Pygmy resonances probed with electron scattering

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Pygmy resonances in light nuclei excited in electron scattering are discussed. These collective modes will be explored in future electron-ion colliders such as ELISe/FAIR (spokesperson: Haik Simon - GSI). The response functions for direct breakup are described here with few-body models, exploring the dependence upon final state interactions. A hydrodynamical model for collective pygmy resonances is developed for light, neutron-rich nuclei. A comparison between direct breakup and collective models is performed.

1. INTRODUCTION

Reactions with radioactive beams have attracted great experimental and theoretical interest during the last two decades [1]. Progresses of this scientific adventure were reported on measurements of nuclear sizes [2], the use of secondary radioactive beams to obtain information on reactions of astrophysical interest [3, 4], fusion reactions with neutron-rich nuclei [5, 6], tests of fundamental interactions [7], dependence of the equation of state of nuclear matter upon the asymmetry energy [8], and many other research directions. Studies of the structure and stability of nuclei with extreme isospin values provide new insights into every aspect of the nuclear many-body problem.

New research areas with nuclei far from the stability line will become possible with newly proposed experimental facilities. Among these I quote the FAIR facility at the GSI laboratory in Germany. One of the projects for this new facility is the study of electron scattering off unstable nuclei in an electron-ion collider mode [9]. A similar proposal exists for the RIKEN laboratory facility in Japan [10]. By means of elastic electron scattering, these facilities will become the main tool to probe the charge distribution of unstable nuclei [11, 12]. This will complement studies of matter distribution which have been performed in other radioactive beam facilities using hadronic probes. Inelastic electron scattering will test the nuclear response to electric and magnetic fields.

Up to now, the electromagnetic response of unstable nuclei far from the stability line has been studied with Coulomb excitation of radioactive beams impinging on a heavy target [4]. This method has been very useful in determining the electromagnetic response in light nuclei [13]. For neutron-rich isotopes [14] the resulting photo-neutron cross sections are characterized by a pronounced concentration of low-lying $E1$ strength. The onset of low-lying $E1$ strength has been observed not only in exotic nuclei with a large neutron excess, but also in stable nuclei with moderate proton-neutron asymmetry. The problem with such experiments is that the probe is not very clean. It is well known that the nuclear

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interaction between projectile and target as well as the long range Coulomb distortion of the energy of the fragments interacting with the target (see, e.g. ref. [15]) are problems of a difficult nature. The nuclear response probed with electron does not suffer from these inconveniences.

The interpretation of the low-lying $E1$ strength in neutron-rich nuclei engendered a debate: are these "soft dipole modes" just a manifestation of the loosely-bound character of light neutron-rich nuclei, or are they a manifestation of the excitation of a resonance? [16, 17, 18, 19]. As far as I know, there has not been a definite answer to this simple question. The electromagnetic response of light nuclei, leading to their dissociation, has a direct connection with the nuclear physics needed in several astrophysical sites [3, 4, 15]. In fact, in has been shown by Goriely [20] that the existence of pygmy resonances have important implications on theoretical predictions of radiative neutron capture rates in the r-process nucleosynthesis, and consequently on the calculated elemental abundance distribution in the universe.

2. Inelastic scattering in electron-ion colliders

Here, J_i (J_f) is the initial (final) angular momentum of the nucleus, $(E, \mathbf{p})$ and $(E\prime, \mathbf{p}')$ are the initial and final energy and momentum of the electron, and $(q_0, \mathbf{q}) = ((E-E')/\hbar c, (\mathbf{p}-\mathbf{p}')/\hbar)$ is the energy and momentum transfer in the reaction. For low energy excitations such that $E, E' \gg \hbar c q_0$, which is a good approximation for electron energies $E \simeq 500$ MeV and small excitation energies $\Delta E = \hbar c q_0 \simeq 1 - 10$ MeV. These are typical values involved in the dissociation of nuclei far from the stability line.

