A multichannel algebraic scattering approach to astrophysical reactions

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Abstract. The investigation of many astrophysical processes is dependent upon an understanding of nuclear reaction rates. However, nuclear capture reactions of astrophysical interest occur at extremely low energies, taking place at the Gamow energy within the stellar environment. Hence, they are hard to study experimentally due to Coulomb repulsion. They may also involve compound resonances stemming from a delicate interplay of many quantum states in the colliding bodies. The multi-channel algebraic scattering (MCAS) method is one that addresses both of these challenges; it has a history of successfully modelling narrow compound resonance structures, incorporating as many channels as are important for a given problem, but is also proven in recreating the low-energy, non-resonant elastic scattering cross sections needed for these astrophysics problems. We provide an overview of MCAS' techniques of modelling elastic scattering reactions, how these may be extended to capture reactions, and current work in this area.

1 Introduction

Many astrophysical processes are governed by nuclear capture reaction rates. Examples include X-ray bursts from accreting neutron stars [1] and white dwarfs in binary star systems, which detonate nova or supernova explosions, and may rotate in decreasing orbits, producing gravitational waves [2].

The principle challenge in measuring these reactions is that they occur at extremely low energies, taking place at the Gamow energy within the stellar environment, where Coulomb

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repulsion dominates. Thus, to understand these reactions, theoretical methods must be employed to match the data that exists at higher energies (though these come with increasing uncertainty as energy reduces), and extrapolate down into the Gamow window. For example, a recent overview of existing data for the important reaction $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ can be found in Ref. [3], and in the near future many laboratories across the world will begin measuring lower energy data, pushing towards but never reaching astrophysical energies [4–11].

The challenge for calculating observables is that the reaction may also involve compound resonances stemming from a delicate interplay of many quantum states in the colliding bodies. For example, whereas the $^{8}\mathrm{Be}(\alpha,\gamma)^{12}\mathrm{C}$ reaction famously depends mainly on the Hoyle state, the $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ rate is known to depend on many broad and thus overlapping resonances as well as non-resonant reaction components [3].

The multichannel algebraic scattering (MCAS) method of calculating nuclear scattering observables addresses the challenge of complex state interplay and accuracy in recreating low-energy cross sections. MCAS may consider as many channels as are necessary, and has a method of calculating both broad and narrow compound-state resonances [12]. Recent calculations have proven its capacity to recreate low-energy, non-resonant elastic scattering cross sections within keV of the scattering threshold [13].

2 Multi-channel Algebraic Scattering (MCAS)

MCAS calculates scattering observables in the low-energy range for nucleons and α-particles scattering with target nuclei where the low-energy spectrum may be described by a predominant mode of excitation. It does this by first solving the coupled-channels Lippmann-Schwinger equations in momentum space:

$$T_{cc'}(p,q;E) = V_{cc'}(p,q) \;+\; \frac{2\mu}{\hbar^2}\left[\sum_{c''=1}^{\text{open}} \int_0^\infty \frac{V_{cc''}(p,x)\;T_{c''c'}(x,q;E)}{k_{c''}^2 - x^2 + i\epsilon}\,x^2\,dx \right.$$
$$\left. - \sum_{c''=1}^{\text{closed}} \int_0^\infty \frac{V_{cc''}(p,x)\;T_{c''c'}(x,q;E)}{h_{c''}^2 + x^2}\,x^2\,dx\right], \qquad (1)$$

where the c are indicies denoting the quantum numbers that identify each channel uniquely. The open and closed channels have wave numbers k_c and h_c for $E > \epsilon_c$ and $E < \epsilon_c$, respectively, and μ is the reduced mass.

The $V_{cc'}$ are interaction potentials between the target and projectile, derived from models of nuclear structure for the target. In principle, any structure model may be used, but to date we have used Tamura collective models of rotor or vibrator character [14]. It is this nuclear structure information which sets the method apart from methods like the R-matrix, which relies on a simple fit to compound-system resonance widths without a description of the underlying nuclear potential.

