Supplemental material to:

Relativistic Coulomb excitation within Time Dependent Superfluid Local Density Approximation

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Density functional in TDSLDA and coupling to electromagnetic field

Here we present various definitions and conventions which we have used in the manuscript. The density functional is constructed from the following local densities and currents:

- density: $\rho(\mathbf{r}) = \rho(\mathbf{r},\mathbf{r}')|_{r=r'}$
- spin density: $\vec{s}(\mathbf{r}) = \vec{s}(\mathbf{r},\mathbf{r}')|_{r=r'}$
- current: $\vec{j}(\mathbf{r}) = \frac{1}{2i}(\vec{\nabla} - \vec{\nabla}')\rho(\mathbf{r},\mathbf{r}')|_{r=r'}$
- spin current (2nd rank tensor): $\mathbf{J}(\mathbf{r}) = \frac{1}{2i}(\vec{\nabla} - \vec{\nabla}') \otimes \vec{s}(\mathbf{r},\mathbf{r}')|_{r=r'}$
- kinetic energy density: $\tau(\mathbf{r}) = \vec{\nabla} \cdot \vec{\nabla}'\rho(\mathbf{r},\mathbf{r}')|_{r=r'}$
- spin kinetic energy density: $\vec{T}(\mathbf{r}) = \vec{\nabla} \cdot \vec{\nabla}'\vec{s}(\mathbf{r},\mathbf{r}')|_{r=r'}$
- anomalous density: $\chi(\mathbf{r}) = \chi(\mathbf{r}\sigma,\mathbf{r}'\sigma')|_{r=r',\sigma=1,\sigma'=-\sigma}$

where

$$
\begin{aligned}
\rho(\mathbf{r},\mathbf{r}') &= \sum_\mu \left(V_\mu^*(\mathbf{r}+)V_\mu(\mathbf{r}'+) + V_\mu^*(\mathbf{r}-)V_\mu(\mathbf{r}'-)\right)\\
s_x(\mathbf{r},\mathbf{r}') &= \sum_\mu \left(V_\mu^*(\mathbf{r}+)V_\mu(\mathbf{r}'-) + V_\mu^*(\mathbf{r}-)V_\mu(\mathbf{r}'+)\right)\\
s_y(\mathbf{r},\mathbf{r}') &= i\sum_\mu \left(V_\mu^*(\mathbf{r}+)V_\mu(\mathbf{r}'-) - V_\mu^*(\mathbf{r}-)V_\mu(\mathbf{r}'+)\right)\\
s_z(\mathbf{r},\mathbf{r}') &= \sum_\mu \left(V_\mu^*(\mathbf{r}+)V_\mu(\mathbf{r}'+) - V_\mu^*(\mathbf{r}-)V_\mu(\mathbf{r}'-)\right)\\
\tau(\mathbf{r},\mathbf{r}') &= \sum_\mu \left(\vec{\nabla}V_\mu^*(\mathbf{r}+)\cdot\vec{\nabla}V_\mu(\mathbf{r}'+) + \vec{\nabla}V_\mu^*(\mathbf{r}-)\cdot\vec{\nabla}V_\mu(\mathbf{r}'-)\right)\\
\vec{j}(\mathbf{r}) &= -Im\left(\sum_\mu V_\mu^*(\mathbf{r}+)\cdot\vec{\nabla}V_\mu(\mathbf{r}+) + V_\mu^*(\mathbf{r}-)\cdot\vec{\nabla}V_\mu(\mathbf{r}-)\right) =\\
&= Im\left(\sum_\mu V_\mu(\mathbf{r}+)\cdot\vec{\nabla}V_\mu^*(\mathbf{r}+) + V_\mu(\mathbf{r}-)\cdot\vec{\nabla}V_\mu^*(\mathbf{r}-)\right)\\
J_x(\mathbf{r}) &= Im\left(\frac{\partial}{\partial y}s_z(\mathbf{r},\mathbf{r}') - \frac{\partial}{\partial z}s_y(\mathbf{r},\mathbf{r}')\right)|_{r=r'}\\
J_y(\mathbf{r}) &= Im\left(\frac{\partial}{\partial z}s_x(\mathbf{r},\mathbf{r}') - \frac{\partial}{\partial x}s_z(\mathbf{r},\mathbf{r}')\right)|_{r=r'}\\
J_z(\mathbf{r}) &= Im\left(\frac{\partial}{\partial x}s_y(\mathbf{r},\mathbf{r}') - \frac{\partial}{\partial y}s_x(\mathbf{r},\mathbf{r}')\right)|_{r=r'}\chi(\mathbf{r}\sigma,\mathbf{r}'\sigma') = \sum_\mu V_\mu^*(\mathbf{r}\sigma)U_\mu(\mathbf{r}'\sigma')
\end{aligned}
\tag{4}
$$

The coupling of the nuclear system to the electromagnetic field:

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t} \tag{5}$$
$$\vec{B} = \vec{\nabla} \times \vec{A} \tag{6}$$
$$\tag{7}$$

requires the following transformation of proton densities and currents:

- density: $\rho_A(\mathbf{r}) = \rho_A(\mathbf{r})$
- spin density: $\vec{s}_A(\mathbf{r}) = \vec{s}(\mathbf{r})$
- current: $\vec{j}_A(\mathbf{r}) = \vec{j}(\mathbf{r}) - \frac{e}{\hbar c}\vec{A}\rho(\mathbf{r})$
- spin current (2nd rank tensor): $\mathbf{J}_A(\mathbf{r}) = \mathbf{J}(\mathbf{r}) - \frac{e}{\hbar c}\vec{A} \otimes \vec{s}(\mathbf{r})$
- spin current (vector): $\vec{J}_A(\mathbf{r}) = \vec{J}(\mathbf{r}) - \frac{e}{\hbar c}\vec{A} \times \vec{s}(\mathbf{r})$
- kinetic energy density: $\tau_A(\mathbf{r}) = \left(\vec{\nabla} - i\frac{e}{\hbar c}\vec{A}\right) \cdot \left(\vec{\nabla}' + i\frac{e}{\hbar c}\vec{A}\right) \rho(\mathbf{r}, \mathbf{r}')|_{r=r'}$
  $= \tau(\mathbf{r}) - 2\frac{e}{\hbar c}\vec{A} \cdot \vec{j}(\mathbf{r}) + \frac{e^2}{\hbar^2 c^2}|\vec{A}|^2\rho(\mathbf{r}) = \tau(\mathbf{r}) - 2\frac{e}{\hbar c}\vec{A} \cdot \vec{j}_A(\mathbf{r}) - \frac{e^2}{\hbar^2 c^2}|\vec{A}|^2\rho(\mathbf{r})$
- spin kinetic energy density: $\vec{T}_A(\mathbf{r}) = \left(\vec{\nabla} - i\frac{e}{\hbar c}\vec{A}\right) \cdot \left(\vec{\nabla}' + i\frac{e}{\hbar c}\vec{A}\right) \vec{s}(\mathbf{r}, \mathbf{r}')|_{r=r'}$
  $= \vec{T}(\mathbf{r}) - 2\frac{e}{\hbar c}\vec{A}^T \cdot \mathbf{J}(\mathbf{r}) + \frac{e^2}{\hbar^2 c^2}|\vec{A}|^2\vec{s}(\mathbf{r}) = \vec{T}(\mathbf{r}) - 2\frac{e}{\hbar c}\vec{A}^T \cdot \mathbf{J}_A(\mathbf{r}) - \frac{e^2}{\hbar^2 c^2}|\vec{A}|^2\vec{s}(\mathbf{r})$