In the plane wave Born approximation (PWBA) and using the Siegert's theorem, one can show that [21]

$$\frac{d\sigma}{d\Omega dE_\gamma} = \sum_L \frac{dN_e^{(EL)}(E, E_\gamma, \theta)}{d\Omega dE_\gamma}\ \sigma_\gamma^{(EL)}(E_\gamma), \tag{1}$$

where $\sigma_\gamma^{(EL)}(E_\gamma)$, with $E_\gamma = \hbar c q_0$, is the photo-nuclear cross section for the EL-multipolarity, given by [4]

$$\sigma_\gamma^{(EL)}(E_\gamma) = \frac{(2\pi)^3 (L+1)}{L\left[(2L+1)!!\right]^2} \left(\frac{E_\gamma}{\hbar c}\right)^{2L-1} \frac{dB(EL)}{dE_\gamma}. \tag{2}$$

In the long-wavelength approximation, the response function, $dB(EL)/dE_\gamma$, in eq. 2 is given by

$$\frac{dB(EL)}{dE_\gamma} = \frac{\left|\langle J_f \| Y_L(\widehat{\mathbf{r}}) \| J_i \rangle\right|^2}{2J_i + 1} \left[\int_0^\infty dr\ r^{2+L}\ \ \delta\rho_{if}^{(EL)}(r)\right]^2 \rho(E_\gamma), \tag{3}$$

where $\rho(E_\gamma)$ is the density of final states (for nuclear excitations into the continuum) with energy $E_\gamma = E_f - E_i$. The geometric coefficient $\langle J_f \| Y_L(\widehat{\mathbf{r}}) \| J_i \rangle$ and the transition density $\delta\rho_{if}^{(EL)}(r)$ will depend upon the nuclear model adopted.

One can also define a differential cross section integrated over angles so that

$$\frac{d\sigma_e}{dE_\gamma} = \sum_L \frac{dN_e^{(EL)}(E, E_\gamma)}{dE_\gamma}\ \sigma_\gamma^{(EL)}(E_\gamma), \tag{4}$$

where

$$\frac{dN_e^{(EL)}(E,E_\gamma)}{dE_\gamma} = 2\pi \int_{E_\gamma/E}^{\theta_m} d\theta \ \sin\theta \ \frac{dN_e^{(EL)}(E,E_\gamma,\theta)}{d\Omega dE_\gamma}, \tag{5}$$

and θ_m is the maximum electron scattering angle, which depends upon the experimental setup. Notice that the lowest limit in the above integral is $\theta_{\min} = E_\gamma/E$, and not zero. This is equivalent to the condition that the minimum momentum transfer in electron scattering is given by $\Delta E/\hbar c$.

Eqs. 1-5 show that under the conditions of the proposed electron-ion colliders, electron scattering will offer the same information as excitations induced by real photons. The reaction dynamics information is contained in the virtual photon spectrum, $N_e^{(EL)}(E,E_\gamma,\theta)$, while the nuclear response dynamics information will be contained in eq. 3. The quantities $dN_e^{(EL)}/d\Omega dE_\gamma$ can be interpreted as the number of equivalent (real) photons incident on the nucleus per unit scattering angle Ω and per unit photon energy E_γ. Note that $E0$ (monopole) transitions do not appear in this formalism. As immediately inferred from eq. 3, for $L=0$ the response function $dB(EL)/dE_\gamma$ vanishes because the volume integral of the transition density also vanishes in the long-wavelength approximation. But for larger scattering angles the Coulomb multipole matrix elements in are in general larger than the electric (EL) multipoles, and monopole transitions become relevant [22]. Eq. 1 will not be valid under these conditions.

It is found that that the spectrum $dN_e^{(EL)}(E,E_\gamma)/dE_\gamma$ increases rapidly with decreasing energies. Also, for $E = 500$ MeV and excitation energies $\Delta E = 1$ MeV, the spectrum yields the ratios $dN_e^{(E2)}/dN_e^{(E1)} \simeq 500$ and $dN_e^{(E3)}/dN_e^{(E2)} \simeq 100$. However, although $dN_e^{(EL)}/dE_\gamma$ increases with the multipolarity L, the nuclear response decreases rapidly with L, and $E1$ excitations tend to dominate the reaction. For larger electron energies the ratios $N^{(E2)}/N^{(E1)}$ and $N^{(E3)}/N^{(E1)}$ decrease rapidly. A similar relationship as eq. 1 also exists for Coulomb excitation [4] in heavy ion scattering. But for Coulomb excitation this factorization is exact for the reason that Coulomb excitation occurs when the nuclei do not overlap. In the electron scattering case, because the electron can also scatter through the nuclear interior, the longitudinal and transverse components of the interaction acquire different weights.

