PreLab 3 - Simple Harmonic Motion

A simple bathroom scale measures the weight that is placed on it by equating the amount of weight to the compression of a spring inside the scale. The larger the weight placed on top, the more the spring inside is compressed. The bathroom scale is calibrated to account for the acceleration of gravity at the Earth's surface. If you took the scale to the moon it would give a reading of about one-sixth of what it would show for the same person on Earth, because the acceleration of gravity on the moon is about one-sixth of the gravity on Earth. This lab exercise will show the difference between mass and weight in a way that will explain how a simple scale works.

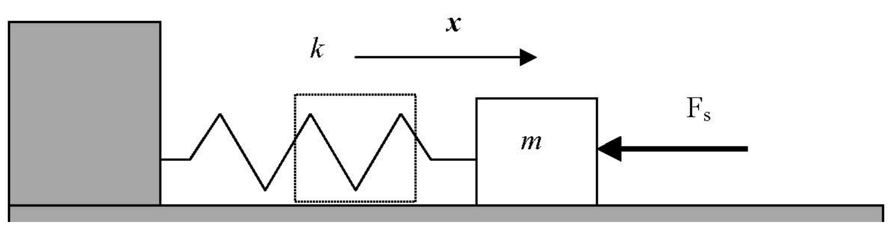
If you apply a force to one end of a spring while holding the other end stationary, the spring will stretch. You will need to apply a larger force to achieve a longer stretch. The spring will exert a force that tries to bring the spring back to its unstretched position. The farther it is stretched, the harder it will pull back against the stretching force. Another way of saying this is that there is a restoring force that is proportional to the stretching force and it is in the opposite direction. The difference between two springs might be described as how "stiff" they are. A "stiffer" spring requires more force to stretch a certain amount that a less stiff spring. The stiffness of a spring is called the **spring constant** and we use the symbol of a lowercase *k*. If *k* is large, it is stiffer than if *k* were small.

Hooke's Law states the relationship between the amount of stretch in a spring and the force required to make it stretch:

$$F = -kx.$$

F is the force that the spring is exerting to try to return it to its unstretched position (i.e. **equilibrium**), and x is the **displacement** or change from equilibrium position of the end of the spring. The *k* is a constant for a given spring, and the negative sign is a result of the force always being directed toward the equilibrium position. The units of the spring constant will be in Newtons per meter (i.e. *N/m*). Note: the force is proportional to the distance the spring is stretched from the equilibrium position.

A very simple oscillatory device is created when a mass is attached at the end of a spring, as shown below. If the mass is then displaced from its equilibrium position, the spring will exert a force on the mass to restore it to its equilibrium position. However, when the mass returns to this position, it will have a velocity and will overshoot the equilibrium position. The force the spring now exerts on the mass will be opposite to the direction of motion. The mass will then slow down and stop away from the equilibrium position. This process will continue to repeat forever in an ideal spring-mass system (defined as a system without friction). In the figure, a mass *m* is attached to an ideal spring with spring constant *k*. The equilibrium position of the mass is indicated by the dotted-line box. The position *x* is shown relative to the equilibrium position.



The resulting motion is given by the following formula:

$$x = A\cos(\omega t),$$

where **A** is the amplitude of the motion. The **period** of the motion is given by

$$T = 2\pi\sqrt{\frac{m}{k}},$$

and the **angular frequency** $\omega = 2\pi/T$. Note, the period of motion does not depend on the amplitude.

LAB 3 & Lab Report

Name: ________________ **Section**: ______________

PART 1 - Measuring the Spring Constant

Equipment: spring, weights, ruler

In this part of the exercise, you will find the spring constant *k* of your spring by hanging masses from it. The apparatus has a spring hanging on a stand. Attach the Scale Indicator at the bottom of the spring and align the zero on the sliding transparent scale with the Stretch Indicator. Connect a mass hanger to the bottom of the Stretch Indicator, and then add mass to stretch the spring. The spring will resist the change in its length with a restoring force proportional to the change in length. As we add to the weight on the spring, it will stretch more. Hang at least 5 masses on the spring and measure the change in length. *Don't forget to include the mass of the hanger!* Since theory involves the use of a massless spring, you can correct for this by adding 1/3 of the mass of the spring to the overall mass.

CAUTION: Stretching the spring more than twice its resting length can damage the spring. Test your mass choice prior to trying the experiment.

The amount of mass times the acceleration of gravity (i.e. g = 9.81 m/s^2) will give us the restoring force.

Trial	**Mass**	**Displacement**	**Force**
Measurement 1	__________	__________	__________
Measurement 2	__________	__________	__________
Measurement 3	__________	__________	__________
Measurement 4	__________	__________	__________
Measurement 5	__________	__________	__________

There are many ways to deal with raw data in order to put it into a form that we can both understand and predict results for other events. A graph is a wonderful tool for interpreting data. Make a plot of the restoring force vs. the displacement of the mass.

(a) There are some basic "rules" for graph construction. Every graph needs a title, and both axes of the graph need labels and units telling what they represent. For each pair of data, we mark a point on the graph. Draw a 'best-fit' straight line, which passes closest to your data set. (Don't connect the dots!) Check with your TA if you are uncertain about this.

(b) The slope of you best-fit line gives the spring constant. The general formula for the slope of a line is:

$$m = \frac{y_2 - y_1}{x_2 - x_1},$$

where *x* and *y* are coordinates on the line. Using your best-fit line, pick any two points to use for the calculation of the spring constant k.

k = ______________

Part II – The Motion of a Mass on a Spring

In this part of the lab you will use the Sonic Ranger (Trademarked by Pasco) to measure the motion of a spring oscillating vertically. The Sonic Ranger is a SONAR device. It sends out ultrasonic pulses of sound that are reflected from the target. By measuring the return time, the ultrasonic ranger can determine the distance of the object. The ranger sends out several sound pulses each second and can therefore track the motion of the mass.

You will hang the mass m_1 from a spring on the stand. To test the setup, collect data while the object is motionless. Record the distance x_1 of the hanging mass from the sensor. Make sure the mass never gets closer to the ranger than 41 *cm* (the minimum range of the sonic ranger). Replace the mass with another mass m_2 and record the distance x_2 of the hanging mass from the sensor.

(1) Using the following equation:

$$k = g\frac{m_1 - m_2}{x_2 - x_1},$$

calculate the spring constant *k*.

(2) Now that you've calculated the spring constant k, calculate the period T of oscillation using the following equation:

$$T_{theoretical} = 2\pi\sqrt{\frac{m}{k}},$$

where *m* is the mass attached to the spring and *k* is the spring constant calculated in part (a).

(3) Next you will put the mass into motion. You will measure the position, velocity, and acceleration as a function of time. As you did in the pendulum lab, use the 'Delta Tool' for the position vs. time plot to find the period of oscillation. Is the mass in simple harmonic motion?
Compare the position versus time, the velocity versus time, and the acceleration versus time to each other. Are they what you expected?

(4) Repeat steps 2-3 for four additional masses.

k = ____________

Trial	Mass	T (theory)	T (experiment)	% Error
Measurement 1	__________	__________	__________	__________
Measurement 2	__________	__________	__________	__________
Measurement 3	__________	__________	__________	__________
Measurement 4	__________	__________	__________	__________
Measurement 5	__________	__________	__________	__________

What were some possible sources of error in your measurements? (Human error does not suffice as an answer!)