As a result the proton single particle hamiltonian has the form:

$$\begin{aligned} h_A(\mathbf{r}) = -\vec{\nabla}_A \cdot \left(B(\mathbf{r}) + \vec{\sigma} \cdot \vec{C}(\mathbf{r})\right)\vec{\nabla}_A + U_A(\mathbf{r}) + \frac{1}{2i}\left(\vec{W}(\mathbf{r}) \cdot (\vec{\nabla}_A \times \vec{\sigma}) + \vec{\nabla}_A \cdot (\vec{\sigma} \times \vec{W}(\mathbf{r}))\right) \\ + \vec{U}^A_\sigma(\mathbf{r}) \cdot \vec{\sigma} + \frac{1}{i}\left(\vec{\nabla}_A \cdot \vec{U}^A_\Delta(\mathbf{r}) + \vec{U}^A_\Delta(\mathbf{r}) \cdot \vec{\nabla}_A\right) \end{aligned} \tag{8}$$

and

$$U_A(\mathbf{r}) = U(\mathbf{r}) - C^{\nabla J}\frac{e}{\hbar c}\vec{\nabla} \cdot [\vec{A} \times \vec{s}(\mathbf{r})] - C^\tau\left(2\frac{e}{\hbar c}\vec{A} \cdot \vec{j}(\mathbf{r}) + \frac{e^2}{\hbar^2 c^2}|\vec{A}|^2\rho(\mathbf{r})\right) \tag{9}$$
$$\vec{U}^A_\sigma(\mathbf{r}) = \vec{U}_\sigma(\mathbf{r}) - C^{\nabla J}\frac{e}{\hbar c}\vec{\nabla} \times [\vec{A}\rho(\mathbf{r})] - C^{sT}\left(2\frac{e}{\hbar c}\vec{A}^T \cdot \mathbf{J}(\mathbf{r}) + \frac{e^2}{\hbar^2 c^2}|\vec{A}|^2\vec{s}(\mathbf{r})\right) \tag{10}$$
$$\vec{U}^A_\Delta(\mathbf{r}) = \vec{U}_\Delta(\mathbf{r}) - C^j\frac{e}{\hbar c}\vec{A}\rho(\mathbf{r}) \tag{11}$$
$$\begin{aligned} &\vec{\nabla}_A \cdot \left(B(\mathbf{r}) + \vec{\sigma} \cdot \vec{C}(\mathbf{r})\right)\vec{\nabla}_A = \left[\vec{\nabla}_A\left(B(\mathbf{r}) + \vec{\sigma} \cdot \vec{C}(\mathbf{r})\right)\right] \cdot \vec{\nabla}_A + \\ &\left(B(\mathbf{r}) + \vec{\sigma} \cdot \vec{C}(\mathbf{r})\right)\left[\Delta - i\frac{e}{\hbar c}\left(\vec{A} \cdot \vec{\nabla}_A + \vec{\nabla}_A \cdot \vec{A}\right) + \frac{e^2}{\hbar^2 c^2}|\vec{A}|^2\right] \end{aligned} \tag{12}$$

Numerical Implementation

We build a spatial three-dimensional Cartesian grid in coordinate space with periodic boundary conditions, and derivatives evaluated in momentum (Fourier-transformed) space. This method represents a flexible tool to describe large amplitude nuclear motion as it contains the coupling to the continuum and between single-particle and collective degrees of freedom. For the present problem, we have considered a box size of 40^3 with the lattice constant 1 fm. The time step has been set to 0.076772 fm/c with a total time interval of about 4000 fm/c. The projectile is initially placed at such a distance from the target nucleus that the collision occurs after $1600-1700$ fm/c. Even though inially the projectle is far enough from the target and hence the EM fields are weak, spurious excitations produced by a sudden switch of the EM interaction at $t=0$ are possible. They were avoided by multiplying the the EM potentials in Eq. (**??**) by the smoothing function $f(t) = 1/\left[1 + \exp((r(t) - R_0)/a_0)\right]$, where $R_0 = 250$ fm, $a_0 = 50$ fm. This ensures that the EM field varies smoothly within the distance a_0, but stay approximately equal to its physical value within the distance $2R_0$.

Coulomb potential on the lattice

Here we describe the method used to describe the Coulomb self-interaction of the target nucleus.

Consider the charge distribution $e\rho(\mathbf{r})$:

$$\nabla^2\Phi(\mathbf{r}) = 4\pi e^2\rho(\mathbf{r}) \tag{13}$$

$$\Phi(\mathbf{r}) = \int d^3r' \frac{e^2\rho(\mathbf{r})}{|\mathbf{r}-\mathbf{r}'|} \tag{14}$$

Note that above we have defined Φ as $e\Phi$ (note e^2 in the formula). After the Fourier transform one gets:

$$\Phi(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3}\frac{e^2\rho(\vec{k})}{k^2}\exp(i\vec{k}\cdot\mathbf{r}) \tag{15}$$

The above prescription generates however the spurious interaction between neighbouring cells.

Therefore we define the modified potential (N_x, N_y, N_z denote number of equidistant lattice points in each direction, $L_i = N_i\Delta x$, $i = x, y, z$, Δx is lattice constant):

$$\begin{aligned} &f(r) = 1/r \text{ for } r < \sqrt{L_x^2 + L_y^2 + L_z^2} \\ &f(r) = 0 \text{ otherwise} \end{aligned} \tag{16}$$

Clearly the Fourier transform is:

$$f(k) = 4\pi\frac{1-\cos(k\sqrt{L_x^2 + L_y^2 + L_z^2})}{k^2} \tag{17}$$

and moreover

$$\Phi(\mathbf{r}) = \int \frac{d^3k}{(2\pi)^3}\frac{e^2\rho(\vec{k})}{k^2}\exp(i\vec{k}\cdot\mathbf{r}) = \frac{1}{27N_xN_yN_z}\sum_{\vec{k}\in L_xL_yL_z} e^2\rho(\vec{k})f(k)\exp(i\vec{k}\cdot\mathbf{r}) \tag{18}$$

where in the last term $\rho(\vec{k})$ is the Fourier transformed density on the lattice $27L_xL_yL_z$. In practice it means that one has to perform forward and backward Fourier transforms on the lattice three times bigger in each direction.

This may however be avoided if one realizes that the Fourier transform of the density in the larger lattice can be expressed through the Fourier transforms in the smaller lattices:

$$\rho_{klm}(\vec{k}) = \sum_{\mathbf{r}\in L^3}\rho(x,y,z)\exp\left(-i\left(k\frac{2\pi}{3L_x}x + l\frac{2\pi}{3L_y}y + m\frac{2\pi}{3L_z}z\right)\right)\exp(-i\vec{k}\cdot\mathbf{r}) \tag{19}$$

and we need to perform 27 FFTs on the smaller lattice L for $k, l, m = 0, 1, 2$ of the following quantities: $\rho(x,y,z)\exp\left(-i\left(k\frac{2\pi}{L_x}x + l\frac{2\pi}{L_y}y + m\frac{2\pi}{L_z}z\right)\right)$

Subsequently we obtain the potential through the relation:

$$\begin{aligned} &\Phi(\mathbf{r}) = \\ &= \frac{1}{27N_xN_yN_z}\sum_{k,l,m=0}^{2}\left[\sum_{\vec{k}\in L^3} e^2\rho_{klm}(\vec{k})f\left(\vec{k} + \left(k\frac{2\pi}{3L_x}, l\frac{2\pi}{3L_y}, m\frac{2\pi}{3L_z}\right)\right)\exp(i\vec{k}\cdot\mathbf{r})\right] \\ &\times\exp\left(i\left(k\frac{2\pi}{3L_x}x + l\frac{2\pi}{3L_y}y + m\frac{2\pi}{3L_z}z\right)\right) \end{aligned} \tag{20}$$

Dipole component of the electromagnetic field produced by the projectile

Coordinates (see fig. 7):

$$\mathbf{R} = (b, 0, vt), \qquad \mathbf{r} = (x, y, z). \tag{21}$$

Interaction potential:

$$V_E(\mathbf{r}, t) = \Phi(\mathbf{r}, t)\rho_c(\mathbf{r})d^3r, \tag{22}$$

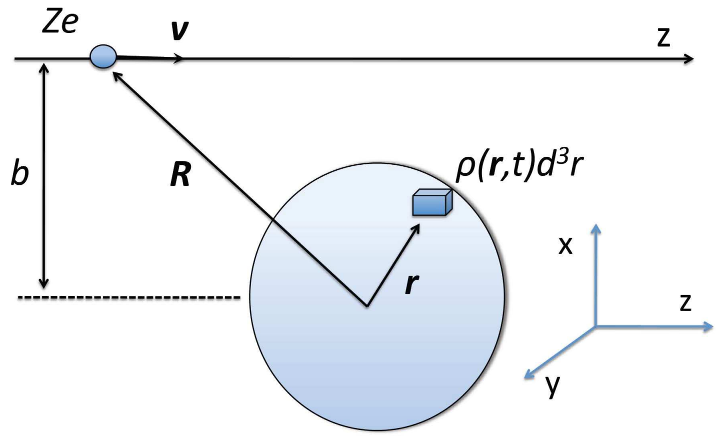

FIG. 4. Coordinate system used to describe the reaction

where

$$\Phi(\mathbf{r},t) = \frac{\gamma Z e}{\sqrt{(x-b)^2 + y^2 + \gamma^2(z-vt)^2}}, \tag{23}$$

and $\gamma = (1 - v^2/c^2)^{-1/2}$. $\rho_c(\mathbf{r}) = e\Psi^*(\mathbf{r})\Psi(\mathbf{r})$ is the charge density of the nucleus at location $\mathbf{r}$ and $\Psi(\mathbf{r})$ are proton wavefunctions . The vector potential is given by

$$\mathbf{A}(\mathbf{r},t) = \frac{\mathbf{v}}{c}\phi(\mathbf{r},t). \tag{24}$$

In order to extract the dipole component we used the interaction Hamiltonian:

$$\mathcal{H}_{int}(\mathbf{r},t) = \frac{\gamma Z e^2}{\sqrt{(x-b)^2 + y^2 + \gamma^2(z-vt)^2}} - \frac{\gamma Z e^2}{\sqrt{b^2 + \gamma^2 v^2 t^2}}, \tag{25}$$

where one subtracts the second term which is responsible for the c.m. scattering (i.e. monopole field).