A comparison between the $E1$ virtual photon spectrum, dN_e/dE_γ, of 1 GeV electrons with the spectrum generated by 1 GeV/nucleon heavy ion projectiles ws done in ref. [21]. In the case of Coulomb excitation, the virtual photon spectrum was calculated in ref. [4], eq. 2.5.5a. The spectrum for the heavy ion case was found to be much larger than that of the electron for large projectile charges. For ^{208}Pb projectiles it can be of the order of 1000 times larger than that of an electron of the same energy. As a natural consequence, reaction rates for Coulomb excitation are larger than for electron excitation. But electrons have the advantage of being a clean electromagnetic probe, while Coulomb excitation at high energies needs a detailed theoretical analysis of the data due to contamination by nuclear excitation. The virtual spectrum for the electron contains more hard photons, i.e. the spectrum decreases slower with photon energy than the heavy ion photon spectrum. This is because, in both situations, the rate at which the spectrum decreases depends on the ratio of the projectile kinetic energy to its rest mass, E/mc^2, which is much larger for the electron ($m = m_e$) than for the heavy ion ($m =$ nuclear mass).

3. Dissociation of weakly-bound systems

3.1. One-neutron halo

In a two-body model, the single-particle picture has been used previously to study Coulomb excitation of halo nuclei with success [23, 24, 25, 26, 27, 28]. The initial wavefunction can be written as $\Psi_{JM} = r^{-1}u_{ljJ}(r)\mathcal{Y}_{lJM}$, where $R_{ljJ}(r)$ is the radial wavefunction and $\mathcal{Y}_{lJM}$ is a spin-angle function [29]. The radial wavefunction, $u_{ljJ}(r)$, can be obtained by solving the radial Schrödinger equation for a nuclear potential, $V^{(N)}_{Jlj}(r)$. Some analytical insight may be obtained using a simple Yukawa form for an s-wave initial wavefunction, $u_0(r) = A_0 \exp(-\eta r)$, and a p-wave final wavefunction, $u_1(r) = j_1(kr)\cos\delta_1 - n_1(kr)\sin\delta_1$. In these equations η is related to the neutron separation energy $S_n = \hbar^2\eta^2/2\mu$, μ is the reduced mass of the neutron + core system, and $\hbar k = \sqrt{2\mu E_r}$, with E_r being the final energy of relative motion between the neutron and the core nucleus. A_0 is the normalization constant of the initial wavefunction. The transition density is given by $r^2\delta\rho_{if}(r) = e_{ff}A_i u_i(r)u_f(r)$, where i and f indices include angular momentum dependence and $e_{eff} = -eZ_c/A$ is the effective charge of a neutron+core nucleus with charge Z_c. The $E1$ transition integral $\mathcal{I}_{l_il_f} = \int_0^\infty dr\, r^3\ \delta\rho_{if}(r)$ is

$$\mathcal{I}_{s\to p} \simeq \frac{e_{ff}\hbar^2}{2\mu}\frac{2E_r}{(S_n+E_r)^2}\left[1+\left(\frac{\mu}{2\hbar^2}\right)^{3/2}\frac{\sqrt{S_n}\,(S_n+3E_r)}{-1/a_1+\mu r_1 E_r/\hbar^2}\right], \tag{6}$$

where the effective range expansion of the phase shift, $k^{2l+1}\cot\delta \simeq -1/a_l + r_l k^2/2$, was used..

The energy dependence of eq. 6 has a few unique features. As shown in previous works [23, 24], the matrix elements for electromagnetic response of weakly-bound nuclei present a small peak at low energies, due to the proximity of the bound state to the continuum. This peak is manifest in the response function of eq. 3:

$$\frac{dB(EL)}{dE} \propto \left|\mathcal{I}_{s\to p}\right|^2 \propto \frac{E_r^{L+1/2}}{(S_n+E_r)^{2L+2}}. \tag{7}$$

It appears centered at the energy [24] $E_0^{(EL)} \simeq (L+1/2)S_n/(L+3/2)$ for a generic electric response of multipolarity L. For $E1$ excitations, the peak occurs at $E_0 \simeq 3S_n/5$.

