INFLUENCE OF PARALLEL DARK MATTER SECTORS ON BIG BANG NUCLEOSYNTHESIS

A THESIS by VENKATA SAI SREEHARSHA CHALLA

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ABSTRACT

INFLUENCE OF PARALLEL UNIVERSES OF DARK MATTER ON BIG BANG NUCLEOSYNTHESIS

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Big Bang Nucleosynthesis (BBN) is a phenomenological theory that describes the synthesis of light nuclei after a few seconds of the cosmic time in the primordial universe. The twelve nuclear reactions in the first few seconds of the cosmic history are constrained by factors such as baryon to photon ratio, number of neutrino families, and present day element abundances. The belief that the expansion of the universe must be slowed down by gravity, was defeated by the recent observation of an accelerated expansion of the universe. Friedmann equations, which describe the cosmic dynamics, need to be revised considering also the existence of dark matter, another recent astronomical observation. The effects of multiple parallel universes of dark matter (dark sectors) on the accelerated expansion of the universe are studied. Collectively, these additional effects will lead to a new cosmological model. We had developed a numerical code on BBN to address the effects of such dark sectors on the abundances of all the light elements. We have studied the effect of degrees of freedom of dark-matter in the early universe on primordial abundances of light elements. The predicted abundances of light elements are compared with observed constraints to obtain bounds on the number of dark sectors, N_{DM} . Comparison of the obtained results with the observations during the BBN epoch shows that the number of dark matter sectors are only loosely constrained, and the dark matter sectors are colder than the ordinary matter sectors. Also, we verified that the existence of parallel dark matter sectors with colder temperatures does not affect the constraints set by observations on the number of neutrino families, N_{ν} .

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1. Introduction

1.1 Statement of the Problem

Big Bang Nucleosynthesis (BBN) deals with the production of light nuclei during the early moments of existence of the universe. To present date, our understanding of the evolution of the universe and the fundamental forces that explain cosmodynamics is incomplete. Numerous theories aimed at explaining the rapid rate of expansion of the universe include the creation and destruction of matter and its energy density changes. BBN serves as a fingerprint of such physical evolution of the universe. The effects of dark matter and dark energy on the expansion of the universe have changed the entire view of the predicted cosmological model. With limited experimental data available, one is compelled to revive the cosmological constant model introduced by Einstein in order to explain the accelerated expansion rate. But this scenario is not ideal and defies our understanding of the physical reason for the cosmological constant. This constant might be for example related to super-symmetric particles, but we do not know for sure.

1.2 Purpose of the Study

Fusion of primordial elements started roughly a millisecond after the Big Bang. It produced helium (He), which amounts to one quarter of the total matter in the visible universe by mass. Deuterium, Tritium, Lithium, and Beryllium are observed in considerably smaller amounts. But they are also important fingerprints of the BBN model, which is a remarkably successful theory in matching the predictions to observations of elements in the universe. The two important aspects of the big bang theory of relevance for BBN are the thermal physics and the rate of occurrence of nuclear reactions [1].

To solve the equations for the BBN one also needs Friedmann equations to predict how the universe cooled down. With the assumption of dark matter sectors, such equations are modified, leading to a plausible explanation for some of the few issues remaining with the predictions of the BBN such as the *Lithium problem*. The Lithium problem is related to the fact that the BBN predicts a larger amount of the Lithium-7 isotope than is observed in astronomy. This is the last remaining puzzle associated with

the BBN. We had assessed this problem by calculating the effects of new degrees of freedom induced by the dark sectors on the prediction of the light elements following the BBN. A comparison of theoretical predictions with observational constraints on primordial abundances of the light elements, we thereby obtain bounds on the number of dark sectors N_{DM} [2].

1.3 Hypothesis

Based on the limited experimental evidence on the dark particles, we will assume that dark matter consists of particles which are copies of the known particles in the Standard Particle Model. The dark particles interact as usual within their sectors, but very weakly and mostly gravitationally interacting with particles in the other sectors. Despite the interactions being very weak between the dark sectors, they still have had an appreciable effect in changing the number of degrees of freedom in the early universe. This obviously had affect the BBN and its predictions for the abundance of light elements.

1.4 Basic Definitions

- Baryons : A particle with spin half odd integers with mass equal to or greater than that of proton
 [3] . It is a composite quark based subatomic particle.
- 2. Gluons: The particle that gets exchanged between two quarks in a strong process [3, 4].
- 3. Photons : Quantum of light and other forms of electro magnetic radiation [3, 4].
- Bosons : Bosons are particles with integer spin which are characterized by Bose-Einstein statistics. They are force carrier particles [5]. Gluons and photons are elementary bosons and mesons are quark composite bosons.
- 5. **Inflation :** Inflationary epoch refers to the period of rapid expansion of the universe due to a phase transition when the nuclear force broke away from weak and electromagnetic forces which were unified at Planck's era as electronuclear force. This phase transition resulted in a scalar field which filled the entire universe with vacuum energy [1]. This cosmological type of energy density dominated the energy density of the universe where gravitation became repulsive for a period of

 10^{-32} s during which universe had expanded at an astonishing rate increasing its size with a factor of 10^{50} . This enormous expansion was much more than predicted by big bang theory.

6. Scale Factor : Scale factor is a expansion parameter that represents size of the universe [1, 6]. It refers to the proper distance (a function of time) between two celestial objects at any time t with respect to a reference distance. Time t is measured from birth of universe whereas reference time t_0 is the present age of universe

$$a(t) = \frac{d(t)}{d(t_0)} \tag{1}$$

7. Hubble's Law : Edwin Hubble found out that galaxies are moving away from the Earth which is referred as Hubble's expansion law. This is also observed as red shift [1, 6, 7] of galaxies spectrum. Hubble's constant is defined as the ratio of the radial outward velocity of a galaxy to the distance of that galaxy from the earth. The Hubble's constant (H) is a measure of the expansion of the universe. The value of the velocity can be obtained from spectroscopic observations (red shift).

$$H = \frac{v}{d} \tag{2}$$

Friedmann equations: Friedmann equation governs the dynamics of the expansion of the universe.
 For homogeneous and isotropic universe [6–8] ,

$$H^{2} = \frac{8}{3}\pi G\rho - \frac{kc^{2}}{R^{2}} + \frac{\wedge c^{2}}{3}$$
(3)

The first term in the equation represents the effect of matter/energy density and second term represents curvature density and final term is due to a cosmological constant.

9. **Big bang nucleosynthesis (BBN) and the baryon to photon ratio :** The observed abundances of ⁴He,³He,²H,⁷Li are compared with predictions from BBN theory. Helium isotopes cannot be destroyed easily and they are continuously produced in stellar cores whereas deuterium is easily

destroyed to He isotopes due to less stability. BBN depends on baryon to photon ration (η). η constraints the total number of particles contributing to the energy density during BBN epoch.