Consequently the dipole term reads:

$$\mathcal{H}_{\mathrm{E}1m}(\mathbf{r},t) = \sqrt{\frac{2\pi}{3}} r Y_{1m}(\hat{\mathbf{r}}) \frac{\gamma Z e^2}{(b^2 + \gamma^2 v^2 t^2)^{3/2}} \begin{cases} \mp b, & (\text{if} \quad m = \pm 1) \\ \sqrt{2} vt & (\text{if} \quad m = 0) \end{cases}, \tag{26}$$

where $\mathbf{r}$ is the coordinate of one of the protons in the target. A sum over m has to be performed, i.e.

$$\mathcal{H}_{E1}(\mathbf{r},t) = \sum_{i=protons} \sum_m \mathcal{H}_{E1m}(\mathbf{r_i},t) = \sum_{i=protons} \sum_m r_i^l Y_{1m}(\hat{\mathbf{r}}_i) f_{1m}(t), \tag{27}$$

where $f_{1m}(t)$ is the part of the interaction which does not involve the intrinsic structure of the nucleus:

$$f_{1m}(t) = \sqrt{\frac{2\pi}{3}} \frac{\gamma Z e^2}{(b^2 + \gamma^2 v^2 t^2)^{3/2}} \begin{cases} \mp b, & (\text{if} \quad m = \pm 1) \\ \sqrt{2} vt & (\text{if} \quad m = 0) \end{cases}. \tag{28}$$

Electromagnetic radiation from a nucleus described within TDSLDA

Let us consider the proton density and current (we use Gauss units):

$$\rho(\mathbf{r},t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \rho(\mathbf{r},\omega) \exp(-i\omega t) \tag{29}$$

$$\vec{j}(\mathbf{r},t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \vec{j}(\mathbf{r},\omega) \exp(-i\omega t) \tag{30}$$

where

$$\rho(\mathbf{r},\omega) = \int \frac{d^3k}{(2\pi)^3} \rho(\vec{k},\omega) \exp(i\vec{k}\cdot\mathbf{r}) \tag{31}$$

$$\vec{j}(\mathbf{r},\omega) = \int \frac{d^3k}{(2\pi)^3} \vec{j}(\vec{k},\omega) \exp(i\vec{k}\cdot\mathbf{r}) \tag{32}$$

Maxwell equations:

$$\nabla \cdot \vec{E}(\mathbf{r},t) = 4\pi e \rho(\mathbf{r},t) \tag{33}$$
$$\nabla \cdot \vec{B}(\mathbf{r},t) = 0 \tag{34}$$
$$\nabla \times \vec{E}(\mathbf{r},t) = -\frac{1}{c}\frac{\partial}{\partial t}\vec{B}(\mathbf{r},t) \tag{35}$$
$$\nabla \times \vec{B}(\mathbf{r},t) = \frac{1}{c}\left(\frac{\partial}{\partial t}\vec{E}(\mathbf{r},t) + 4\pi e \vec{j}(\mathbf{r},t)\right) \tag{36}$$

and spatial Fourier transforms:

$$i\vec{k} \cdot \vec{E}(\vec{k},t) = 4\pi e \rho(\vec{k},t) \tag{37}$$
$$i\vec{k} \cdot \vec{B}(\vec{k},t) = 0 \tag{38}$$
$$i\vec{k} \times \vec{E}(\vec{k},t) = -\frac{1}{c}\frac{\partial}{\partial t}\vec{B}(\vec{k},t) \tag{39}$$
$$i\vec{k} \times \vec{B}(\vec{k},t) = \frac{1}{c}\left(\frac{\partial}{\partial t}\vec{E}(\vec{k},t) + 4\pi e \vec{j}(\vec{k},t)\right) \tag{40}$$

Hence clearly:

$$\vec{E} = \vec{E}_{||} + \vec{E}_{\perp} \tag{41}$$
$$\vec{B} = \vec{B}_{\perp} \tag{42}$$

where

$$\nabla \times \vec{E}_{||}(\mathbf{r},t) = 0 \tag{43}$$
$$\nabla \cdot \vec{E}_{\perp}(\mathbf{r},t) = 0 \tag{44}$$
$$\nabla \cdot \vec{B}_{\perp}(\mathbf{r},t) = 0 \tag{45}$$

and

$$\vec{E} = -\nabla\phi(\mathbf{r},t) - \frac{1}{c}\frac{\partial}{\partial t}\vec{A}(\mathbf{r},t) \tag{46}$$
$$\vec{B} = \nabla \times \vec{A}(\mathbf{r},t) \tag{47}$$

Clearly

$$\vec{E}_{||}(\mathbf{r},t) = -\nabla\phi(\mathbf{r},t) - \frac{1}{c}\frac{\partial}{\partial t}\vec{A}_{||}(\mathbf{r},t) \tag{48}$$
$$\vec{E}_{\perp}(\mathbf{r},t) = -\frac{1}{c}\frac{\partial}{\partial t}\vec{A}_{\perp}(\mathbf{r},t) \tag{49}$$
$$\vec{B} = \nabla \times \vec{A}_{\perp}(\mathbf{r},t) \tag{50}$$

Therefore one has a freedom to choose $\vec{A}_{||}(\mathbf{r},t)$ (gauge transformation) whereas $\vec{A}_{\perp}(\mathbf{r},t)$ is the gauge invariant part of the vector potential.

We choose the Coulomb gauge:

$$\vec{A}_{||}(\mathbf{r},t) = 0 \Leftrightarrow \vec{A}(\mathbf{r},t) = \vec{A}_{\perp}(\mathbf{r},t) \tag{51}$$

Hence

$$\vec{E}_{||}(\mathbf{r},t) = -\nabla\phi(\mathbf{r},t) \tag{52}$$
$$\vec{E}_{\perp}(\mathbf{r},t) = -\frac{1}{c}\frac{\partial}{\partial t}\vec{A}_{\perp}(\mathbf{r},t) \tag{53}$$
$$\vec{B} = \nabla \times \vec{A}_{\perp}(\mathbf{r},t) \tag{54}$$

and only perpendicular components of electric and magnetic fields are responsible for emission of radiation. The important equation in this case is the fourth Maxwell equation:

$$\nabla \times \vec{B}_{\perp}(\mathbf{r},t) = \frac{1}{c}\left(\frac{\partial}{\partial t}\vec{E}_{\perp}(\mathbf{r},t) + 4\pi e \vec{j}_{\perp}(\mathbf{r},t)\right) + \frac{1}{c}\left(\frac{\partial}{\partial t}\vec{E}_{||}(\mathbf{r},t) + 4\pi e \vec{j}_{||}(\mathbf{r},t)\right) \tag{55}$$

Since the lhs represents the vector of type $\perp$ therefore:

$$\frac{\partial}{\partial t}\vec{E}_{||}(\mathbf{r},t) + 4\pi e\vec{j}_{||}(\mathbf{r},t) = 0 \tag{56}$$

and

$$\nabla \times \vec{B}_{\perp}(\mathbf{r},t) = \frac{1}{c}\left(\frac{\partial}{\partial t}\vec{E}_{\perp}(\mathbf{r},t) + 4\pi e\vec{j}_{\perp}(\mathbf{r},t)\right) \tag{57}$$

Substituting the potential $\vec{A}$:

$$\nabla \times \nabla \times \vec{A}_{\perp}(\mathbf{r},t) = -\nabla^2\vec{A}_{\perp}(\mathbf{r},t) = \frac{1}{c^2}\left(-\frac{\partial}{\partial t^2}\vec{A}_{\perp}(\mathbf{r},t) + 4\pi ce\vec{j}_{\perp}(\mathbf{r},t)\right) \tag{58}$$

$$\nabla^2\vec{A}_{\perp}(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial}{\partial t^2}\vec{A}_{\perp}(\mathbf{r},t) = -\frac{1}{c}4\pi e\vec{j}_{\perp}(\mathbf{r},t) \tag{59}$$

where

$$\vec{j}_{\perp}(\mathbf{r},t) = \vec{j}(\mathbf{r},t) - \vec{j}_{||}(\mathbf{r},t) = \vec{j}(\mathbf{r},t) + \frac{1}{4\pi e}\frac{\partial}{\partial t}\vec{E}_{||}(\mathbf{r},t) = \vec{j}(\mathbf{r},t) - \frac{1}{4\pi e}\frac{\partial}{\partial t}\nabla\phi(\mathbf{r},t) \tag{60}$$