The second term inside brackets in eq. 6 is a modification due to final state interactions. This modification becomes important in many situations [21]. In fact, the strong dependence of the response function on the effective range expansion parameters makes it a good tool to study the scattering properties of light nuclei which are of interest for nuclear astrophysics.

3.2. Two-neutron halo

Many weakly-bound nuclei, like ^{6}He or ^{11}Li, require a three–body treatment. In one of these models, the bound–state wavefunction in the center of mass system is written as an expansion over hyperspherical harmonics (HH), see e.g. [30],

$$\Psi(\mathbf{x},\mathbf{y}) = \frac{1}{\rho^{5/2}}\sum_{KLSl_xl_y}\Phi^{l_xl_y}_{KLS}(\rho)\left[\mathcal{J}^{l_xl_y}_{KL}(\Omega_5)\otimes\chi_S\right]_{JM}. \tag{8}$$

Here $\mathbf{x}$ and $\mathbf{y}$ are Jacobi vectors where $\mathbf{x}=(\mathbf{r}_1-\mathbf{r}_2)/\sqrt{2}$ and $y=\sqrt{\frac{2(A-2)}{A}}\left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2}-\mathbf{r}_c\right)$, where A is the nuclear mass, $\mathbf{r}_1$ and $\mathbf{r}_2$ are the position of the nucleons, and $\mathbf{r}_c$ is the position of the core. The hyperradius ρ determines the size of a three-body state: $\rho^2=x^2+y^2$. The five angles $\{\Omega_5\}$ include usual angles (θ_x,ϕ_x), (θ_y,ϕ_y) which parametrize the direction of the unit vectors $\widehat{\mathbf{x}}$ and $\widehat{\mathbf{y}}$ and the hyperangle θ, related by $x=\rho\sin\theta$ and $y=\rho\cos\theta$, where $0\leq\theta\leq\pi/2$. The hyperspherical harmonics have the explicit form$\mathcal{J}_{KLM_L}^{l_xl_y}(\Omega_5)=\phi_K^{l_xl_y}(\theta)\,Y_{LM_L}^{l_xl_y}(\widehat{\mathbf{x}},\widehat{\mathbf{y}})$, where $K=l_x+l_y+2\nu$ $(\nu=0,1,2,\cdots)$, and $Y_{LM_L}^{l_xl_y}(\widehat{\mathbf{x}},\widehat{\mathbf{y}})=\left[Y_{l_x}(\widehat{\mathbf{x}})\otimes Y_{l_x}(\widehat{\mathbf{y}})\right]_{LM_L}$. $Y_{lm}(\hat{\mathbf{x}})$ are the usual spherical harmonics. The hyperangular functions are given by$\phi_K^{l_xl_y}(\theta)=N_K^{l_xl_y}\sin^{l_x}\theta\cos^{l_y}\theta P_n^{l_x+1/2,l_y+1/2}(\cos 2\theta)$, where $P_n^{\alpha,\beta}$ are the Jacobi polynomials, $n=(K-l_x-l_y)/2$ and $N_K^{l_xl_y}$ are normalization constants. The hyperspherical harmonics are orthonormalized using the volume element $d\Omega_5=\sin^2\theta\cos^2\theta d\widehat{\mathbf{x}}d\widehat{\mathbf{y}}$. The insertion of the three-body wavefunction, eq. 8, into the Schrödinger equation yields a set of coupled differential equations for the hyperradial wavefunction $\Phi_{KLS}^{l_xl_y}(\rho)$.

For weakly-bound systems having no bound subsystems the hyperradial functions entering the expansion 8 behave asymptotically as [31] $\Phi_a(\rho)\longrightarrow\exp(-\eta\rho)$ as $\rho\longrightarrow\infty$, where the two-nucleon separation energy is related to η by $S_{2n}=\hbar^2\eta^2/(2m_N)$. This wavefunction has similarities with the two-body case, when ρ is interpreted as the distance r between the core and the two nucleons, treated as one single particle. But notice that the mass m_N would have to be replaced by $2m_N$ if a simple two-body (the dineutron-model [4, 32]) were used for ^{11}Li or ^{6}He.