- 10. Black Body Spectrum : A black body is that absorbs all the electromagnetic radiation incident upon it and emits radiation to stay in thermal equilibrium [9]. The thermal radiation emitted is a characteristic of the temperature. The intensity of the emitted radiation is proportional to the fourth power of the absolute kelvin temperature. The thermal radiation produced by hot bodies has a continuous spectrum. The intensity of the cosmic background radiation at different wave lengths is consistent with the thermal radiation distribution of a black body. This in fact is a valid evidence that our universe had started with a singularity [10], and was infinitely hot and dense at the beginning followed by rapid expansion and cooling ever since.
- 11. Gauge Theory : Gauge field theory is a special quantum field theory in which matter fields (hadrons-baryons and mesons, leptons) interact between themselves with forces mediated by the exchange of vector bosons (photons and gluons). Such gauge particles include higgs particles. No experimental proof validating the existence of higgs bosons was put forth except theoretical requirements by gauge theories. Gauge symmetry refers to invariance of the theory under transformations by a symmetry group [11]. These are local symmetries that act differently at each point of space and time. Gauge theories is called *Abelian*, if their gauge fields obey the additional commutative laws. The triumph of gauge theories is that they provide a unified framework describing the quantum mechanical behavior of three of the four fundamental forces of nature namely electromagnetic, strong and weak forces. This gauge theory is referred as the standard model, with an overall symmetry denoted by SU(3)*SU(2)*U(1) [11, 12].

U(1): U(1) refers to unitary and complex matrices of unit norm.

$$U = Z = \exp(i\psi),\tag{4}$$

where ψ is real.

A gauge symmetry transformation of a given quantum state $\psi(x, t)$ is given by :

$$\psi'(x,t) = \exp\left(i\psi(x,t)\right)\dot{\psi}(x,t) \tag{5}$$

SU(n): SU(n) denotes complex and unitary n * n matrices with determinant 1.

SU(2): SU(2) denotes isospin symmetry of weak interactions mediated by W^+ , W^- and Z particles. Isospin symmetry in nuclei refers to symmetry invariance under interchange of neutrons and protons. It refers to a quantum number related to the strong interaction. Particles involved in strong interactions are considered as a doublet, that is, the same particle with different isospins.

SU(3): SU(3) is a color symmetry of strong interaction (hadrons) [13]. Hadrons are made up of three different flavours namely up, down and strange quarks. SU(3) acts on quarks in a similar way that isospin acts on neutrons and protons.

SU(3) flavor symmetry was considered an incomplete theory after the discovery of other three quarks (charm, top and bottom). The theory of QCD which assumes that quarks have three different colors as well as flavors accounts for the present way of understanding strong interactions. It is based on color SU(3) gauge symmetry.

SU(3)*SU(2)*U(1) is a standard model that encompasses color, isospin and hypercharge and is a most successful model that unified strong, weak and electromagnetic forces under one roof. The agreement of color gauge theories with experiment is impressive in concordance the of standard model with experiments and observations.

12. **Red Shift :** Hubble found a remarkable discovery that light from distant stars from distant galaxies appear to have a longer wavelength. The spectra was appeared to be shifted towards the red end of the spectrum [7]. By using Doppler shift measurements, it is evident that distant galaxies are moving away from us with increasing speeds which is a first experimental evidence that the universe

is expanding.

13. The QCD Lagrangian : The QCD Lagrangian is given by ,

$$L_{QCD} = \psi'(i\gamma^{\mu}D_{\mu} - m)\psi - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}$$
(6)

where, ψ represents the quark field, $G_{\mu\nu}$ represents the gluon field tensor and γ^{ν} are the Dirac matrices [12].

2. Literature Survey

A leading explanation about the beginning of the universe from a small singularity and inflating over 13.8 billion years to the present day cosmos is the Big bang theory [1, 6, 7, 14]. It is the most successful cosmological model for the evolution of the universe from high dense regime to galactic clusters. The timeline of the theory adopts a temperature dependence to describe the initial expansion, formation of elementary particles, their interactions and the synthesis of elements to form matter that later coalesced through gravity to form stars and galaxies. The big bang theory was a physical model developed from the observations as the sensitivity of current instruments constrains us to explore back further to understand the prevailing conditions at the universe's birth. The standard theories of particles and Einstein's general theory of relativity laid the mathematical foundations of the theory. The analysis of observations from Hubble on the universe expansion and the discovery of the Cosmic Microwave Background (CMB) provided an experimental evidence validating the theory universally. The consistency of predicted abundances of light elements with observations is a potential evidence for the big bang theory, as it is the only theory known that can explain the synthesis of elements at early stages. The most recent observational evidences include the formation and evolution of galaxies and distribution of large-scale cosmic structures [15]. These observational evidences are considered as the main pieces that support the most dominant cosmological model of the universe. Though the big bang theory is a well established model, it is under intense investigation due to newly revealed mysteries like dark matter as they significantly influence the cosmo dynamics.

2.1 Big Bang Theory

The observations of dynamics of all other galaxies moving relative to ours with increasing speeds with distance lead to the conclusion that they have been propelled by an ancient explosive force. This observation lead to birth of the most successful theory that explains the origin and fate of our universe. A leading explanation that the universe had started with a very small singularity [10], inflated over a period of time, and finally to the cosmos today is the big bang theory. It estimates the age of universe to be around 13.8 billion years based upon temperature fluctuations in the CMB, the present value of the

Hubble constant, and many other observations.

The singularities are considered as zones of extreme gravitational pressure. The pressure was so intense that finite matter is squeezed into infinite density. It was believed that the origin of the universe is from a singularity. The singularity refers to a infinitesimally small, infinitely hot and dense zone. The singularity apparently inflated and the temperature fluctuations lead to a detailed picture of sequential steps that explained the evolution of matter. The universe followed a rapid expansion state under the inflationary epoch due to quantum perturbations. However, this interpretation of quantum perturbations during inflationary period had no experimental evidence to support this theory. A proper investigation in the direction of singularities leading to the origin of the universe was achieved by Stephen Hawking and George Ellis [16]. They turned their attention to the theory of relativity and its implications over notions of time, extending the theory of general relativity to include observations of time and space. The observations reflected that time and space had finite origin that lead to the beginning of matter and energy. Prior to the singularity, nothing existed and singularity refers to origin of space and time. There is a lot of misconceptions about the existence of singularity but a logical explanation is to consider that the singularity did not appear in space, rather, the space began inside a singularity. This singularity, normally referred to as *Big Bang* Singularity or Initial Singularity [10, 17], was a gravitational singularity that followed rapid expansion by quantum fluctuations and expanded over years creating the present day universe.

The direct observational evidence for the big bang was the Hubble's law of expansion of the universe and the CMB that fills the present universe. The glow of CMB is the surviving trace of left over light from the big bang. The left over radiation (CMB) consists of abundant amount of information about primordial conditions prevailing at the birth of the universe.

2.1.1 Cosmic Wave Background Spectrum (CMB) and Planck's spectrum

George Gamow was the cosmologist who investigated the early effects of big bang considering the consequences of the huge amount of heat dissipated after the explosion. The researchers Ralph Alpher and Robert Herman asserted that the total amount of energy dissipated during the big bang had been distributed homogeneously in all directions in the form of a cosmic wave background radiation [18]. The

first experimental proof of the microwave background was provided by Amo Penzias and Robert Wilson. The microwave antenna sensed microwave signals from all directions with equal magnitude proving the homogeneity. Later evidence was provided by two satellites Cosmic Background Explorer (COBE) and Wilkinson Microscopy Anisotropic Probe (WMAP) with great accuracy. The cosmic wave background probes has provided both observational evidence and the way to investigate further into the early universe. When the size of the universe was one hundred millionth of its present size, the universe was filled with ionized hydrogen with densities around 1000 ions per cubic centimeter. Due to the high temperatures ranging around 278 million degrees above absolute zero, electrons and protons were in ionic state. The scattering of free electrons and CMB photons resulted in the blackbody spectrum of photons.



Figure 1. Cosmic Microwave Spectrum from FIRAS Experiment [19].