Solutions of these Lippmann-Schwinger equations are found by expanding the potential matrix elements into a finite number of energy-independent separable terms called sturmians, χ. The details of how these terms are defined in terms of the potential, preserving the nuclear structure information, is outlined in Ref. [15]. This method allows the location of all resonance centroids and widths of the compound system (as needed for certain astrophysical reactions).

In terms of these sturmians, the S-matrix is defined as follows. It is from this that observables which may be experimentally measured are calculated.

$$S_{cc'} = \delta_{cc'} \;-\; i^{l_{c'}-l_c+1}\pi\mu \sum_{n,n'=1}^{N} \sqrt{k_c}\hat{\chi}_{cn}(k_c)\left(\left[\boldsymbol{\eta} - \mathbf{G}_0\right]^{-1}\right)_{nn'}\hat{\chi}_{c'n'}(k_{c'})\sqrt{k_{c'}} \qquad (2)$$

where

$$[\mathbf{G}_0]_{nn'} = \mu \left[\sum_{c=1}^{\text{open}} \int_0^\infty \hat{\chi}_{cn}(x) \frac{x^2}{k_c^2 - x^2 + i\epsilon} \hat{\chi}_{cn'}(x)\, dx \right. \\ \left. - \sum_{c=1}^{\text{closed}} \int_0^\infty \hat{\chi}_{cn}(x) \frac{x^2}{h_c^2 + x^2} \hat{\chi}_{cn'}(x)\, dx. \right] \quad (3)$$

This has the ability to determine all subthreshold bound states by taking advantage of the momentum-space formalism wherein the use of negative projectile energies is possible. Importantly, there is a mechanism to incorporate the Pauli principle, even with collective models [15]. This becomes necessary for coupled-channels calculations, as deformation only accounts for the Pauli principle in single-channel calculations with the target ground state. Additionally, MCAS has a means of accounting for particle-instability of the target nucleus' states [16].

3 Preliminary results of nucleon-^{56}Ni elastic scattering

MCAS, to date, has been designed to account for elastic scattering and (energetically) inelastic scattering. Immediate future development will be to expand it to account for radiative capture reactions.

^{56}Ni$(p,\gamma)^{57}$Cu has been identified as a reaction whose uncertainty will, alone, affect the interpretation of data for X-ray bursts on the surface of accreting neutron stars, when using the Single-zone X-Ray Burst Model [1]. ^{56}Ni has a low-energy spectrum suggestive of collective excitation. These features makes it a good candidate for a future MCAS capture-rate calculation.

The process for studying such a reaction will be to first study the elastic scattering case in order to calibrate the parameters of the collective-model interaction potential. This will then generate sturmians for use in the subsequent capture reaction calculation. If the reaction of interest is (p,γ), as for this example, we first perform the (n,n) calculation with the mirror nucleus of the target (which in this case is itself), allowing for the nuclear potential to be determined without a Coulomb potential. Preliminary results for the spectrum of the ^{57}Ni compound nucleus are shown in Fig. 1, using an interaction of vibrator character. The neutron emission threshold of ^{57}Ni is $\sim$10.2 MeV, so the states shown here represent negative projectile energies.

To model the (p,p) reaction, a Coulomb potential is applied, based upon a 3pF charge distribution [18]. While the 3pF model includes three parameters, it has been shown [19] that the values of these do not affect the resulting spectrum of compound states, so long as the chosen values reproduce the experimentally measured root-mean-square charge radius [20]. Preliminary results for the spectrum of the ^{57}Ni compound nucleus are shown in Fig. 2. ^{57}Cu is only bound to proton emission by 0.69 MeV, so the states shown are all resonances of the compound system, except the ground state which represents a negative projectile energy in the calculation.

The next step to be taken in the study of this system is to more extensively search the parameter space of the nuclear interaction potential for a better match to data. Once finalised, this will yield a calculated elastic scattering cross section. However, as the ground and first two excited state centroids are well-placed in this preliminary study, we may expect an accurate elastic and subsequent capture cross sections.