Since only the core carries charge, in a three-body model the $E1$ transition operator is given by $M\sim yY_{1M}(\widehat{\mathbf{y}})$ for the final state (see also [34]). The $E1$ transition matrix element is obtained by a sandwich of this operator between $\Phi_a(\rho)/\rho^{5/2}$ and scattering wavefunctions. In ref. [33] the scattering states were taken as plane waves. I will use distorted scattering states, leading to the expression

$$\mathcal{I}(E1)=\int dxdy\frac{\Phi_a(\rho)}{\rho^{5/2}}\;y^2xu_p(y)\,u_q(x)\,, \tag{9}$$

where $u_p(y)=j_1(py)\cos\delta_{nc}-n_1(py)\cos\delta_{nc}$ is the core-neutron asymptotic continuum wavefunction, assumed to be a p-wave, and $u_q(x)=j_0(qx)\cos\delta_{nn}-n_0(qx)\cos\delta_{nn}$ is the neutron-neutron asymptotic continuum wavefunction, assumed to be an s-wave. The relative momenta are given by $\mathbf{q}=(\mathbf{q}_1-\mathbf{q}_2)/\sqrt{2}$ and $\mathbf{p}=\sqrt{\frac{2(A-2)}{A}}\left(\frac{\mathbf{k}_1+\mathbf{k}_2}{2}-\mathbf{k}_c\right)$.

The $E1$ strength function is proportional to the square of the matrix element in eq. 9 integrated over all momentum variables, except for the total continuum energy $E_r=\hbar^2(q^2+p^2)/2m_N$. This procedure gives

$$\frac{dB(E1)}{dE_r}=\text{constant}\times\int\left|\mathcal{I}(E1)\right|^2E_r^2\cos^2\Theta\sin^2\Theta d\Theta d\Omega_q d\Omega_p, \tag{10}$$

where $\Theta=\tan^{-1}(q/p)$.

Most integrals in eqs. 9 and 10 can be done analytically, leaving two remaining integrals which can only be performed numerically. In ref. [21] it was shown that the calculation follwoing the above prescription is able to reproduce the available scattering data, and that final state interactions are of extreme relevance.

As pointed out in ref. [33], the $E1$ three-body response function of ^{11}Li can still be described by an expression similar to eq. 7, but with different powers. Explicitly,

$$\frac{dB\left(E1\right)}{dE_r} \propto \frac{E_r^3}{\left(S_{2n}^{eff}+E_r\right)^{11/2}}. \tag{11}$$

Instead of S_{2n}, one has to use an effective $S_{2n}^{eff} = aS_{2n}$, with $a \simeq 1.5$. With this approximation, the peak of the strength function in the three-body case obtained from eq. 11 is situated at about three times higher energy than for the two-body case, eq. 7. In the three-body model, the maximum is thus predicted at $E_0^{(E1)} \simeq 1.8S_{2n}$, which fits the experimentally determined peak position for the ^{11}Li $E1$ strength function very well [33]. It is thus apparent that the effect of three-body configurations is to widen and to shift the strength function $dB\left(E1\right)/dE$ to higher energies.

4. Collective excitations: the pigmy resonance

4.1. The hydrodynamical model

As with giant dipole resonances (GDR) in stable nuclei, one believes that pygmy resonances at energies close to the threshold are present in halo, or neutron-rich, nuclei. This was proposed by Suzuki et al. [37] using the hydrodynamical model for collective vibrations. We will use the method of Myers et al. [40], who considered collective vibrations in nuclei as an admixture of Goldhaber-Teller (GT) and Steinwedel-Jensen (SJ) modes. For light nuclei they found that Goldhaber-Teller modes dominate. But in order to reproduce the correct position of the GDR along the periodic table both modes have to be included.

