich nuclei are produced in laboratories and it is now possible to measure reaction cross-sections of very short half-life halo nuclei. They are composed by a core plus one or two loosely bound nucleons like the neutron-rich carbon isotopes, which have been observed within last couple of years [1, 2, 3]. In addition to experimental efforts, theoretical groups, in the last decades, have been making efforts in order to obtain indirect informations about neutron halo from observables such as the core momentum distributions [4, 5, 6]. This is important since it provides model constraints to make possible the prediction of other halo-nuclei properties, like the matter radius, see for example [7].

The differential cross-sections for the stripping processes (inelastic breakup) for two-body projectile on a target was discussed in Ref. [8]. We follow the theory presented in that work and compute the core longitudinal momentum distribution of the bound neutron-rich carbon isotopes, in particular, for the collision of beams of $^{20}$C and $^{22}$C with energy of 240 A MeV, with a target of stable $^{12}$C. We approximate the three-body projectile wave function by a two-body one, namely an effective core-dineutron wave function, by integrating the three-body wave function over the neutron-neutron relative coordinate.

## 2. Three-body zero-range wave function

To describe the two-neutron halo nucleus as a three-body system (for example $^{20}\mathrm{C} = {}^{18}\mathrm{C}+n+n$, where $n$ represents one neutron of the halo), the wave function can be written as a possible set of the relative coordinates $\Psi(\mathbf{r}, \mathbf{r}_{nn})$ as defined in Fig. 1. The wave function is an eigenstate of the three-body zero-range Hamiltonian where $S_{2n} = -E$ is the two-neutron separation energy. The neutrons are supposed to be in a spin singlet state and the configuration space zero-range model wave function [9, 10] is:

$$\Psi(\mathbf{r}, \mathbf{r}_{nn}) = \int d\mathbf{q} \, \frac{e^{-\kappa_{nn}|\mathbf{r}_{nn}|}}{|\mathbf{r}_{nn}|} \, e^{i\mathbf{q}.\mathbf{r}} \, f_{nn}(\mathbf{q}) + \left\{ \int d\mathbf{q} \, \frac{e^{-\kappa_{nc}|\mathbf{r}_{nc}|}}{|\mathbf{r}_{nc}|} \, e^{i\mathbf{q}.\mathbf{r}_{n',nc}} f_{nc}(\mathbf{q}) + (n \leftrightarrow n') \right\}, \quad (1)$$

where the last term $(n \leftrightarrow n')$ means the symmetry under exchange of the neutrons. The relative coordinate of the core to the neutron-neutron center of mass is $\mathbf{r}$. The absolute value for vector $\mathbf{r}_{nc} = \mathbf{r} + \frac{\mathbf{r}_{nn}}{2}$ is the distance between the core and the neutron. The relative coordinate of $n'$ to the neutron-core center of mass is $\mathbf{r}_{n',nc} = \frac{A}{A+1}\mathbf{r} - \frac{A+2}{2(A+1)}\mathbf{r}_{nn}$ and the $\kappa$'s in the two-body subsystems wave functions are: $\kappa_{nn} = \sqrt{2\frac{\mu_{nn}}{\hbar^2}\left(S_{2n} + \frac{\hbar^2 q^2}{2\mu_{nn,c}}\right)}$, and $\kappa_{nc} = \sqrt{2\frac{\mu_{nc}}{\hbar^2}\left(S_{2n} + \frac{\hbar^2 q^2}{2\mu_{nc,n}}\right)}$, with the reduced masses, $\mu_{nn} = \frac{m}{2}, \mu_{nn,c} = m\frac{2A}{A+2}, \mu_{nc} = m\frac{A}{A+1}, \mu_{nc,n} = m\frac{A+1}{A+2}$, where $A$ is the mass number of the core and the neutron mass is $m$. The zero-range three-body wave function is obtained by solving the coupled integral equations for $f_{nn}(\mathbf{q})$ and $f_{nc}(\mathbf{q})$, which are spectator functions. These integral equations are solved having as inputs the $n-n$ and $n-c$ scattering lengths, and the value of $S_{2n}$ (See details in Ref. [9]).