2.1.2 Hubble Expansion

Assuming an isotropic and homogeneous universe, Einstein formulated his theory of general relativity to describe space time relative to cosmo dynamics. Einstein's theory had put down the Newtonian view of infinite and static universe when he realized that the the equations of general relativity do not describe a static universe [20]. When the universe is considered as a whole, the space-time as a whole must be warped and start curving on itself, wherein matter will be essentially in motion, and thereby shrinking on its own gravity. To counter act gravity, Einstein introduced a cosmological constant in order to have a stable and static universe. A clear picture of the modern expansion paradigm was developed by George Lemaitre that the universe could not be static, but expanding. The observational evidence for an expanding universe was put forth by Edwin Hubble, which formed the basis for modern expansion paradigm.

During 1920s and 30s, Edwin Hubble discovered that the domain of the universe is much larger than our milky way galaxy. He demonstrated that there are millions of galaxies in this universe, many of those are at huge distances from our own. Later, he noticed that the light from the other galaxies is moving towards the red end of the spectrum indicating that they are moving away from us. This is referred to as Red Shift [7] and is due to the Doppler effect. The observed red shifts are exceptionally isotropic and homogeneous supporting the cosmological theory that the universe is isotropic from all the directions. Hubble showed that the speed of the moving galaxy is in direct proportion to its distance. The proportionality is called as Hubble's law. Hubble's law states that the universe is growing in size and that a galaxy twice as far away as another is receding twice as fast.

$$v = H_0 d. \tag{7}$$

This expansion is referred as a metric expansion which means that the individual galaxies are not growing themselves but the that chunks of separated matter (cluster of galaxies) at the beginning of the universe are getting widely separated. Thus the universe is not expanding into the existing space but the space itself is expanding by relative separation of the cluster of galaxies [21]. Under such an expansion, all other galaxies see the others moving away from themselves. Thus the framework of galactic dispersion is homogeneous from anywhere in the cosmos.

As the density of the universe was falling as a function of time [22], so there was a period when the universe was infinitely dense and extremely hot. This logical interpretation leads to the first observational evidence that the universe is expanding with time and serves as a solid evidence most cited in support of the big bang theory.

2.2 Timeline of Big Bang

The timeline of the big bang follow distinct epochs that define the origin of the universe from early periods to the present day.

- 1. Planck radiation epoch: All known four forces were unified. Density and temperature nearly infinite.
- 2. Grand unified era: Gravity freezes out and breaks from unification.
- 3. Electroweak era: The strong nuclear force freezes out and the fundamental forces (weak and electromagnetic forces) are defined under the single name of Electroweak force.
- 4. **Particle formation epoch:** Particles are formed during this time. The formation of neutrons and protons took place and are in thermal equilibrium.
- 5. Nuclear epoch: Helium and Deuterium were formed by the fusion of protons and neutrons.
- 6. Atomic epoch: In this epoch, atoms are formed after the fusion of leptons with nuclei. During this period first stars were formed.
- 7. Galactic epoch: Formation of large scale structures like galaxies occurred.
- 8. **Stellar epoch:** The star density is maximum and the galaxies are combined to form extra galactic clusters leading up to the present.

The first few epochs are defined to be very dense and short lasting. The laws of physics were considerably different in first few eras but a significant amount of information can be inferred by investigating the later eras. As discussed, nothing existed prior to singularity before, no matter and no time [10, 17]. This singularity cannot be understood solely on the theory of general relativity [23]. However, a detailed conjuncture during succeeding intervals after the big bang depicts a clear picture of the evolution of the universe from the singularity to the entire cosmos including the formation of clusters, galaxies and so on.

 $t \sim 10^{-43}$ s. This time can be considered as early phase of development of the universe. There are many speculations about the conditions prevailed at this period. As the energy densities are very high, the classical treatment of general relativity during this period is no longer valid. The existing temperature of the universe was 10^{32} K. It is believed that quantum fluctuations are responsible for the formation of matter known as the quantum birth of the universe. Due to the extremely high energy densities and temperatures, no plausible evidence or successful explanation can provide further information except a quantum theory of gravity.

 $t \sim 10^{-38}$ s. Strong and weak interactions are indistinguishable. At this period, Grand Unified Theories(GUT) phase transition occurs which is highly speculative. The temperature was around 10^{29} K.

 $t \sim 10^{-34}$ s. This refers to the inflationary period [1, 24]. The universe followed a rapid expansion increasing its size by factor of 10^{30} . The temperature was 10^{27} K. Quantum fluctuations are responsible for the formation and distribution of matter in space at a given time. The universe was filled with photons, leptons and quarks that constraints the formation of protons and neutrons due too high temperature.

 $t \sim 10^{-30}$ s. Peccei-Quinn phase transition occurred when the free quarks combine to form hadrons. This transition resulted in the formation of axions. Axions are the cold dark matter particles.

 $t \sim 10^{-8}$ s. Dark matter interaction freeze out period. If the dark matter consists of super-symmetric particles or WIMPS, the interaction between them is decoupled fixing their cosmological abundance.

 $t \sim 10^{-4}$ s. Quark-hadron phase transition where quarks and gluon become bound into neutrons, protons and their anti particles. Due to a rapid expansion, the temperature cooled down and the resulting photons do not have any excess energy to break up the forming new particles. Matter and antimatter annihilate each other and the slight excess of matter is what we see today. Though there is no signature of a phase transition, it is believed that the universe went through this phase due to the formation of neutrons and protons leading to the synthesis of heavy nuclei.

 $t \sim 1$ min. The universe had cooled down so that the protons and neutrons first combine to form D, ⁴He, ²H, ³He and ⁷Li. This is known as Big Bang Nucleosynthesis, a remarkably successful theory which agrees impressively with the element abundances we observe today. A complete description of BBN is discussed in the coming sections.

 $t \sim 10^5$ s. The photon density falls out due to the freeze out of reactions that change the photon density in comparison with the expansion rate. Photons fall out of equilibrium as they are neither created nor destroyed.

 $t \sim 10^{-4}$ yrs. An inequality arises between matter and radiation density. The gravitational potential wells appear to grow due to perturbations in the dark matter density. These potential wells grow in size becoming dark matter halos.

 $t \sim 379000$ yrs. As a result of expansion, the universe cooled down to 2970 K, when electrons and protons combined to form hydrogen atoms . At higher temperatures, the CMB photons were tightly coupled to matter by Thompson scattering [25] from free leptons but due to the decreasing temperature, these CMB photons decouple. Atoms of helium and hydrogen agglomerate under influence of gravity forming higher structures like stars and galaxies.

 $t \sim 10^6$ yrs. Perturbations in baryons grows as the CMB photons are no longer coupled to the baryon fluid. This is referred to as baryon drag.

 $t \sim 10^{10}$ yrs. Baryons and CMB are completely decoupled. All visible and invisible matter from sub atomic to galactic scale is uniformly distributed in the universe. The uniformity of matter in the universe is constrained as the visible matter is collected in form of galaxies and super clusters of galaxies. It is assumed by the big bang theory that this occurred when the universe was 400,000 yrs. If so, the CMB should not be perfectly distributed today due to non-uniform distribution of matter [18, 26]. The experimental evidence for non-uniform CMB distribution was later provided by NASA's COBE and WMAP measurements. An increased resolution in the picture of non-uniformity is provided effectively by images resulting from WMAP observations of the universe when the universe was 379,000 yrs old. This is a triumph for the big bang theory and the inflation theory as the validity is tested accurately by the measurements from WMAP.