For spherically symmetric densities, the transition density, $\delta\rho_p\left(\mathbf{r}\right) = \delta\rho_p\left(r\right)Y_{10}\left(\hat{\mathbf{r}}\right)$, can be calculated assuming a combination of the SJ and GT distributions [21],

$$\delta\rho_p\left(r\right) = \sqrt{\frac{4\pi}{3}}R\left\{Z_{eff}^{(1)}\alpha_1\frac{d}{dr} + Z_{eff}^{(2)}\alpha_2\frac{K}{R}j_1\left(kr\right)\right\}\rho_0(r), \tag{12}$$

where R is the mean nuclear radius of the nucleus, and α_i is the percent displacement of the center of mass of the neutron and proton fluids in the GT ($i=1$) and SJ ($i=2$) modes. For light, weakly-bound nuclei, it is appropriate to assume that the neutrons inside the core (A_c, Z_c) vibrate in phase with the protons. The neutrons and protons in the core are tightly bound. Calling the excess nucleons by $(A_e, Z_e) = (A - A_c, Z - Z_c)$, the effective charge for the GT mode is $Z_{eff}^{(1)} = \left(Z_cA_e - A_cZ_e\right)/A$. This effective charge is zero if $(A_c, Z_c) = (A, Z)$ and no pigmy resonance is possible in this model, only the usual GDR. In eq. 12, $j_1(kr)$ is the spherical Bessel function of first order, α_2 the percent displacement of the center of mass of the neutron and proton fluids in the Steinwedel-Jensen mode, $kR = a = 2.081, \quad$ and $K = 2a/j_0\left(a\right) = 9.93$. These relations are obtained by the condition that the radial velocity of the SJ fluid vanishes at the nuclear surface.

The hydrodynamical model can be further explored to obtain the energy and excitation strength of the collective excitations. This can be achieved by finding the eigenvalues of the Hamiltonian $\mathcal{H} = \frac{1}{2}\dot{\alpha}\mathcal{T}\dot{\alpha} + \frac{1}{2}\alpha\mathcal{V}\alpha + \dot{\alpha}\mathcal{F}\dot{\alpha}$, where $\alpha = (\alpha_1, \alpha_2)$ is now a vector containing the GT and SJ contributions to the collective motion. $\mathcal{T}$ and $\mathcal{V}$ are the kinetic and potential energies 2×2 matrices [40]. The kinetic term can be calculated from the GT and SJ velocity fields, $\mathbf{v}_{1p}$ and $\mathbf{v}_{2p}$: $T = \frac{1}{2}m_N^* \int \left[\rho_p\left(\mathbf{v}_{1p}+\mathbf{v}_{2p}\right)^2 + \rho_n\left(\mathbf{v}_{1n}+\mathbf{v}_{2n}\right)^2\right] d^3r$, where the effective nucleon mass m_N^* accounts for the meson exchange effects. The potential term can be related to the stiffness parameters of the liquid-drop model adjusted to a best fit to the nuclear masses. The stiffness of the system is due to the change in symmetry energy of the system as it goes out of the equilibrium position: $V = -\kappa \int d^3r\, (\rho_p - \rho_n)^2/(\rho_p + \rho_n)$, where κ can be estimated from the semi-empirical mass formula ($\kappa \simeq 30 - 40$ MeV). The last term in the Hamiltonian is the Rayleigh dissipation term, which can be related to the Fermi velocity of the nucleons [40] and yields the width of the eigenstate.

As shown by Myers et al. [40], the liquid drop model predicts an equal admixture of SJ+GT oscillation modes for large nuclei. The contribution of the SJ oscillation mode decreases with decreasing mass number, i.e. $\alpha \longrightarrow (\alpha_1,\ 0)$ as $A \longrightarrow 0$. This is even more probable in the case of halo nuclei, where a special type of GT mode (oscillations of the core against the halo nucleons) is likely to be dominant. For this special collective motion an approach different than those used in refs. [40] and [37] has to be considered. It is easy to make changes in the original Goldhaber and Teller [38] formula to obtain the energy of the collective vibrations. One has to account for the effective mass of our modified GT model. The resonance energy formula derived by Goldhaber and Teller [38] changes to