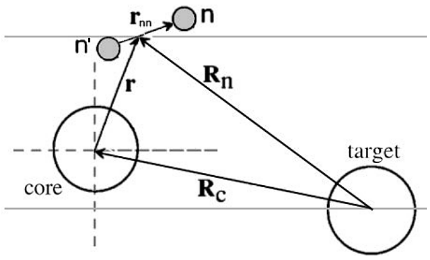


**Figure 1.** Coordinates for two-neutron stripping reaction.

## 3. Differential two-neutron stripping cross-section

The cross-section for the stripping reaction $(n+n+c) + target \to c + X$, where $(n+n+c)$ is the initial state of the projectile and $c$ corresponds to a final state of the core, is given in [8]:

$$\frac{d\sigma_{str}}{d^3k_c} = \frac{1}{(2\pi)^3}\frac{1}{2l+1}\sum_m \int d^2b_n \, \left[1 - |S_n(\mathbf{b}_n)|^2\right] \left| \int d^3r \, e^{i\mathbf{k}_c.\mathbf{r}} S_c(\mathbf{b}_c)\Psi_{lm}(\mathbf{r}) \right|^2, \quad (2)$$

where $\mathbf{k}_c$ is the core momentum, $\mathbf{b}_c$ and $\mathbf{b}_n$ are the impact parameter vectors referring to the transverse components of $\mathbf{R}_c$ and $\mathbf{R}_n$ as in Fig. 1. $S_c$ and $S_n$ are the scattering matrix of the $c + target$ and $(n+n) + target$, respectively. We introduce the three-body bound state wave function, Eq. (1), which has the predominance of s-wave $\ell = 0$. However, we approximate the wave function from three- to two-body by considering the two-neutron stripping. For that reduction to an effective core-dineutron wave function we integrate the three-body wave function over $\mathbf{r}_{nn}$ as follows:

$$\Psi(\mathbf{r}) := \int d^3r_{nn} \, \Psi(\mathbf{r}, \mathbf{r}_{nn}). \quad (3)$$

### 3.1. Eikonal approximation

The Eikonal approximation is a semiclassical method to obtain the S-matrix, $S(b)$, as a function of the impact parameter $b$, where one neglects the excitation energies of the projectile which moves along a straight-line trajectory at high energy. By considering that the projectile beam propagate towards the target with the initial momentum on the z-axis, the $S$-matrix can be written as $S(b) = \exp[i\chi(b)]$, and the eikonal phase is given by:

$$\chi(b) = -\frac{1}{\hbar v}\int_{-\infty}^{\infty} U_{opt}(b,z)dz, \tag{4}$$

with $v$ the fragment-target relative velocity and $U_{opt}$ is the optical potential.

The São Paulo optical potential (SPP) is used to describe the interaction between the core and the carbon target. This potential consists on a theoretical energy-dependent model that has been successful in describing the elastic and inelastic scattering for weakly bound nuclei based on a double folding potential for target and the projectile [11, 12]. It is built with the fundamental nucleon-nucleon interaction folded into a product of the nucleon densities of the nuclei and a polarization potential that carries the nonelastic contributions.

The dineutron-target interaction is described by a Woods-Saxon potential that is given by:

$$V^{WS}(r) = \frac{V_0 - iV_{0_I}}{1 + \exp\left((r - R_0)/a\right)}, \tag{5}$$

where $R_0$ is the target nuclear radius of $^{12}$C and $a = 0.676$ fm determines the diffuseness of the nuclear surface. The parameters used for real and imaginary parts are $V_{0_R} = 49.9395$ MeV and $V_{0_I} = 1.8256$ MeV, respectively [13].

## 4. Core longitudinal momentum distributions

We decompose the cross-section in momentum components and integrate over the perpendicular momentum component $k_{c_\perp}$ as,

$$\frac{d\sigma}{d\,k_{c_\parallel}} = \int \frac{d\sigma}{d^3k_c}\, d^2k_{c_\perp}, \tag{6}$$

the longitudinal momentum distribution is given by

$$\frac{d\sigma}{dk_{c_\parallel}} = \frac{1}{2\pi}\int_0^{\infty} d^2b_n\Big[1 - |S_n(b_n)|^2\Big]\int_0^{\infty} d^2\rho\, |S_c(\rho, b_n, \phi)|^2 \left|\int_{-\infty}^{\infty} dz \exp[-ik_\parallel z]\Psi(\rho, z)\right|^2, \tag{7}$$

where $\phi$ is the core angle around the z-axis and $|\mathbf{b}_c| = \sqrt{b_n^2 + \rho^2 - 2\rho b_n \cos(\phi - \phi_n)}$, with the dineutron position vector was fixed at angle $\phi_n = 2\pi$.