2.2.1 Thermal History of Early Universe

The early phases of big bang are very speculative. At the beginning, the universe was homogeneously and isotropically filled with a high energy density. After going through a sequential phase transitions, the density and temperature decrease and it expanded at an accelerated rate in all directions equally. The phase transition occurred at 10^{37} s and caused a cosmic inflation [24] leading to an exponential growth of the universe. After the inflation, the primordial soup consists of vast number of different species of elementary particles. The temperature of different species is not all same. The temperatures of individual species depend on their reactions to reach equilibrium and thermal history of the universe. As the universe got expanded and cooled down, the dominant number of species became non-relativistic and got involved in annihilation process at different times.

During the radiation dominated era [27], the energy density of relativistic particles is much greater than the energy density of non-relativistic particles. The total energy density of the universe is,

$$\rho(T) = \sum_{n} \rho_n(T) \tag{8}$$

where, the suffix runs over number of different particle species.

When the temperature of the universe fallen to around 100 GeV, the electromagnetic and weak force were differentiated by symmetry breaking providing grounds for matter-antimatter symmetry. As the temperatures had fallen to around 100 MeV, a QCD phase transition occured where quarks lost their asymptotic freedom and confined to hadrons. The free quarks and gluons after the transition formed three-quark systems called *Baryons(Fermions)* and quark-antiquark pairs called *Mesons(Bosons, Pions-\pi^{\pm}, \pi^{0})*, etc. The major contribution to the particle density at this temperature were electrons, neutrinos, photons, pions and muons. At temperatures around 20 MeV, pions and muons annihilate and as the temperatures falls further, the weak interaction became much weaker. When the temperature is close to 1 MeV, neutrinos decouple ceasing further interactions. The temperature had fallen and the universe continued to expand. In terms of the phase elements, as the volume of the universe increased (V), the momentum (p) space available for the particle species decreased but the phase space element ($d^{3}p \, dV$) stayed constant.

If N is the number of particles, the phase element $(d^3p \, dV)$ consists of [28],

$$dN = \frac{g}{2\pi^3} f(\vec{p_1}) d^3 p_1 dV_1$$
(9)



Figure 2. As the universe expands this increases the volume element dV and decreases the momentum space element d^3p but the phase space $(d^3p dV)$ remains constant.

where, g is effective degrees of freedom.

For relativistic particles [29],

$$f(\vec{p_1}) = \frac{1}{\exp\left(\frac{p_1 - \mu_1}{T_1}\right) \pm 1}$$
(10)

is the distribution function at time t_1 where μ is the chemical potential. At a later time t_2 , the same number of particles were now distributed in a phase space element $d^3p_2 dV_2$.

Substituting the following relation in the above equation results in ,

$$d^3 p_2 = \left(\frac{a_1}{a_2}\right)^3 d^3 p_1 \tag{11}$$

$$dV_2 = \left(\frac{a_2}{a_1}\right)^3 dV_1 \tag{12}$$

and the number of particles in the phase element are,

$$dN = \frac{g}{(2\pi)^3} \frac{d^3 p_1 \, dV_1}{\exp\left(\frac{p_1 - \mu_1}{T_1}\right) \pm 1},\tag{13}$$

16

$$dN = \frac{g}{(2\pi)^3} \frac{\left(\frac{a_2}{a_1}\right)^3 (d^3 p_2) \left(\frac{a_1}{a_2}\right)^3 (dV_2)}{\exp\left(\frac{\frac{a_2 p_2}{a_1} - \mu_1}{T_1}\right) \pm 1}.$$
(14)

where a_1 and a_2 are the scale factors corresponding to times t_1 and t_2 . Considering that $\mu_2 = (\frac{a_1}{a_2})\mu_1$, and $T_2 = (\frac{a_1}{a_2})T_1$ results in,

$$dN = \frac{g}{(2\pi)^3} \frac{d^3 p_2 \, dV_2}{\exp\left(\frac{p_2 - \mu_2}{T_2}\right) \pm 1}$$
(15)

Thus, the temperature and chemical potential follow the red shift pattern proportional to a^{-1} and the thermal distribution remains unchanged. So, the neutrino distribution is the same as if it is in thermal equilibrium with the temperature of the universe.

From the laws of thermodynamics,

$$E = TS - pV + \sum_{n} \mu_n N_n \tag{16}$$

Rearranging terms,

$$s = \left(\frac{S}{V}\right) = \frac{\rho + p - \sum \mu_n N_n}{T} \tag{17}$$

where *s* is the entropy density.

For relativistic particles,

$$s_f = \frac{\rho + p}{T} = \frac{7\pi^2}{180} g T^3 \tag{18}$$

$$s_b = \frac{\rho + p}{T} = \frac{2\pi^2}{45} g T^3$$
(19)

where the subscripts f(b) represents fermions (bosons).

As the temperature of thermal baths of different species (additional component due to kinetic temperature) differs from the species that are in thermal equilibrium (T), the energy and entropy densities [30] are given by,

$$\rho(T) = \frac{\pi^2}{30} g_*(T) T^4, \tag{20}$$

$$s(T) = \frac{2\pi^2}{45} g_s(T) T^3.$$
(21)

where,

$$g_*(T) = \sum_b g_b \left(\frac{T_b}{T}\right)^4 + \frac{7}{8} \sum_f g_f \left(\frac{T_f}{T}\right)^4,$$
(22)

$$g_s(T) = \sum_b g_b \left(\frac{T_b}{T}\right)^3 + \frac{7}{8} \sum_f g_f \left(\frac{T_f}{T}\right)^3$$
(23)

are the effective degrees of freedom during nucleosynthesis, $g_{b(f)}$ is the number of degrees of freedom of bosons and fermions, $T_{b(f)}$ is the temperature of thermal bath of bosons (fermions), T is the temperature of the photon thermal bath.

As long as the temperature of all the species remains the same and $p = \frac{\rho}{3}$ [21], both fermion and bosons have the same effective degrees of freedom. The electron annihilation process $e^+ - e^-$ allows the equilibrium of stable particles such as p, n, e^- and γ but force a distinction between $g_*(T)$ and $g_s(T)$. During the electron annihilation phase, $g_s(T)$ changes from

$$g_s(T) = g_*(T) = 2(\gamma) + 3.5(e^{\pm}) + 5.25(\nu) = 10.75$$
(24)

to

$$g_s(T) = 2 + 5.25 \left(\frac{T_v}{T}\right)^3$$
 (25)

During the annihilation phase, the energy density and entropy are transferred from electrons an positrons to photons but not to neutrinos. However, in the ultrarelativisitc limit ($m_v \ll T$), neutrinos are basically

massless or their masses can be ignored. As the universe cooled down, the rest mass energy density of matter gravitationally overcomes the photon radiation. After 400,000 years, the radiation is decoupled from matter as the electrons fused with nuclei forming atoms (mostly hydrogen). The decoupled relic radiation continued through space without impediment and this relic radiation is the CMB.

2.2.2 Decoupling

Analogous to photon scattering, neutrinos scattered through their interaction with electrons and positrons influencing cosmo dynamics [21]. This scattering occurred when the neutrinos were in thermal equilibrium with protons and electrons, maintained through weak interactions. When the rate of these weak interactions are slower than rate of expansion of the universe, neutrinos ceased interacting with baryonic matter. Though no direct observational evidence regarding neutrino decoupling have been found, it is expected to lead to a cosmic neutrino background analogous to the cosmic microwave background.

$$e^- + e^+ \Leftrightarrow v_e + \overline{v_e} \tag{26}$$

Recent neutrino oscillation experiments conclude that neutrino have a mass in MeV range and neutrino background is thought to be non relativistic.