Therefore in the Coulomb gauge

$$\vec{A}(\mathbf{r},t) = \vec{A}_{\perp}(\mathbf{r},t) = \frac{1}{c}\int d^3r'\frac{e\vec{j}_{\perp}(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/c)}{|\mathbf{r}-\mathbf{r}'|} = \frac{1}{c}\int d^3r'\frac{e\vec{j}_{\perp}(\mathbf{r}',\omega)\exp(ik|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|} \tag{61}$$

and in the far zone $r >> r'$:

$$\vec{A}(\mathbf{r},\omega) = \frac{\exp(ikr)}{r}\frac{1}{c}\int d^3r' e\vec{j}_{\perp}(\mathbf{r}',\omega)\exp(-i\vec{k}\cdot\mathbf{r}') = \frac{\exp(ikr)}{r}\frac{1}{c}e\vec{j}_{\perp}(\vec{k},ck) \tag{62}$$

where $\vec{k} = k\frac{\mathbf{r}}{r}$ and $\omega = ck$ and consequently:

$$\vec{A}(\mathbf{r},t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\vec{A}(\mathbf{r},\omega)\exp(-i\omega t) = \frac{e}{2\pi}\int_{-\infty}^{\infty} dk\vec{j}_{\perp}(\vec{k},ck)\frac{\exp(ik(r-ct)}{r} \tag{63}$$

Consequently since $\vec{B} = \nabla \times \vec{A}$ we get:

$$\begin{aligned}
\vec{B}(\mathbf{r},\omega) &= \nabla \times \vec{A}(\mathbf{r},\omega) = \nabla \times \frac{1}{c}\int d^3r'\frac{e\vec{j}_{\perp}(\mathbf{r}',\omega)\exp(ik|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|} = \\
&= \frac{1}{c}\int d^3r'\frac{ik(\mathbf{r}-\mathbf{r}')\times e\vec{j}_{\perp}(\mathbf{r}',\omega)\exp(ik|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|^2} - \frac{1}{c}\int d^3r'\frac{(\mathbf{r}-\mathbf{r}')\times e\vec{j}_{\perp}(\mathbf{r}',\omega)\exp(ik|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|^3} \\
&= \frac{1}{c}\int d^3r'\frac{ik(\mathbf{r}-\mathbf{r}')\times e\vec{j}(\mathbf{r}',\omega)\exp(ik|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|^2} - \frac{1}{c}\int d^3r'\frac{(\mathbf{r}-\mathbf{r}')\times e\vec{j}(\mathbf{r}',\omega)\exp(ik|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|^3},
\end{aligned} \tag{64}$$

where in the last line we have used the fact that rotation of the vector of type $||$ is zero. For the electric field:

$$\vec{E}_{\perp}(\mathbf{r},\omega) = \frac{i\omega}{c}\vec{A}(\mathbf{r},\omega) = \frac{i\omega}{c^2}\int d^3r'\frac{e\vec{j}_{\perp}(\mathbf{r}',\omega)\exp(ik|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|} \tag{65}$$

$$\vec{E}_{||}(\mathbf{r},\omega) = -\nabla\phi(\mathbf{r},\omega) = -\nabla\int d^3r'\frac{e\rho(\mathbf{r}',\omega)}{|\mathbf{r}-\mathbf{r}'|} \tag{66}$$

Hence in the far zone $r >> r'$ one gets:

$$\vec{B}(\mathbf{r},\omega) = \frac{ie}{c}\frac{\exp(ikr)}{r}\int d^3r'\vec{k}\times\vec{j}_{\perp}(\mathbf{r}',\omega)\exp(-i\vec{k}\cdot\mathbf{r}') = \frac{ie}{c}\frac{\exp(ikr)}{r}\vec{k}\times\vec{j}(\vec{k},\omega) \tag{67}$$

$$\vec{E}(\mathbf{r},\omega) = \vec{E}_{\perp}(\mathbf{r},\omega) + \vec{E}_{||}(\mathbf{r},\omega) = \frac{\exp(ikr)}{r}\frac{i\omega}{c^2}e\vec{j}_{\perp}(\vec{k},ck) = \frac{ie}{c}\frac{\exp(ikr)}{r}k\vec{j}_{\perp}(\vec{k},ck) \tag{68}$$

$$= \frac{ie}{c}\frac{\exp(ikr)}{r}k(\vec{j}(\vec{k},ck) - \vec{j}_{||}(\vec{k},ck)) = \frac{ie}{c}\frac{\exp(ikr)}{r}\left((\vec{k}\cdot\frac{\mathbf{r}}{r})\vec{j}(\vec{k},ck) - (\vec{k}\cdot\vec{j}(\vec{k},ck))\frac{\mathbf{r}}{r}\right) \tag{69}$$

$$= \frac{ie}{c}\frac{\exp(ikr)}{r}\frac{\mathbf{r}}{r}\times\left(\vec{j}(\vec{k},\omega)\times\vec{k}\right) \tag{70}$$

and consequently:

$$\vec{B}(\mathbf{r},t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \vec{B}(\mathbf{r},\omega)\exp(-i\omega t) = \frac{ie}{2\pi}\int_{-\infty}^{\infty} dk\vec{k}\times\vec{j}(\vec{k},ck)\frac{\exp(ik(r-ct))}{r} \tag{71}$$

$$= \frac{ie}{c}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\vec{k}\times\vec{j}(\vec{k},\omega)\frac{\exp(-i\omega(t-r/c))}{r} \tag{72}$$

$$\vec{E}(\mathbf{r},t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \vec{E}(\mathbf{r},\omega)\exp(-i\omega t) = \frac{ie}{2\pi}\frac{\mathbf{r}}{r}\times\int_{-\infty}^{\infty} dk\left(\vec{j}(\vec{k},ck)\times\vec{k}\right)\frac{\exp(ik(r-ct))}{r} \tag{73}$$

$$= \frac{ie}{c}\frac{\mathbf{r}}{r}\times\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\left(\vec{j}(\vec{k},\omega)\times\vec{k}\right)\frac{\exp(-i\omega(t-r/c))}{r} \tag{74}$$

Note that in the above expressions $\vec{k}$ and ω are related: $\omega = c|\vec{k}|$.

Poynting vector reads $\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{B}$ and thus:

$$\vec{S}(t) = \frac{c}{4\pi(2\pi)^2}\int_{-\infty}^{\infty} d\omega\int_{-\infty}^{\infty} d\omega'\vec{E}(\mathbf{r},\omega)\times\vec{B}(\mathbf{r},\omega)\exp(-i(\omega+\omega')t)$$

$$= \frac{c}{4\pi(2\pi)^2}\int_{-\infty}^{\infty} d\omega\int_{-\infty}^{\infty} d\omega'\vec{E}(\mathbf{r},\omega)\times\vec{B}^*(\mathbf{r},\omega')\exp(-i(\omega-\omega')t)$$

$$= \frac{e^2}{4\pi(2\pi)^2cr^2}\frac{\mathbf{r}}{r}\int_{-\infty}^{\infty} d\omega\int_{-\infty}^{\infty} d\omega'(\vec{k}\times\vec{j}(\vec{k},\omega))\cdot(\vec{k'}\times\vec{j}^*(\vec{k'},\omega'))\exp(-i(\omega-\omega')t+i(k-k')r) \tag{75}$$

$$= \frac{e^2}{4\pi cr^2}\frac{\mathbf{r}}{r}\left|\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}(\vec{k}\times\vec{j}(\vec{k},\omega))\exp(-i\omega t+ikr)\right|^2 \tag{76}$$

$$= \frac{c}{4\pi}\frac{\mathbf{r}}{r}\left|\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\vec{B}(\mathbf{r},\omega)\exp(-i\omega t)\right|^2 \tag{77}$$

Energy per unit time emitted to the angle $d\Omega$ reads:

$$dP(t) = \vec{S}(t)\cdot\frac{\mathbf{r}}{r}r^2 d\Omega = \frac{e^2}{4\pi c}\left|\int\frac{d\omega}{2\pi}(\vec{k}\times\vec{j}(\vec{k},\omega))\exp(-i\omega t+ikr)\right|^2 d\Omega \tag{78}$$

Hence

$$\frac{dP}{d\Omega}(t) = \frac{e^2}{4\pi c}\left|\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}(\vec{k}\times\vec{j}(\vec{k},\omega))\exp(-i\omega(t-r/c))\right|^2 = \frac{c}{4\pi}r^2\left|\vec{B}(\mathbf{r},t)\right|^2 \tag{79}$$