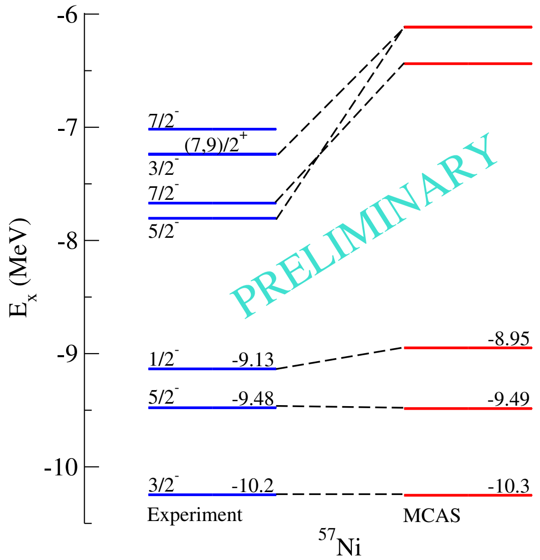

Figure 1. Experimental spectrum of ^{57}Ni [17] compared to the subtheshold bound states of a preliminary MCAS ^{56}Ni$(n, n)^{56}$Ni calculation.

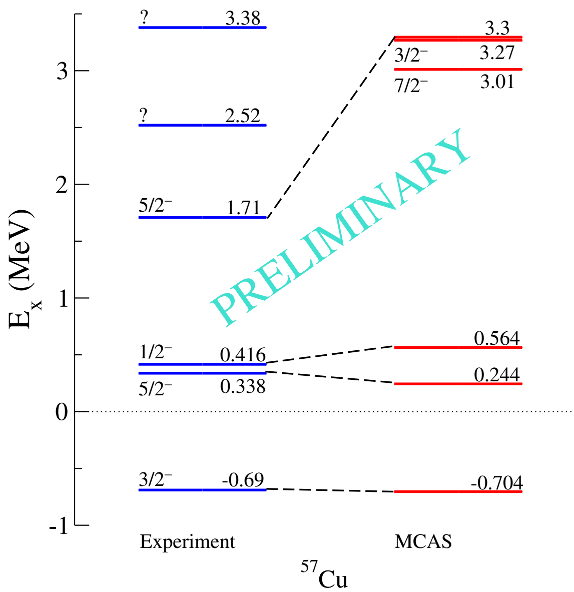

Figure 2. Experimental spectrum of ^{57}Cu [17] compared to the resonances and subtheshold bound states of a preliminary MCAS ^{56}Ni$(p, p)^{56}$Ni calculation.

4 Methods of calculating electromagnetic transition in single photon emission

In this section, we briefly outline how we may incorporate radiative capture in MCAS. First, we require co-ordinate space wave functions to be generated from the sturmians of above-threshold resonances and the subthreshold bound states they will decay to in the event of capture. These are defined in Ref. [21],

$$\Psi^{J(+)}_{cc"}(R) = i^{L"} \sqrt{\frac{2}{\pi}} \frac{\exp\left[i\sigma_L(P)\right]}{\sqrt{P"P}R} \left[F_L(P,R)\,\delta_{c"c} - \pi\mu\,\sqrt{P"P}\,\Phi^{J(+)}_{cc"}(R)\right] , \quad (4)$$

where

$$\Phi^{J(+)}_{cc"}(R) = i^{L-L"} \sum_{n,n'=1}^{N} \Big(F_{L"}(P"R)\,\chi^{G}_{c"n}(P"R) \; - \; G_{L"}(P"R)\,\chi^{F}_{c"n}(P"R) + O^{R(+)}_{L"}(P"R)\,\hat{\chi}_{c"n}(P")\Big) \left[\eta - \mathcal{G}^{(+)}_0\right]^{-1}_{nn'} \hat{\chi}_{cn'}(P) \quad (5)$$