2.2.3 Matter Era

Due to asymmetrical pair annihilation process and low energy density, the rest masses of the particles left over from annihilation dominated the radiation density leading to matter-dominated era. During the matter era,

$$\rho(T) = \frac{\pi^2}{30} g_*(T) T^4 \tag{27}$$

where,

$$g_*(T) = g_b(T) + \left(\frac{7}{8}\right)g_f(T)$$
 (28)

where, $g_b(T)$ is the effective degrees of freedom for relativisitic bosons and $g_f(T)$ is that of relativisitic fermions.

2.2.4 Big Bang Nucleosynthesis

The term Big Bang Nucleosynthesis refers to formation of elements towards the end of the first three minutes of cosmic time. The physics involved in the formation of all the light elements is well understood but the final abundances of the elements faced some uncertainties to that predicted by theory. The final abundances are sensitive to many constraints like baryon to photon ratio (η), and the abundance of ⁴He. The expected abundances of elements matches well with predicted abundances of the BBN model and the uncertainties in the observed abundances are attributed to creation and destruction of light elements in cosmological environments leading to the synthesized abundances. Due to the consistency of the predicted and observed values, it is important to consider the validity of the BBN model and then to discuss the corrections in the final abundances.

The big bang theory predicts that the early universe when started was hot and dense. Roughly a second after big bang, the temperature was around 10 billion degrees and it was filled with neutrons, positrons, protons, electrons and photons (radiation). At such higher temperatures, neutrons and protons were maintained in kinetic equilibrium [31],

$$T_n = T_p = T_e = T_{\nu_e} = T,$$
(29)

and chemical equilibrium [31],

$$\mu_n - \mu_p = \mu_e - \mu_{\nu_e} = \mu_{\bar{\nu}_e} - \mu_{e^+}.$$
(30)

Through weak process, neutrons decayed into protons and electrons.

$$n + v_e \rightleftharpoons p + e^- \tag{31}$$

$$n + e^+ \rightleftharpoons p + \bar{v_e} \tag{32}$$

$$n \rightleftharpoons p + e^- + v\bar{e} \tag{33}$$

The fractional neutron abundance is obtained by finding the equilibrium solution to the balance equation [31].

$$\frac{dX_n(t)}{dt} = \lambda_{pn}(t)[1 - X_n(t)] - \lambda_{np}(t)X_n(t)$$
(34)

where, X_n is fractional neutron abundance given by,

$$X_n = \frac{n_n}{n_N} \tag{35}$$

where, n_N is the total nucleon density at this time, $n_N = n_n + n_p$.

The equation states clearly that the reaction rate of neutron to proton decreases sufficiently to be in comparison with Hubble parameter, so as to be in a state of chemical equilibrium. Now neutrons freeze out making the exponential part to reach zero and the chemical equilibrium is broken. The synthesis of helium in the early universe is resulted by the surviving neutrons after the equilibrium is broken. Breaking of weak interaction from neutron to proton, neutrons and protons combined to form light nuclei following a sequence of two-body reactions:

 $p(n,\gamma) D,$ $D(p,\gamma)^{3}He, D(D,n)^{3}He, D(D,p)T,$ $T(D,n)^{4}He, T(^{4}He,\gamma)^{7}Li,$ $^{3}He(n,p)T, ^{3}He(D,p)^{4}He, ^{3}He(^{4}He,\gamma)^{7}Be,$ $^{7}Li(p,^{4}He)^{4}He,$



Figure 3. Primordial synthesis Reaction Network [33].

 $^{7}\text{Be}(n,p)^{7}\text{Li}$

Direct many - body reactions to form nuclei such as,

are not allowed due to the very small individual density. Therefore, neutrons combined with protons to make deuterium, an isotope of hydrogen. During the first three minutes, most of the deuterium combined to form helium and traces of lithium [32]. Approximately all of the surviving neutrons are bound to form ⁴He due to its large binding energy through the reactions shown above. Heavier nuclei do not form due to the large Coulomb barrier but reactions including T(⁴He, γ) ⁷Li and ³He(⁴He, γ) ⁷Be do occur. Figure 3 depicts the 12 fundamental process for the synthesis of all the lighter elements.

The predicted abundance of helium is also constrained by the small neutron life time and the BBN theory negates that by considering a large beta-decay lifetime. Considering a larger number of neutrino species will substantially increase the number of degrees of freedom, leading to speed up the expansion and resulting in early freeze out. Therefore, as the nucleon to photon ratio (η) increases, the formation of deuterium is faster allowing a large fraction of neutrons to survive beta-decay. Thus the predicted and observed abundances of helium will be consistent as more number of neutrons will be burnt to form ⁴He rising its abundance logarithmically as η increases. The following picture depicts the observed abundances relative to density of ordinary matter over that of photons (η).



Figure 4. A comparison of predicted abundances (BBN) and observed abundances (WMAP) [34].

The predicted abundances of elements depend on the density of ordinary matter relative to photons (η) at the early phase. The relative abundance of helium is invariant to the abundance of ordinary matter relative to photons (η) after a certain threshold. The deuterium and ³He abundances drop significantly with increasing η promising significant formation of ⁴He. The abundance of lithium face a decrease with increasing η from 10⁻¹¹ to 10⁻⁹ due to the synthesis through ⁴He(T, γ)⁷Li and ⁷Li(p,⁴He)⁴He. However for η greater than 3 * 10⁻¹⁰, the abundance of ⁷Li increases due to the production of ⁷Be through ⁴He(³He, γ)⁷Be making ⁷Li through the process of electron capture. The observed abundances of elements from the WMAP satellite were in good agreement with the predicted abundances from the BBN model which is a great triumph for the big bang theory.

2.3 Dark-matter and Dark-energy

A remarkable success of the Hubble expansion law describes the dynamics of the universe after the big bang had taken place. After the period of rapid expansion (inflation), the universe continued to expand at a slower rate while gravity of all the matter existing in this universe worked to slow down and reverse the expansion. The possibilities of slowing down and reverse or even accelerating the expansion depends upon the amount of available matter in the universe. If the universe consists of sufficient matter (called critical mass), the gravity will reverse the expansion leading to a collapse of the entire universe on its own gravity. This is a logical back explanation for big bang called the Big Crunch [24] . If the universe does not have sufficient amount of matter, gravity can no longer overcome the expansion rate causing universe to expand forever but at slower rate. However in 1998, a plausible explanation to cosmo dynamics was put forth as a result of parallel discoveries from two separate astronomers observing a distant type 1a supernova. The observations from light curves of supernova explosions indicate that they are moving at rapid speeds than theorized suggesting the expansion of universe to be in an accelerated state nailing down all the predicted theories. The underlying reason for this accelerated expansion can be the virtue of the space itself suggesting that the space is not empty but filled up with some vacuum energy causing an accelerated expansion. It is believed that this vacuum energy covers two-thirds of the entire universe and took the name of Dark energy [35].

By comparing theoretical calculations of the composition of the universe with observations, it is believed that 68 % of universe is filled with dark energy, approximately 27 % with dark matter and 4 % with the visible matter, including everything on earth and we observe. Vera Rubin and Kent Ford measured rotation rates of different galaxies by measuring the Doppler shifts of clusters of stars located within each galaxy at different distances from galactic center. The observations conclude that the orbital speed of stars at galactic center remains unchanged when measured with respect to any further distance from center. A consistent explanation to the findings considering Lagrangian mechanics was that the galaxy contains much more matter than that is visible. The other studies on galactic rotations suggest that the universe abounds matter that we cannot see. This invisible matter does not emit light meaning that it is not in the form of stars and planets that we can see. It is referred to as dark matter and it covers approximately 27 % of total universe. It is believed that dark matter is not baryonic as normal matter. If it is baryonic, the detection of baryonic clouds by the absorption of radiation could make it visible. Dark matter is not antimatter as no sign of gamma ray detection was obtained due to matter-anti matter annihilation. The only known possible member of dark matter are neutrinos [36]. Despite of the fact that neutrinos have very small relative masses, the total mass of neutrinos in the galaxy is significant due to very high neutrino density. But the common view about dark matter is that is made up of probably more exotic particles like Axions or WIMPS.