Note that the radiation at time t is given by the current at time $t-r/c$, thus a simple time shift, which we can discard. Therefore the total amount of radiated energy at the angle $d\Omega$ reads:

$$\int_{-\infty}^{\infty}\frac{dP}{d\Omega}(t)dt = \frac{c}{4\pi}r^2\int_{-\infty}^{\infty}\left|\vec{B}(\mathbf{r},t)\right|^2 dt = \frac{c}{4\pi}r^2\int_{-\infty}^{\infty}\vec{B}(\mathbf{r},t)\cdot\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\vec{B}(\mathbf{r},\omega)\exp(-i\omega t)dt$$

$$= \frac{c}{4\pi}r^2\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\vec{B}(\mathbf{r},-\omega)\cdot\vec{B}(\mathbf{r},\omega) = \frac{c}{4\pi}r^2\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\vec{B}^*(\mathbf{r},\omega)\cdot\vec{B}(\mathbf{r},\omega) \tag{80}$$

which gives the spectral decomposition of emitted radiation:

$$\int_{-\infty}^{\infty}\frac{dP}{d\Omega}(t)dt = \frac{c}{8\pi^2}r^2\int_{-\infty}^{\infty} d\omega\left|\vec{B}(\mathbf{r},\omega)\right|^2 = \frac{c}{4\pi^2}r^2\int_{0}^{\infty} d\omega\left|\vec{B}(\mathbf{r},\omega)\right|^2 \tag{81}$$

Hence the energy emitted at the angle Ω at frequency ω reads:

$$\frac{dE}{d\Omega d\omega}(\omega) = \frac{e^2}{4\pi^2 c}\left|\vec{k}\times\vec{j}(\vec{k},\omega)\right|^2 = \frac{e^2}{4\pi^2 c}\left|\int d^3r\left(\nabla\times\vec{j}(\mathbf{r},\omega)\right)\exp(-i\vec{k}\cdot\mathbf{r})\right|^2 \tag{82}$$

In order to calculate the quantities given by the expressions: (79) (82) we use the multipole expansion. Namely, let us consider eq. (82):

$$\frac{dE}{d\omega} = \int\frac{dE}{d\Omega d\omega}(\omega)d\Omega = \int\frac{e^2}{4\pi^2 c}\left|\vec{k}\times\vec{j}(\vec{k},\omega)\right|^2 d\Omega \tag{83}$$

$$= \frac{e^2}{4\pi^2 c}\int\left|\int d^3r\int_{-\infty}^{\infty} dt\left(\vec{\nabla}\times\vec{j}(\mathbf{r},t)\right)\exp(-i\vec{k}\cdot\mathbf{r}+i\omega t)\right|^2 d\Omega \tag{84}$$

Let us denote:

$$\vec{\nabla} \times \vec{j}(\mathbf{r}, t) = \vec{b}(\mathbf{r}, t) \tag{85}$$

$$\vec{\nabla} \times \vec{j}(\mathbf{r}, \omega) = \vec{b}(\mathbf{r}, \omega) \tag{86}$$

We expand $\exp(-i\vec{k} \cdot \mathbf{r})$:

$$\exp(-i\vec{k} \cdot \mathbf{r}) = 4\pi \sum_{l,m} (-i)^l j_l(kr) Y_{lm}(\hat{k}) Y^*_{lm}(\hat{r}) \tag{87}$$

and consequently we get

$$\frac{dE}{d\omega} = \frac{e^2}{4\pi^2 c} \int \left| \int d^3r \int_{-\infty}^{\infty} dt \left(\vec{b}(\mathbf{r}, t) 4\pi \sum_{l,m} (-i)^l j_l(kr) Y_{lm}(\hat{k}) Y^*_{lm}(\hat{r}) \right) \exp(i\omega t) \right|^2 d\Omega \tag{88}$$

$$= \frac{e^2}{4\pi^2 c} \int \left| \int_{-\infty}^{\infty} dt 4\pi \sum_{l,m} (-i)^l \vec{b}_{lm}(\vec{k}, t) Y_{lm}(\hat{k}) \exp(i\omega t) \right|^2 d\Omega \tag{89}$$

$$= \frac{e^2}{4\pi^2 c} \int \left| 4\pi \sum_{l,m} (-i)^l \vec{b}_{lm}(\vec{k}, \omega) Y_{lm}(\hat{k}) \right|^2 d\Omega \tag{90}$$

where

$$\vec{b}_{lm}(k, t) = \int d^3r \vec{b}(\mathbf{r}, t) j_l(kr) Y^*_{lm}(\hat{r}) \tag{91}$$

$$\vec{b}_{lm}(k, \omega) = \int_{-\infty}^{\infty} \vec{b}_{lm}(k, t) \exp(i\omega t) dt \tag{92}$$

Note that $\vec{b}_{lm}$ is a function of k (not $\vec{k}$) and

$$\frac{dE}{d\omega} = \frac{e^2}{4\pi^2 c} (4\pi)^2 \int \left(\sum_{l,m,l',m'} (-i)^l i^{l'} \left(\vec{b}_{lm}(k, \omega) \cdot \vec{b}^*_{l'm'}(k, \omega) \right) Y_{lm}(\hat{k}) Y^*_{l'm'}(\hat{k}) \right) d\Omega \tag{93}$$

$$= \frac{e^2}{4\pi^2 c} (4\pi)^2 \sum_{l,m} |\vec{b}_{lm}(k, \omega)|^2 = \frac{4e^2}{c} \sum_{l,m} |\vec{b}_{lm}(k, \omega)|^2 \tag{94}$$

The above equation is used to calculate the spectrum of emitted radiation. In practice one needs only few multipoles. The contribution coming from $l = 4$ term is already negligibly small.

In order to determine the rate of emitted radiation let us consider eq. (79):

$$P(t + r/c) = \int \frac{dP}{d\Omega}(t + r/c) d\Omega = \frac{e^2}{4\pi c} \int \left| \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} (\vec{k} \times \vec{j}(\vec{k}, \omega)) \exp(-i\omega t) \right|^2 d\Omega \tag{95}$$

$$= \frac{e^2}{4\pi c} \int \left| \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left(\int d^3r (\vec{\nabla} \times \vec{j}(\mathbf{r}, \omega)) \exp(-i\vec{k} \cdot \mathbf{r}) \right) \exp(-i\omega t) \right|^2 d\Omega \tag{96}$$

$$= \frac{e^2}{4\pi c} \int \left| \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left(\int d^3r 4\pi \sum_{l,m} (-i)^l \vec{b}(\mathbf{r}, \omega) j_l(kr) Y_{lm}(\hat{k}) Y^*_{lm}(\hat{r}) \right) \exp(-i\omega t) \right|^2 d\Omega \tag{97}$$

$$= \frac{e^2}{4\pi c} \int \left| \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left(4\pi \sum_{l,m} (-i)^l \vec{b}_{lm}(k, \omega) Y_{lm}(\hat{k}) \right) \exp(-i\omega t) \right|^2 d\Omega \tag{98}$$

$$= \frac{e^2}{4\pi c} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left((4\pi)^2 \int d\Omega \sum_{l,m,l',m'} (-i)^l i^{l'} \left(\vec{b}_{lm}(k, \omega) \cdot \vec{b}^*_{l'm'}(k', \omega') \right) Y_{lm}(\hat{k}) Y^*_{l'm'}(\hat{k}) \right) \times$$
$$\times \exp(-i(\omega - \omega')t) \tag{99}$$

Note that in the last two lines of the above expression $\hat{k} = \hat{k}'$ because the vectors $\vec{k}, \vec{k}'$ differ only by length ($\omega = ck, \omega' = ck'$) but have the same direction specified by the angle Ω. Therefore:

$$P(t + r/c) = \frac{e^2}{4\pi c} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \left((4\pi)^2 \sum_{l,m} \left(\vec{b}_{lm}(k,\omega) \cdot \vec{b}^*_{lm}(k',\omega') \right) \right) \exp(-i(\omega - \omega')t) \tag{100}$$

$$= \frac{e^2}{\pi c} \sum_{l,m} \left| \int_{-\infty}^{\infty} \vec{b}_{lm}(k,\omega) \exp(-i\omega t) d\omega \right|^2 \tag{101}$$

The last equation is used in practice to calculate the rate of emitted radiation.