The two-neutron differential stripping cross-sections for the momentum distributions of the core $P_\parallel = \hbar k_{c_\parallel}$ are shown in Fig. 2 for the projectiles of $^{22}$C (left-frame) and $^{20}$C (right-frame) colliding at 240 A MeV on the carbon target. The distributions have been convoluted with the experimental resolution and normalized to the cross-sections data. The distributions are convoluted as $n_{\rm conv}(q) = \int_{-\infty}^{\infty} dq'\ \exp\big(-(q-q')^2/2\sigma^2\big)\, n(q')$, where $n(q')$ is the cross-sections calculated with Eq. (2) and are represented by full lines in both frames. The theoretical results for the $^{22}$C are convoluted with the experimental resolution $\sigma = 27$ MeV/c and added to a wide experimental distribution $\sigma_{\rm wide} = 89.6$ MeV/c (brown dashed line) associated to a full width at half maximum of FWHM = 211 MeV/c from Ref. [2]. The distribution for the $^{18}$C core in the reaction $^{12}$C($^{20}$C,$^{18}$C)X is computed for an experimental resolution of $\sigma = 28$ MeV/c. The red dashed line in the right-frame represents the approximation so-called the "transparent limit" where $S_c$=1 and it neglects the effect of the interaction between the observed fragment and the target nucleus. The distribution without the experimental resolution is shown as well. The three-body wave functions for the halo nuclei are computed by using two-neutron separation energies with $S_{2n} = 3.5$ MeV($S_{2n} = 0.396$ MeV [3]) for the $^{20}$C($^{22}$C). The bound subsystem $^{19}$C has one-neutron separation energy of $S_{1n} = 0.58$ MeV and for the borromean system $^{22}$C, we use the neutron-core system in the unitary limit, where the neutron-core virtual state energy vanishes, $E_{nc}^{virtual} = 0$.

The wide distribution added in the case of $^{22}$C is associated with neutrons in the inner orbits of the core. In the case of $^{20}$C, we have not taken into account such contributions, but as one see in Fig. 2, our calculations lacks strength for $|P_{\|}| > 100$ MeV/c, due possibly to neutrons emitted from such inner configurations, which should be considered in a future investigation.

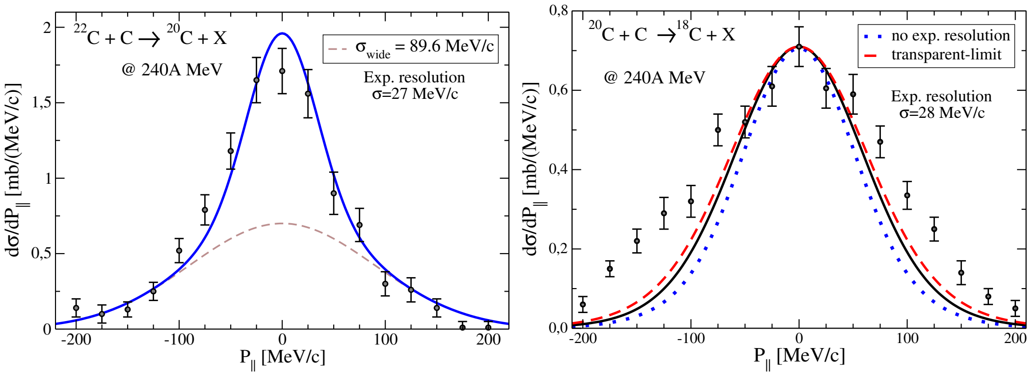


**Figure 2.** The core longitudinal momentum distributions of $^{22}$C (left-frame) and $^{20}$C (right-frame) obtained with a beam energy of 240 A MeV form the collision with the carbon target are represented by full lines in both panels. The distributions are convoluted with the experimental resolutions, and for $^{22}$C it is added to a wide normal distribution represented by dashed line. The blue-dotted line is the distribution without the experimental resolution for $^{20}$C. The red-dashed line is the transparent-limit in which $S_c$=1. The experimental results are from Ref. [2].

## 5. Discussion and Outlook

The theory of one-neutron halo stripping reaction has been extended to calculate the cross-sections of two-neutron stripping processes. The core momentum distribution of $^{22}$C, computed by using the known low-energy parameters, shows a fair consistency with the experimental results. The nucleus $^{20}$C requires a more accurate analysis, since we believe that a wider experimental resolution should be considered. In general, the results show that the zero-range model is appropriate to describe the three-body projectile in that stripping process. Despite we have only computed the momentum density for the neutron rich isotopes of carbon, we plan in the future calculate the breakup of other exotic two-neutron halo nuclei, using the same approximation to obtain the nuclear distortion of the fragments in the stripping reaction . In addition, Coulomb interaction should be taken account to complete this work when considering more heavier targets.

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