2.3.1 Axions

Axions are hypothetical particles that have been postulated to solve the strong CP violation problem [37]. CP-symmetry means charge parity symmetry that states that when a particle is interchanged with its anti-particle(C-symmetry) and the the coordinates are swapped (P-symmetry), the laws of physics should remain invariant. The strong force that holds nuclei together and the weak force responsible for nuclear decay differ significantly in the amount of CP-violation. Electro-weak interactions violates CP-symmetry which is not the case with strong interactions. QCD predicts CP-violation in strong interaction which is not experimentally known yet. A neutron dipole moment must be induced due to large CP-violating interactions which is not experimentally observed as predicted by QCD. The neutron dipole moment is caused by the distributions of positive and negative charges inside a neutron. If the center of positive and negative charge distribution coincides, it results in zero net dipole moment which is as observed. The lack of observational evidence imposes several constraints on the input parameters of the standard model making the theory incomplete. This strong CP-violation is the most challenging and unsolved problem in physics. Among several proposed solutions to the problem, the well known was Peccei-Quinn mechanism to introduce a pseudo particle, the AXION [29]. Axion is a neutral and very light particle. It does not interact (or only weakly) with conventional matter making it nearly impossible to detect. Axions are called strange photons due to their ability to transform into photons (viceversa) in the presence of an electromagnetic field. Figure 5 is the Feynmann diagram showing the conversion process of axion into a photon. The dashed line is the axion which is converted into photons (wavy line on right) in the presence of electromagnetic field (wavy line vertically). Axions would be an ideal dark matter candidate along with WIMPS. Due to their exceptionally low mass, they belong to CDM (Cold Dark Matter) moving slowly and thereby forming dark matter halos.

Though the detection of axions is impossible, the property of conversion into photons provides a path way for their detection. This effect is called as *Primakoff effect*. Based on the Primakoff effect, axions are believed to be produced in the solar core when axions scatter in strong electric fields. Axion heliscopes like CERN Axion Solar Telescope (CAST) and its extension International Axion Observatory (IAXO) are specifically designed to detect axions basing on this technique which is independent on axion being



Figure 5. Feynmann diagram showing conversion of axions into photons in presence of strong electromagnetic field [38].

a dark matter candidate. Considering the axion as the main component of dark matter, the detection based on a microwave resonant cavity in presence of magnetic fields is the another process by ADMX (Axion Dark Matter Experiment). Other experiments includes Polarization changes of light propagating in a magnetic field (PVLAS) and dark matter cryogenic detectors for the detection of axions which could plausibly help to explain the dark matter problem.

2.3.2 WIMPS (weakly interacting massive particles)

The mass discrepancies from extragalactic systems lead to the inference of the existence of some invisible matter that predominantly affects the Cosmo dynamics. This invisible matter covers around 27 % of entire mas of the universe and is referred to as dark matter. Although no experimental evidence for its existence is known, it is a fact that the dark matter is not baryonic. MACHO's gained initial reputation and later they are ruled out by WIMPS. WIMPS are particles emerged from super symmetry and are believed to be the solution to dark matter. Supersymmetry refers to the notion that all the funadamental particles must have supersymmetric partners [39]. The lightest supersymmetric particle known is NEUTRALINO, which is possibly the WIMP that makes up dark matter. It is stable and is not likely to perform any further decay processes.

Though the existence of WIMP is hypothetical, it fits perfectly for relic dark matter particle type from early universe. As the early universe was hot and dense, the energy density was so high that dark matter particles and their anti-particles would form and annihilate into lighter particles. As the universe had expanded, it cooled down and the energy density is no longer sufficient to create particle-anti-particles pair. However, the remaining dark matter particle and anti-particles annihilate each other decreasing the number density exponentially. After the freeze out (ceasing of any further interactions), the density of dark matter particles remained constant. The interaction cross-section of these particles was believed to be so low that particles and anti-particles could no longer interact to annihilate completely [2]. This relic density depends on properties of the particles. The interaction cross-section that governs dark matter particle-anti-particle annihilation, comes out about that of weak force. Relic density refers to particles interacting with the weak nuclear force. The coincidence of weak interactions and the relic density is referred to as WIMP Miracle, a hypothesis supporting WIMP as a candidate for cosmological dark matter. WIMPS as relic particles form cold dark matter is a research hypothesis which is subject to experimental detection for further understanding. As they interact through weak forces and also due to the extremely high temperatures required, the detection of WIMPS is highly difficult and likely less probabilistic. WIMPS are like neutrinos which can pass through baryonic matter without experiencing any electromagnetic interaction. Due to the low interaction cross section, the detection of WIMPS is possible only if they come close to a nucleus and interact with it through the weak nuclear force which is a highly sensitive measurement. Several techniques like cryogenic crystal detectors, noble gas scintillators and bubble chambers were employed through different experiments like the PICASSO (project in Canada to search for super symmetrical objects), DEAP (Dark Matter Experiment using Argon Pulse-shape), WARP (WIMP Argon Programme) to detect WIMPS and the only positive signature was published by the CDMS (Cryogenic Dark matter search) [40] in collaboration with DAMA (Data management Association). There are several next generation experiments like LUX-ZEPLIN (LZ) [40] and DARWIN to detect WIMPS on the basis of liquid xenon detection experiments.

Dark candidates arise frequently in physics beyond the standard model such as extra dimensions and supersymmetry. Dark matter candidates are light enough to be produced at colliders. The existence of dark matter is observed only from the gravitational effects on the visible matter and the dark matter particles are pseudo particles with no experimental evidence found.

3. Big-bang nucleosynthesis constraints on dark-matter sectors and their temperatures

There is an overwhelming amount of evidence that the universe is dominated by dark energy and dark matter, with the visible matter amounting to only 5% of its total energy/matter content. Dark energy takes about 70% of it, while dark matter is responsible for the other 25%. The evidences for the existence of Dark Matter (DM) are based on galaxy rotation curves [41], gravitational lensing [42, 43], and other recently observed phenomena [44]. The only thing we know so far about DM is that it interacts gravitationally, but there is a growing consensus in the scientific community that it might consist of a new particle beyond the standard model of particle physics. The most popular candidate for this particle is known as WIMP (Weakly Interacting Massive Particle), whose properties are basically unknown and left to wide theoretical speculation. Direct detection of such particles based on WIMP scattering induced nuclear recoil have been pursued experimentally, but has yield negative results so far (see, e.g., Ref. [45]).