The above prescriptions work efficiently if one considers the radiation emitted due to internal nuclear excitation. However in order to determine the contribution coming from the CM motion of the nucleus the simpler formula can be derived. In this case the proton current reads:

$$\vec{j}_p(\mathbf{r}, t) = \vec{V}(t) \delta(\mathbf{r} - \mathbf{r}_0(t)) \tag{102}$$

Then

$$\vec{A}(\mathbf{r}, \omega) = \frac{\exp(ikr)}{r} \frac{Ze}{c} \int d^3r' \vec{j}_p(\mathbf{r}', \omega) \exp(-i\vec{k} \cdot \mathbf{r}') \tag{103}$$

$$= \frac{\exp(ikr)}{r} \frac{1}{c} Ze \int_{-\infty}^{\infty} dt \exp(i\omega t) \vec{V}(t) \exp(-i\vec{k} \cdot \mathbf{r}_0(t)) \tag{104}$$

where

$$\vec{j}_p(\vec{k}, \omega) = \int_{-\infty}^{\infty} dt \exp\left(i\omega \left(t - \frac{1}{c} \vec{n} \cdot \mathbf{r}_0(t) \right) \right) \vec{V}(t) \tag{105}$$

$$\approx \int_{-\infty}^{\infty} dt \exp(i\omega t) \vec{V} \left(t + \frac{1}{c} \vec{n} \cdot \mathbf{r}_0(t) \right) \tag{106}$$

$$\approx \int_{-\infty}^{\infty} dt \exp(i\omega t) \vec{V}(t) = \vec{V}(\omega), \tag{107}$$

where $\omega = ck$. The approximation was made above that the velocity is small and the movement of the nucleus is negligible. Therefore the possible perturbation of the radiation due to the change of nucleus position can be neglected.

Consequently:

$$\vec{B}(\mathbf{r}, \omega) = \frac{iZe}{c} \frac{\exp(ikr)}{r} \vec{k} \times \vec{V}(\omega) \tag{108}$$

and

$$\vec{B}(\mathbf{r}, t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \vec{B}(\mathbf{r}, \omega) \exp(-i\omega t) \tag{109}$$

$$= \frac{iZe}{c^2} \frac{1}{r} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \exp\left(-i\omega \left(t - \frac{r}{c} \right) \right) \omega \vec{n} \times \vec{V}(\omega) \tag{110}$$

$$= -\frac{Ze}{c^2} \frac{1}{r} \frac{d}{dt} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \exp\left(-i\omega \left(t - \frac{r}{c} \right) \right) \vec{n} \times \vec{V}(\omega) \tag{111}$$

$$= -\frac{Ze}{c^2} \frac{1}{r} \vec{n} \times \frac{d}{dt} \vec{V} \left(t - \frac{r}{c} \right) \tag{112}$$

Therefore

$$\frac{dP}{d\Omega}(t) = \frac{c}{4\pi} r^2 \left| \vec{B}(\mathbf{r}, t) \right|^2 = \frac{1}{4\pi c^3} (Ze)^2 \left| \frac{dV\left(t - \frac{r}{c} \right)}{dt} \right|^2 \sin^2\theta \tag{113}$$

and

$$P(t) = \frac{2}{3} \frac{(Ze)^2}{c^3} \left| \frac{dV\left(t - \frac{r}{c} \right)}{dt} \right|^2 \tag{114}$$

Spectral decomposition:

$$\frac{dE}{d\Omega d\omega} = \frac{(Ze)^2}{4\pi^2 c}|\vec{k}\times\vec{V}(\omega)|^2 \tag{115}$$

and integrating over angles

$$\frac{dE}{d\omega} = \frac{2}{3\pi}\frac{(Ze)^2}{c}k^2|\vec{V}(\omega)|^2 = \frac{2}{3\pi}\frac{(Ze)^2}{c^3}\omega^2|\vec{V}(\omega)|^2 = \frac{2}{3\pi}\frac{(Ze)^2}{c^3}\left|\frac{d\vec{V}}{dt}(\omega)\right|^2 \tag{116}$$

where $\frac{d\vec{V}}{dt}(\omega)$ is the Fourier transform of acceleration:

$$\frac{d\vec{V}}{dt}(\omega) = \int dt \frac{d\vec{V}}{dt}(t)\exp(i\omega t) \tag{117}$$

The above derivation assumes that the moving nucleus can be treated as a point-like particle. This is a reasonable approximation although it is not difficult to include suitable corrections. Let us consider the proton current in the form:

$$\vec{j}_p(\mathbf{r},t) = \vec{V}(t)\rho(\mathbf{r}-\mathbf{r}_0(t)) \tag{118}$$

Using the same assumption as before, ie. that the motion is nonrelativistic and movement in space is negligible one gets:

$$\vec{j}_p(\vec{k},\omega) = \vec{V}(\omega)\rho(\vec{k}) \tag{119}$$

and (see (82)):

$$\int \frac{dE}{d\Omega d\omega}d\Omega = \frac{(Ze)^2}{4\pi^2 c}|\vec{k}\times\vec{V}(\omega)\rho(\vec{k})|^2 d\Omega = \frac{(Ze)^2}{4\pi^2 c}\frac{1}{c^2}\int|\vec{n}\times\omega\vec{V}(\omega)\rho(\vec{k})|^2 d\Omega \tag{120}$$

$$= \frac{(Ze)^2}{4\pi^2 c^3}\int\left|\vec{n}\times\frac{d\vec{V}}{dt}(\omega)\rho(\vec{k})\right|^2 d\Omega \tag{121}$$

where $\vec{n} = \frac{\mathbf{r}}{r}$. In the case of spherical density distribution it simplifies to:

$$\int \frac{dE}{d\Omega d\omega}d\Omega = \frac{(Ze)^2}{4\pi^2 c^3}\int\left|\vec{n}\times\frac{d\vec{V}}{dt}(\omega)\right|^2|\rho(k)|^2 d\Omega = \frac{2}{3\pi}\frac{(Ze)^2}{c^3}\left|\frac{d\vec{V}}{dt}(\omega)\right|^2|\rho(k)|^2 \tag{122}$$

The above expressions can be used to determine the spectrum of emitted radiation. In the Figures 5 and 6 the contributions to the energy spectrum coming from dipole and quadrupole terms are plotted for 3 values of impact parameter. The difference between the figures originates from two different smoothing widths that have been applied. Namely, the original curves have been convoluted with gaussians of widths 1 MeV (Fig. 5) and 0.5 MeV (Fig. 6).

For the radiation caused by the CM acceleration after collision the decomposition into multipoles is not useful and one can apply instead eqs. (**??**). The emission occurs within much shorter time scale governed by the collision time $\tau_{coll} = \frac{b}{v\gamma}$. The results are plotted in the Figs. 8, 9, 10.

Dipole dynamics and neutron emission

The framework of TDSLDA allows to calculate various one body observables. In this case the most important is the nuclear dipole moment. Only two components of the dipole moment, lying in the reaction plane, can oscillate as a result of collision. In the Figs. 13, 14, 15 these two components of the dipole moment have been plotted.

During the time evolution the nucleus can emit particles. In order to investigate this effect we have calculated the number of neutrons/protons within shells of various radii. As one can see from the Figs. 16, 17, 18, 19 the number of protons in the shells outside the nucleus is negligible. Moreover this proton number is approximately constant which indicate that we rather probe the tale of the proton distribution than the emission process. On the contrary the situation is different for neutrons. The number of neutrons in the smaller shell is much larger, although it is also approximately constant. However in the larger shell the number of neutrons is constantly increasing in time with a fairly constant average rate. It indicates that the neutron emission occurs as a result of Coulomb excitation process.

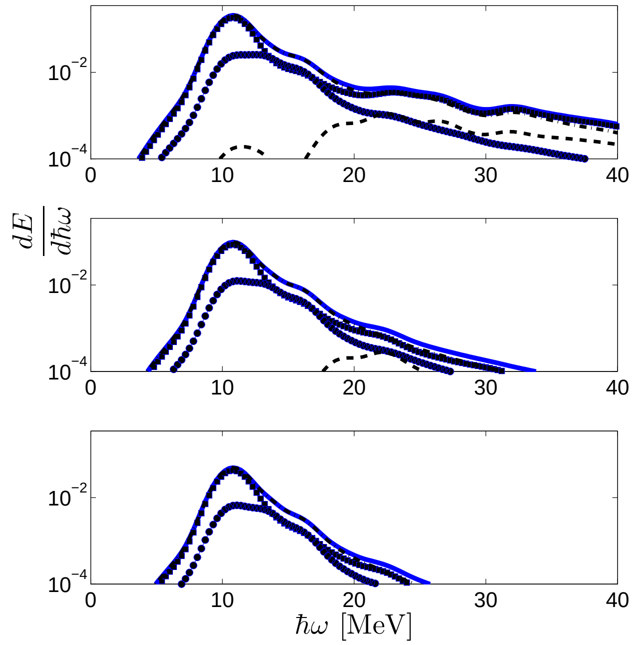

FIG. 5. (color online) The energy spectrum of emitted electromagnetic radiation due to internal excitation of the target nucleus, caused by the collision at the impact parameters b = 12.2 fm (upper subfigure), b = 16.2 and b = 20.2 (lowest subfigure) . The contributions from two orientations of the target nucleus are shown: perpendicular (squares) and parallel (circles) with respect to the incoming projectile. Dotted dashed line represents the dipole component of the radiation. Dashed line represents the quadrupole component of the radiation. In this case the smoothing width of the original curves was set to 1 MeV.