$$E_{PR} = \left(\frac{3\varphi\hbar^2}{2aRm_N A_r}\right)^{1/2}, \tag{13}$$

where $A_r = A_c\left(A - A_c\right)/A$ and a is the length within which the interaction between a neutron and a nucleus changes from a zero-value outside the nucleus to a high value inside, i.e. a is the size of the nuclear surface. φ is the energy needed to extract one neutron from the proton environment. Goldhaber and Teller [38] argued that in a heavy stable nucleus φ is not the binding energy of the nucleus, but the part of the potential energy due to the neutron proton interaction. It is proportional to the asymmetry energy. In the case of weakly-bound nuclei this picture changes and it is more reasonable to associate φ to the separation energy of the valence neutrons, S. I will use $\varphi = \beta S$, with a parameter β which is expected to be of order of one. Since for halo nuclei the product aR is proportional to S^{-1}, we obtain the proportionality $E_{PR} \propto S$. Due to the simplicity of the model, the proportionality factor cannot be trusted. Using eq. 13 for ^{11}Li , with $a = 1$ fm, $R = 3$ fm and $\varphi = S_{2n} = 0.3$ MeV, we get $E_{PR} = 1.3$ MeV. Considering that the pygmy resonance will most probably decay by particle emission, one gets $E_r \simeq 1$ MeV for the kinetic energy of the fragments. This is about a factor 2 larger than what is obtained in a numerical calculation [21]. But it is within the right ballpark. It is possible that formula 13 for the energy of the pigmy collective vibrations can be improved using proton and neutron density profiles obtained from microscopic calculations. It must be remembered, however, that the hydrodynamical model is very unlikely to be an accurate model for light, loosely-bound, nuclei and is significant only in that the correct order of magnitude of the resonance energy is found.

The main decay channel of the pigmy resonance is the breakup of the nucleus. As shown above, both the direct dissociation model and the hydrodynamical model yield a bump in the response function with position proportional to S, the valence nucleon(s) separation energy. In the direct dissociation model the width of the response function obviously depends on the separation energy. But it also depends on the nature of the model, i.e. if it is a two-body model, like the model often adopted for ^{11}Be or ^{8}B, or a three-body model, appropriate for ^{11}Li and ^{6}He. In the two-body model the phase-space depends on energy as $\rho(E) \propto d^3p/dE \propto \sqrt{E}$, while in the three-body model $\rho(E) \sim E^2$. This explains why the peak of dB/dE is pushed toward higher energy values, as compared to the prediction of eq. 7. It also explains the larger width of dB/dE obtained in three-body models. In the case of the pygmy resonance, this question is completely open.

The hydrodynamical model predicts [40] for the width of the collective mode $\Gamma = \hbar\overline{\mathrm{v}}/R$, where $\overline{\mathrm{v}}$ is the average velocity of the nucleons inside the nucleus. This relation can be derived by assuming that the collective vibration is damped by the incoherent collisions of the nucleons with the walls of the nuclear potential well during the vibration cycles. This approach mimics that used in the kinetic theory of gases for calculating the energy transfer of a moving piston to gas molecules in a container. Using $\overline{\mathrm{v}} = 3\mathrm{v}_F/4$, where $\mathrm{v}_F = \sqrt{2E_F/m_N}$ is the Fermi velocity, with $E_F = 35$ fm and $R = 6$ fm, one gets $\Gamma \simeq 6$ fm. This is the typical energy width a giant dipole resonance state in a heavy nucleus. In the case of neutron-rich light nuclei $\overline{\mathrm{v}}$ is not well defined. There are two average velocities: one for the nucleons in the core, $\overline{\mathrm{v}}_c$, and another for the nucleons in the skin, or halo, of the nucleus, $\overline{\mathrm{v}}_h$. One is thus tempted to use a substitution in the form $\overline{\mathrm{v}} = \sqrt{\overline{\mathrm{v}}_c\overline{\mathrm{v}}_h}$. Following ref. [41], the width of momentum distributions of core fragments in knockout reactions, σ_c, is related to the Fermi velocity of halo nucleons by $\mathrm{v}_F = \sqrt{5\sigma_c^2/m_N}$. Using this expression with $\sigma_c \simeq 20$ MeV/c, we get $\Gamma = 5$ MeV (with $R = 3$ fm). This value is much larger than that observed in experiments.

It seems clear that the piston model is not able to describe the width of the response function properly. Microscopic models, such as those based on random phase approximation (RPA) calculations, are necessary to tackle this problem. The halo nucleons have to be treated in an special way to get the response at the right energy position, and with approximately the right width. Right now, the problem remains if the experimentlly observed peak in dB/dE is due to a direct transition to the continuum, weighted by the phase space of the fragments, or if it proceeds sequentially via a soft dipole collective state.

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