Indirect searches for DM include looking for gamma-rays resulting from possible DM-antiDM annihilation or their decay (if they are unstable) into standard model particles. Such experiments are difficult because of the background from other typical astrophysical processes leading the same particles [46]. A far more indirect method is to use well established cosmological models whose predictions can be appreciably altered with the existence of DM. One of such models is the Big Bang Nucleosynthesis (BBN) model, which has been proven to successfully describe the observed abundances of light elements, up to about nuclear masses $A \sim 7$, and trace them back to the primordial epochs of the Universe [47]. The standard BBN predictions is in a good agreement with the observed abundances of light elements. Deviations from these predictions can been used as a test of new physics. The BBN predicted ⁴He abundance has become a benchmark for tests of big bang scenarios, although the BBN predicted abundances of deuterium and ³He are also in good agreement with observations. One exception seems to be the BBN predicted abundance of the ⁷Li isotope which is about a factor 3 larger than the observed values, under reasonable assumptions concerning astration and other possible lithium destruction scenarios. The mismatch between BBN predictions and observations for ⁷Li has become known as the "lithium problem". Several solutions of the lithium problem have been proposed with physics ranging from the destruction of ⁷Be production during the BBN, to the influence of axion particles, and other ideas. Destruction of ⁷Be would possibly solve the lithium puzzle as the electron capture on ⁷Be is responsible for most of the ⁷Li produced in the BBN [48]. Recent analysis of the experimental data seem to rule out this possibility [49, 50]. Many other speculations such as the influence of non-standard model particles in the BBN [51], a hybrid model of axion dark matter [52], or variations of fundamental constants [53] have been proposed. Other effects which have been explored, such as the impact of electron screening on nuclear reactions during the BBN have been shown to be ineffective [54]. More recently, the use of non-extensive statistics and its modification [55] of the usually adopted Maxwell-Boltzmann distribution for the relative velocities of the particles in the plasma has proven to be a possible path to solve the lithium puzzle because it reduces the ⁷Be production without affecting the deuterium and helium abundances [56] (see also [57]).

The influence of dark matter during the BBN has been studied in several publications (see, e.g., Refs. [58–65]). In particular, the gauge model developed in Ref. [30] proposes that the degrees of freedom associated with dark matter can increase the expansion rate of the early Universe. Analogous assumptions have been used in Refs. [63, 66–69], the major difference being the number of dark sectors, which in Ref. [30] amounts to five sectors instead of one as usual. The dark sectors consist of parallel universes of dark matter. implying a larger number of degrees of freedom. The influence of the degrees of freedom of the dark sectors straightforwardly arises from Friedmann equation, e.g. from the relation $H = \dot{R}/R = \sqrt{8\pi G\rho/3}$. Thus, ratio of expansion of the early Universe is proportional to the square root of the energy density which includes dark matter particles. For the temperatures during the BBN the light particles are relativistic and their contribution to the energy density is due to the number of their degrees of freedom displayed through the Stefan-Boltzmann T^4 law dependence on the temperature, and similarly as an entropy density T^3 dependence on the temperature. One does not expect that the temperatures of ordinary matter and those in the dark sectors to be equal and the question is how different they are. The standard BBN model assumes that the effective number of degrees of freedom during the BBN epoch is $g^* = 10.75$, due to photons, e^+e^- pairs, and three neutrino species v_e , v_μ , and v_τ . The model developed in

Ref. [30] assumes that the mirror (dark) sectors are composed of particles which are exact copies of the standard model, but interacting weakly within and across all dark sectors and between the dark sectors and the ordinary one.

Refs. [30, 66] have proposed that after inflation the temperature for the thermal baths associated with each particle species might not be the same. Such temperatures depend on the various possible reactions leading to equilibrium and on the thermal history of the Universe. The assumption used in Ref. [30] relies on the simplest possible picture where all the five dark sectors have the same temperature, different from the ordinary matter thermal bath. This temperature difference might be due to an asymmetric reheating taking place after inflation, as envisaged in Refs. [70–72]. But the mirror model suggested by those authors aimed at explaining the neutrino anomalies, and have mirror baryons about 20 times heavier than the familiar baryons. Such baryons play the role of the cold dark matter and they provide a reasonable explanation of why Ω_{DM} is of the same order as $\Omega_{Baryons}$. In this work we will focus on the influence of the number of dark sectors in BBN and look for the constraints on such a number to test the hypothesis taken in Ref. [30] on the existence of five parallel universes of dark matter.

3.1 Degrees of freedom in dark and ordinary matter sectors

The most constraining parameters of BBN are the ⁴He primordial abundance and the baryon-to-photon ratio $\eta = n_b/n_\gamma$, where n_b is the baryon density and n_γ the photon density in the Universe [73]. They would be the best way to identify the number of possible new particles contributing to the radiation density during the BBN epoch. At very high temperatures, the Universe was dominated by radiation, the particles were all relativistic and their masses can be neglected accordingly. In such a scenario the energy and entropy densities in this epoch are given by [74] ($\hbar = c = k_B = 1$)

$$\rho(T) = \frac{\pi^2}{30} g_*(T) T^4 \quad \text{and} \quad s(T) = \frac{2\pi^2}{45} g_s(T) T^3,$$
(36)

where

$$g_*(T) = \sum_B g_B \left(\frac{T_B}{T}\right)^4 + \frac{7}{8} \sum_F g_F \left(\frac{T_F}{T}\right)^4$$
(37)

and

$$g_{s}(T) = \sum_{B} g_{B} \left(\frac{T_{B}}{T}\right)^{3} + \frac{7}{8} \sum_{F} g_{F} \left(\frac{T_{F}}{T}\right)^{3}$$
(38)

are the effective number of d.o.f. during nucleosynthesis. In these equations, $g_{B(F)}$ is the number of degrees of freedom of bosons (fermions) B(F). The temperature of the thermal bath of each species are denoted by $T_{B(F)}$, respectively, while *T* denotes the temperature of the photon thermal bath.

Just after the reheating we assume that matter and dark matter sectors are nearly decoupled, with different temperatures: T for matter and T' for the dark matter bath. For the dark matter bath, the energy $\rho'(T')$ and entropy s'(T') densities are given as in Eq. (36) but with the effective number of degrees of freedom changed to $g_*(T) \rightarrow g'_*(T')$ and $g_s(T) \rightarrow g'_s(T')$, and replacing T by T'. If entropy in each sector is separately conserved during expansion, then $x = (s'/s)^{1/3}$ is time independent. In this case, unless the temperature of the dark sector T' is much smaller than that of ordinary matter, with the same relativistic particle content for each sector one would reach to the conclusion that the initial condition $g_s(T_0) = g'_s(T'_0)$ implies that x = T'/T. But the dark matter particles might have self-interact via some unknown force besides gravity. Even if it is small compared to the known interactions, the assumption that their masses are the same as particles in the visible sector, with same number of kinds of dark matter particles, same status as fermions or bosons, does not imply that their mean velocity and d.o.f. have to be shared among all of them equally as the universe expands.

Cosmological models have been proposed with self-interacting cold dark matter particles with a large scattering cross section but negligible annihilation or dissipation [75]. Present observations of the dark matter halos seem to suggest that self-interacting dark matter particles have an upper bound for their cross section of the order of $\sigma_x/m_x \sim 10^{-25}$ cm⁻² GeV⁻¹, where σ_x is the cross section and m_x the mass of the dark matter particle [76–79]. Cross sections of weakly interacting particles with energy *E* in a thermal bath with temperature *T* are typically given by $\sigma \approx g^2 E^2 \approx g^2 T^2$, where *g* is the interaction coupling constant and *T* is the temperature. Assuming that *g* is of the same order of magnitude in the dark sectors, one has $\sigma'/\sigma = (T'/T)^2$, where $\sigma'(\sigma)$ is the cross section for the dark (ordinary) particles. Thus, unless $T' \ll T$, the magnitude of the elastic cross sections in the dark sectors would be of similar magnitude as

with the ordinary matter, but with a tendency of dark sectors being less collision intense.