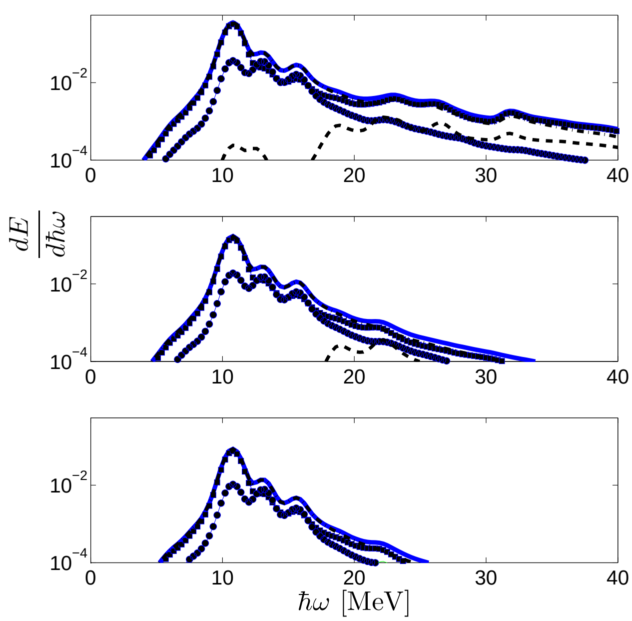

FIG. 6. (color online) The same as in the Fig. 5, but in this case the smoothing width of the original curves was set to 0.5 MeV.

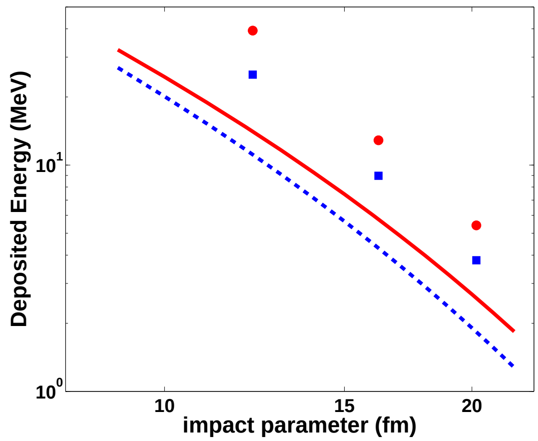

FIG. 7. (color online) Energy deposited in the target nucleus ^{238}U for three values of the impact parameter: $12.2, 16.2, 20.2$ fm and for two nuclear orientations: nuclear symmetry axis being parallel (squares) and perpendicular (circles) to the trajectory of incoming projectile. The same quantity is shown for the Goldhaber-Teller model, assuming that the frequencies of the dipole oscillations are $\hbar\omega = 12$ MeV and $\hbar\omega = 16$ MeV parallel (blue-dashed line) and perpendicular (red-solid line) to the nuclear symmetry axis, respectively.

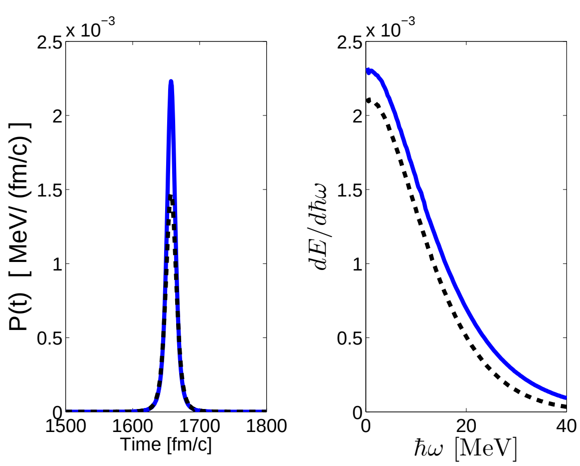

FIG. 8. (color online) The gamma emission rate (left panel) due to bremsstrahlung for the collision at the impact parameter $b = 12.2$fm. The right panel shows the energy spectrum emitted. Solid and dashed lines correspond to the perpendicular and parallel orientation of the target nucleus with respect to incoming projectile.

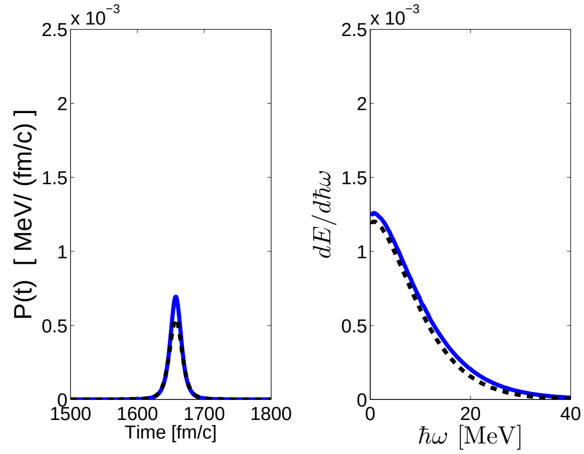

FIG. 9. (color online) The same as in the Fig. 8, but for the impact parameter $b = 16.2$fm.

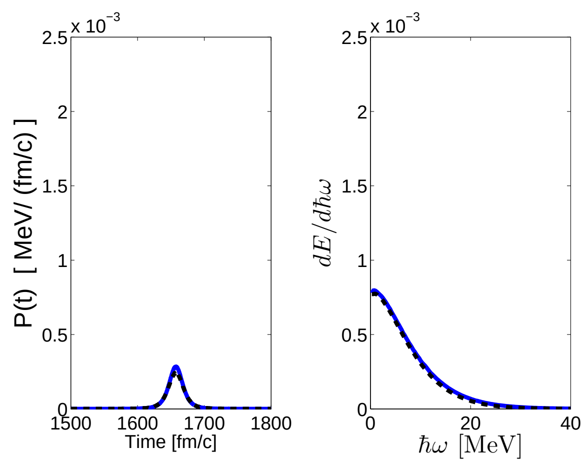

FIG. 10. (color online) The same as in the Fig. 8, but for the impact parameter $b = 20.2$fm.

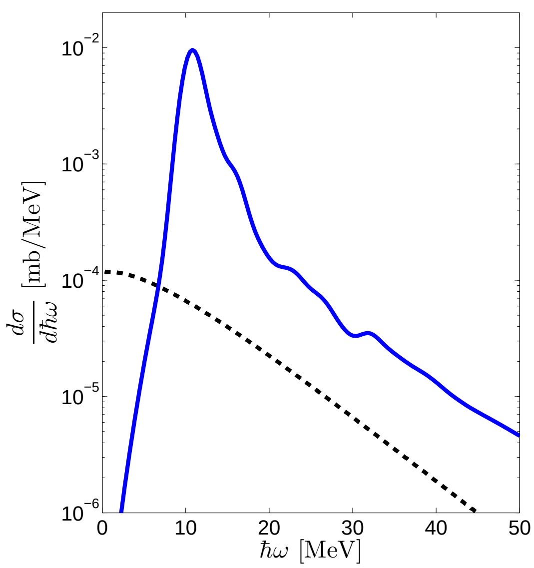

FIG. 11. (color online) The contributions to the cross section with respect to gamma emission during 2500 fm/c after collision. The dashed line represents the Bremsstrahlung contribution. The solid line shows the contribution from intrinsic excitation modes.

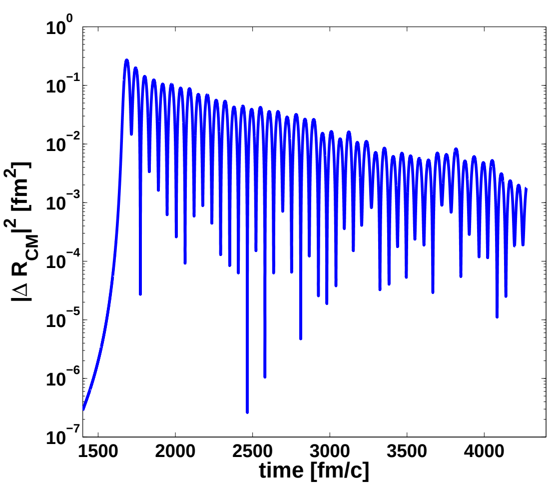

FIG. 12. (color online) The distance squared between the CM of protons and the total nuclear CM: $|\Delta R_{CM}|^2$ as a function of time. The impact parameter is $b = 12.2$ fm, and the nuclear symmetry axis of the target is perpendicular to the projectile's trajectory. The slope does not depend on the orientation and the impact parameter. The numerical fit to the maxima (squared amplitudes of dipole oscillations) with the function $\exp(-t/\tau)$ yields $\tau \approx 500$ fm/c.