incorporates the sturmian form factors, $\hat{\chi}_{cn}$, as per the elastic scattering version of MCAS. c are the channel indices, n is the sturmian index, and N is the number of sturmians used. η_n are sturmian eigenvalues. R is the radius from the scattering site, P is the 2-body centre of mass momentum, and L is the relative orbital angular momentum of the projectile relative to the target. μ is the reduced mass. $\sigma_L(P)$ is the Coulomb phase shift, $F_L(PR)$ are the Ricatti-Bessel or Ricatti-Coulomb functions, $G_L(PR)$ are the Coulomb irregular functions, and $O^{R(+)}_L(PR) = G_L(PR) + iF_L(PR)$. $\chi^F_{cn}(PR)$ are sturmian form factors integrated from $R \to \infty$ rather than $0 \to \infty$, and $\chi^G_{cn}(PR)$ are the same except they involve Coulomb irregular functions rather than Ricatti-Coulomb, and are also integrated from $R \to \infty$. $G^{(+)}_0$ are Greens functions, as per elastic scattering MCAS, and $\left[\eta - \mathcal{G}^{(+)}_0\right]^{-1}_{nn'}$ is an inverse matrix.

From here we may proceed to the calculation of cross sections with an elegant method of separable potentials that was recently used for proton and neutron capture by carbon-12 [22], based upon an earlier elastic scattering formalism [23, 24]. Therein, the co-ordinate space wave functions are ingredients of reduced matrix elements,

$$\langle l j \| O_{\pi L} \| l_0 j_0 \rangle \; = \; \delta_{l+l_0+L,\text{even}}\, (-1)^{l_0+l-j+L-\frac{1}{2}} \frac{e_L}{\sqrt{4\pi}} \sqrt{(2L+1)(2j_0+1)} \times \begin{pmatrix} j_0 & L & j \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix} \int_0^\infty r^L\, \psi^{J_0}_{l_0 j_0}(r)\, \psi^{J}_{lj}(r) , \quad (6)$$

where subscript 0 denotes bound states, and equivalent quantities without denote continuum wavefunctions. e_L is an effective charge:

$$e_L = Z_{\text{targ}} e \left(\frac{-m_{\text{proj}}}{m_{\text{comp}}}\right)^L + Z_{\text{proj}} e \left(\frac{m_{\text{targ}}}{m_{\text{comp}}}\right)^L . \quad (7)$$

These are then used in the scattering cross sections,

$$\sigma_{\pi L} = \frac{(2\pi)^3}{k^2} \left(\frac{E_\gamma}{\hbar c}\right)^{2L+1} \frac{2(2I_{\text{comp}}+1)}{(2I_{\text{proj}}+1)(2I_{\text{targ}}+1)} \frac{L+1}{L\left[(2l+1)!!\right]^2} \times \sum_{ljJ} (2j+1) \begin{Bmatrix} j & J & I_{\text{targ}} \\ J_0 & j_0 & L \end{Bmatrix}^2 |\langle l j \| O_{\pi L} \| l_0 j_0 \rangle|^2 . \quad (8)$$

This will require modification if the projectile is not a nucleon.

5 Conclusions

The multi-channel algebraic scattering formalism has recreated observed elastic scattering observables over many years, including successfully predicting several nuclear states subsequently identified by experiment. The next focus for development is to expand MCAS to evaluate radiative capture reaction data, and make predictions down to the Gamow window for reactions of astrophysical interest. The first planned candidates for this formalism are ^{12}C$(\alpha,\gamma)^{16}$O, ^{56}Ni$(p,\gamma)^{57}$Cu, ^{12}C$(p,\gamma)^{13}$N (which was recently measured at LUNA [25] with their newly-upgraded BGO detector [26], and for which sturmians have already been found [27]) and ^{20}Ne$(p,\gamma)^{21}$Na (currently being measured at LUNA).

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