3.2 Nucleosynthesis constraints on the number of dark sectors

During the BBN and temperatures of the order of 1 MeV, photons, e^+e^- pairs, and three neutrino species v_e , v_μ , and v_τ are in a quasi-equilibrium. The number of degrees of freedom at $m_e < T < m_\mu$ is $g_*(T)|_{T=1MeV} = 43/4 = 10.75$ [1]. Including an yet to be determined number of dark sectors N_{DM} , the Friedman equation during the radiation dominated era is given by

$$H(t) = \sqrt{(8\pi/3c^2) G_N \bar{\rho}}.$$
 (39)

where the total energy density is $\bar{\rho} = \rho + N_{DM}\rho'$, where ρ' is the energy density in the dark matter sectors. Using Eq. (36) for ρ , and ρ' leads to

$$H(t) = \sqrt{\frac{8\pi^3}{90}\bar{g}_*(T)}\frac{T^2}{M_{Pl}},$$
(40)

where $M_{Pl} = \sqrt{\hbar c^5/G}$ is the Planck mass and

$$\bar{g}_*(T) = g_*(T) \left(1 + N_{DM} \xi x^4 \right).$$
(41)

The parameter ξ is given by $\xi = (g'_*/g_*)(g_s/g'_s)^{4/3}$ and is equal to 1, unless T'/T is very small, following the same arguments given before. Therefore, the number of the effective d.o.f. during the BBN in the presence of N_{DM} dark sectors change from g_* to $\bar{g}_* = g_*(1 + N_{DM} x^4)$.

The bounds for N_{DM} and x can be assessed by the relative abundances of the light element isotopes (D, ³He, ⁴He, and ⁷Li). In Figure 6 we show the BBN prediction for the relative abundances of D, ³He, ⁴He and ⁷Li as a function of the ratio of the temperature of dark sectors T' to that of the ordinary matter, T. Here we assume that the number of dark sectors is fixed to $N_{DM} = 5$. The shaded bands represent the uncertainty in the observed values. The most stringent constraint, namely the abundance of helium shows that the dark sectors should have a temperature smaller than the ordinary matter, with $T' \leq 0.4T$.



Figure 6. BBN prediction for the relative abundances of D, ³He, ⁴He and ⁷Li as a function of the ratio of the temperature of dark sectors to that of the ordinary matter. Here we assume that the number of dark sectors is fixed to $N_{DM} = 5$. The shaded bands represent the uncertainty in the observed values.

This is also supported by the relative abundance of deuterium, while the observations on the relative abundance of helium-3 allow for a larger temperature for the dark sectors. Interestingly, the lithium-7 relative abundance is better described by a larger temperature for the dark sectors. This result is rather tempting, due to a possible solution of the so-called lithium puzzle by means of the number of degrees of freedom of particles in the dark sectors. Evidently, the constraints set by the D, ³He, and ⁴He do not allow for this possibility, at least under the scenario all abundances are related by the same reaction BBN networks, with or without dark sectors. We thus conclude that the dark sectors must be cold compared to the ordinary matter during the BBN.

We notice from Eq. (41) that the value of $\bar{g}_*(T)$ is much more sensitive to T' than it is to N_{DM} . In fact, if we set T' = 0.3T as a typical temperature value for cold sectors, we find that here is a large range of $N_{DM} = 1 - 50$ for which the relative abundances of D, ³He and ⁴He would fall within the constraints set by observation. Only if we assume a value $N_{DM} \simeq 100$ would the observed abundance of ⁷Li be explained by BBN theory. However, as in the case displayed in the far right of Figure 6, the BBN predictions of D, ³He and ⁴He would fall off the observation.

Another way to cast the dependence of BBN predictions on the extra degrees of freedoms introduced by the existence of dark sectors is to add the uncertainty in the number of massless neutrinos during nucleosynthesis, which is supposed to be in the interval $2.92 < N_v < 3.38 (1\sigma)$ [43], in agreement with the predictions of the Standard Model (SM). Other probes, with less resolution than the Planck satellite, set this uncertainty within the interval $3.46 < N_v < 5.2$ [80]. Here we will use Planck results to study the impact of the neutrino families on constraining the number and temperature of dark sectors.

The extra degrees of freedom introduced by the dark sectors and the number of neutrino families amounts to

$$\bar{g}_* = g_* (1 + N_{DM} \xi x^4 + 1.75 \Delta N_\nu), \tag{42}$$

where ΔN_{ν} is the variation in equivalent number of neutrinos. Based on the discussion above, we assume T' = 0.3T to reconcile with the BBN data with $N_{\nu} = -3$. We also assume that $N_{DM} = 5$, as required to explain the observed ratio Ω_{DM}/Ω_b . Note that this model does not assume that dark matter couples with neutrinos, as has been a popular approach in recent the literature to account for light ($M \leq 20$ MeV) dark matter particles coupling to neutrinos via dark-matter annihilation. In these models, the neutrino-to-photon temperature ratio can change, as well as the effective number of neutrinos degrees of freedom.

In figure 7 we show the dependence of BBN prediction for the primordial ⁴He as a function of the extra number of neutrino families, including the impact of the degrees of freedom of particles in dark sectors with a temperature $T' = 0.34T_{BBN}$ and the number of dark sectors is $N_{DM} = 5$. One sees that the estimate of number of dark sectors and their temperature does not affect the constraints set by observations (shaded area), unless the number of neutrino families is decreased ($\Delta N_{\nu} < 0$) or increased ($\Delta N_{\nu} > 0$) substantially from the expected value of $N_{\nu} = 3$.

The baryon asymmetry is parameterized by the baryon-to-photon ratio η . The density number of photons n_{γ} is proportional to T³ and, therefore, one can write the density number of dark-photons as



Figure 7. Dependence of BBN prediction for the primordial ⁴He as a function of the extra number of neutrino families, including the impact of the degrees of freedom of particles in dark sectors with a temperature $T' = 0.34T_{BBN}$ and the number of dark sectors is $N_{DM} = 5$.

 $n'_{\gamma} = x^3 n_{\gamma}$. The ratio of dark-baryons to ordinary-baryons is given by $\beta = \Omega'_B / \Omega_B = x^3 \eta' / \eta$ [70]. The bounds from the BBN on x = T'/T imply that the baryon asymmetry in the dark sector is greater than in the ordinary one. Indeed, using as an upper bound the estimate $x \sim 0.8 / N_{DM}^{1/4}$ and assuming that each sector contributes equally to the Universe's energy density $\beta \sim 1$, we obtain $\eta' \sim 2.1 N_{DM}^{3/4} \eta$. For the special where $N_{DM} = 5$ it follows that $\eta' \sim 7\eta$. Asymmetric Dark Matter models, see e.g. [63, 67–69], give similar results for the baryon asymmetry.

3.3 Conclusions

In summary, we have studied effects of dark-matter degrees of freedom in the early universe on the abundances of the light elements synthesized during the BBN epoch. Comparing our theoretical predictions with the observational constraints on the primordial abundances of the light elements, we have obtained bounds on the number of dark matter sectors, N_{DM} . We show that the number of dark sectors is only loosely constrained by the comparison of BBN predictions with observations. The possible value of $N_{DM} = 5$, based on the observed ratio between dark matter and visible matter is a reasonable possibility. The temperature (T') in the dark sectors is colder than that (T_{BBN}) in the ordinary matter sector, albeit not much smaller: $T' \sim 0.3T_{BBN}$.

We also verified that the number of neutrino families, $N_v = 3$ is also compatible with the existence of multiple dark sectors with a colder temperature. The constraints set by the number of neutrino families do not change our predictions for T' and N_{DM} .

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