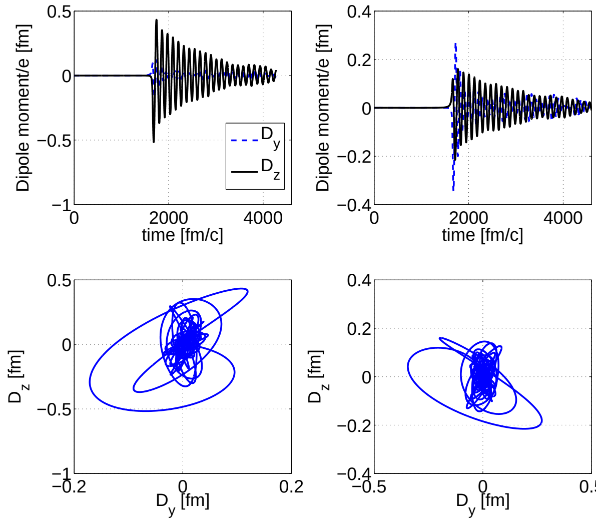

FIG. 13. (color online) Two components of the dipole moment: D_z and D_y as a function of time. The left and right subfigures correspond to the collision with projectile moving along the y-axis and z-axis , respectively. Impact parameter: $b = 12.2$fm.

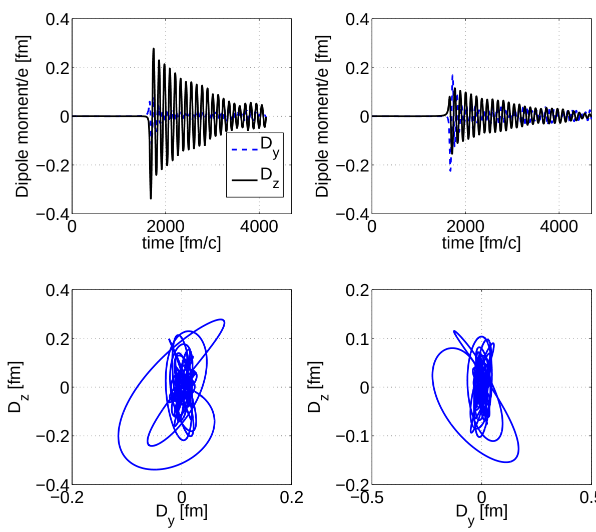

FIG. 14. (color online) The same as in the Fig. 13, but for the impact parameter $b = 16.2$fm.

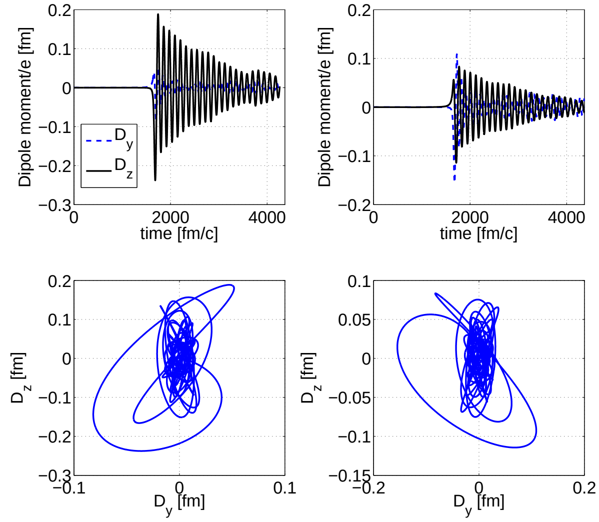

FIG. 15. (color online) The same as in the Fig. 13, but for the impact parameter $b = 20.2$fm.

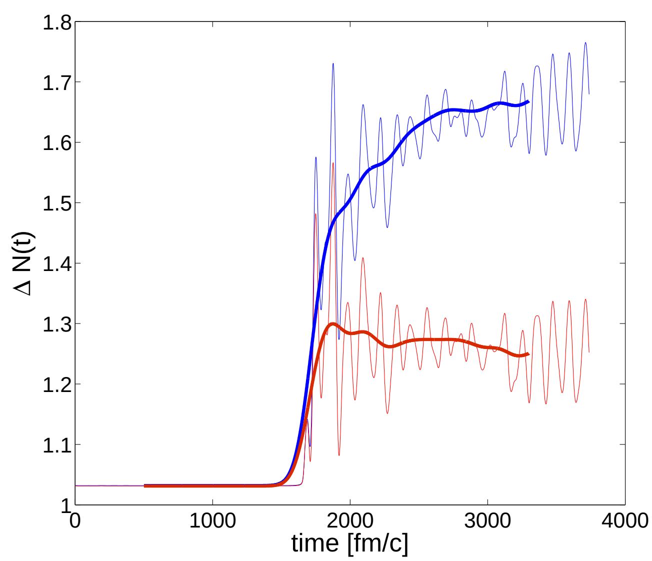

FIG. 16. (color online) The number of neutrons present within the shell with inner radius 10fm and outer radius 15fm (red line). The number of neutrons present within the shell with inner radius 10fm and outer radius 20fm (blue line). Thin line corresponds to the actual number of neutrons, whereas the thick line denotes the average value. The plot corresponds to the collision with the target nucleus symmetry axis perpendicular to the trajectory of the incoming projectile. The impact parameter $b = 12.2$ fm.

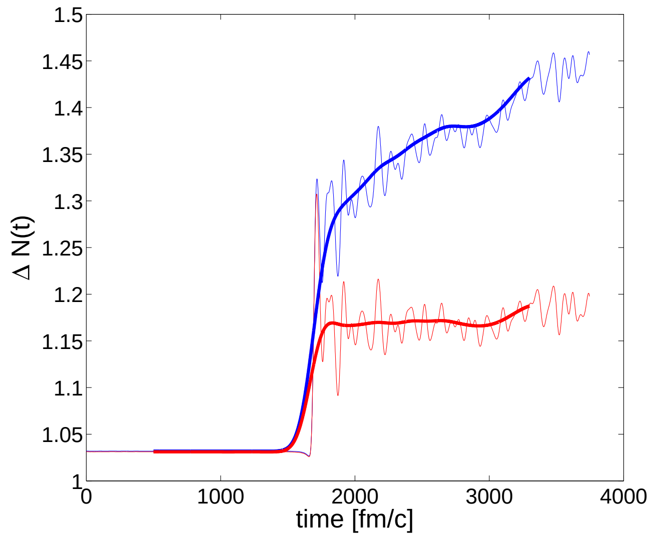

FIG. 17. (color online) The same as in the Fig. 16, but for the nuclear orientation parallel with respect to the incoming projectile.

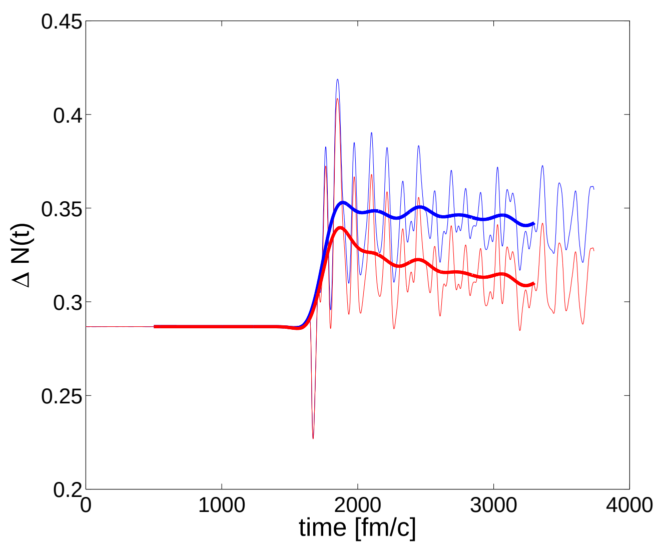

FIG. 18. (color online) The same as in the Fig. 16, but for protons.

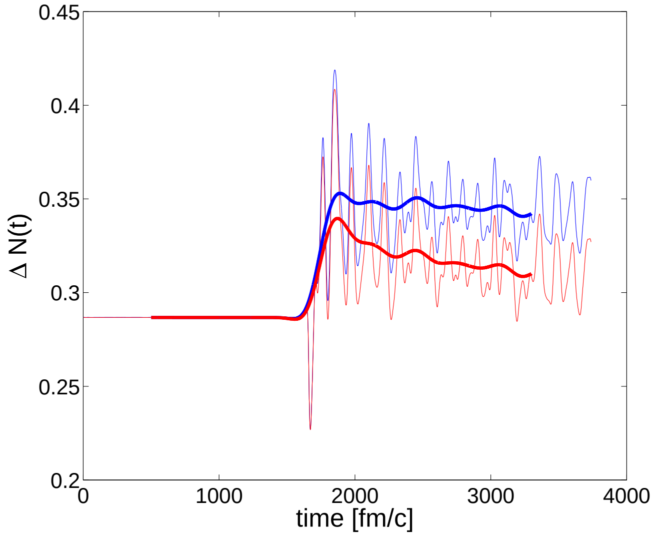

FIG. 19. (color online) The same as in the Fig. 17, but for protons.