Musical Acoustics

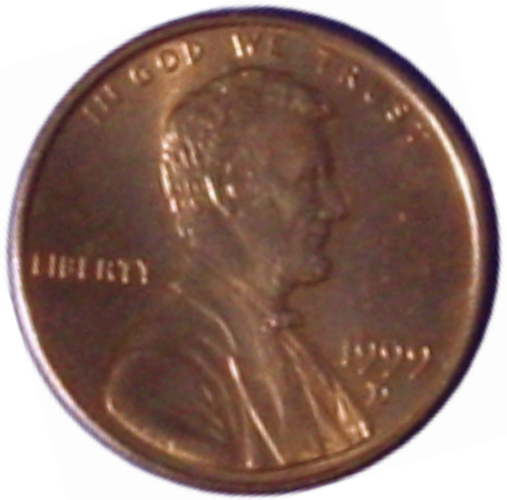
Lecture 1

Basic Mathematics

Scientific Notation

An ordinary penny contains about 20,000,000,000,000,000,000,000 atoms. The average size of an atom is about 0.00000003 centimeters across.

The length of these numbers in standard notation makes them awkward to work with.

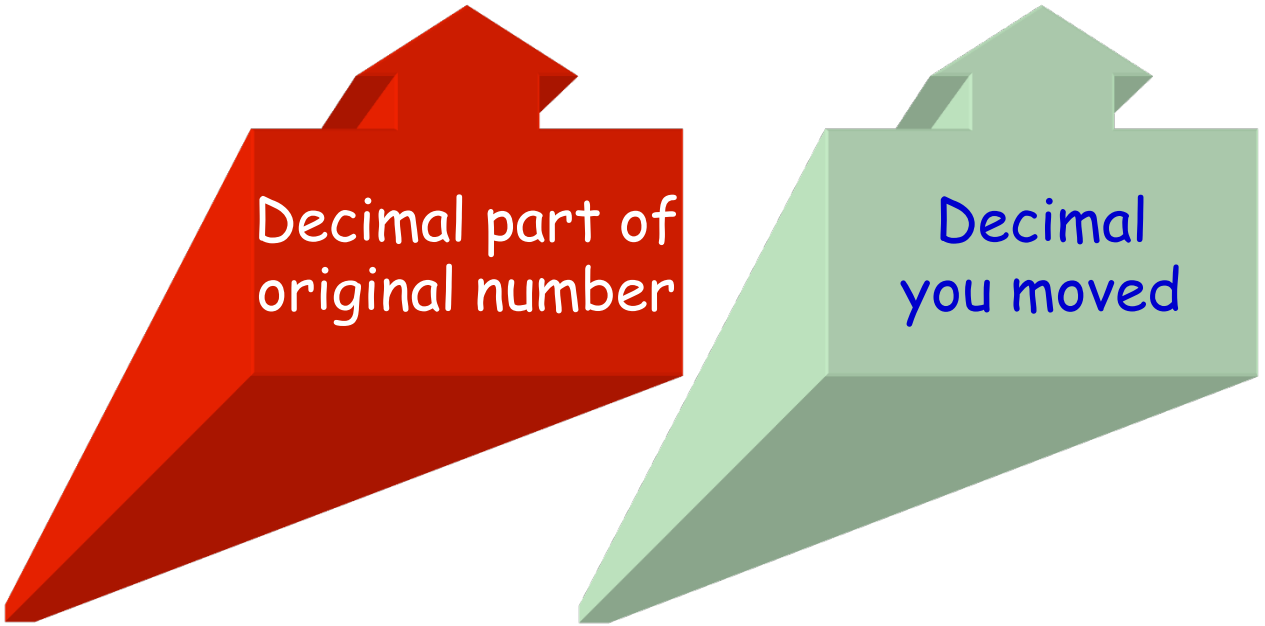


Scientific notation is a shorthand way of writing such numbers.

Scientific Notation

Method to express really big or small numbers.

Format is **Mantissa** **x** **Base** Power



$$6.02 \times 10^{23}$$

We just move the decimal point around.

602000000000000000000000

Scientific Notation

In scientific notation the number of atoms in a penny is 2.0×10^{22}, and the size of each atom is 3.0×10^{-8} centimeters across.

The sign of the exponent tells which direction to move the decimal. A positive exponent means move the decimal to the right, and a negative exponent means move the decimal to the left.

Exponents (or powers)

- $a^1 = a$
- $a^2 = a \times a$
- $a^3 = a \times a \times a$
- $a^4 = a \times a \times a \times a$
- ....
- $a^n = a \times a \times a \times a \times \ldots \times a$

  (n times)

  (base a, exponent n)

Positive Exponents

- $10^1 = 10$
- $10^2 = 10X10 = 100$
- $10^3 = 10X10X10 = 1000$
- $10^4 = 10X10X10X10 = 10{,}000$

Negative Exponents

- $10^{-1} = 1/10 = 0.1$
- $10^{-2} = 1/100 = 0.01$
- $10^{-3} = 1/1000 = 0.001$
- $10^{-4} = 1/10000 = 0.0001$

Scientific Notation

- We use the idea of exponents to make it easier to work with large and small numbers.
- $10{,}000 = 1 \times 10^4$
- Count places to the left until there is one number to the left of the decimal point.
- $250{,}000 = 2.5 \times 10^5$
- $230{,}000 = ?$
- $35{,}000 = ?$

Scientific Notation

- $0.00006 = 6 \times 10^{-5}$
- $0.00045 = 4.5 \times 10^{-4}$
- Count places to the right until there is one number to the left of the decimal point
- $0.003 = ?$
- $0.0000025 = ?$

Multiplying with Scientific Notation

- Add the Exponents
- $10^2 \times 10^3 = 10^5$
- $100 \times 1000 = 100{,}000$

Multiplying with Scientific Notation

$(2.3 \times 10^2)(3.3 \times 10^3)$

- 230 X 3300
- Multiply the Coefficients

  2.3 X 3.3 = 7.59
- Add the Exponents

  $10^2 \times 10^3 = 10^5$
- 7.59×10^5
- 759,000

Dividing with Scientific Notation

- Subtract the Exponents
- $10^4/10^3 = 10^1$
- 10000/1000 = 10

Dividing with Scientific Notation

$(3.3 \times 10^4) / (2.3 \times 10^2)$

- 33000 / 230 = 143.4783
- Divide the Coefficients

  3.3 / 2.3 = 1.434783
- Subtract the Exponents

  $10^4 / 10^2 = 10^2$
- 1.434783×10^2
- 143.4783

Practice

1) Express 0.0000000902 in scientific notation.

Where would the decimal go to make the number be between 1 and 10?

9.02

The decimal was moved how many places?

8

When the original number is less than 1, the exponent is negative.

9.02×10^{-8}

2) Write 28750.9 in scientific notation.

1. 2.87509×10^{-5}
2. 2.87509×10^{-4}
3. 2.87509×10^{4}
4. 2.87509×10^{5}

3) Express 1.8×10^{-4} in decimal notation.

4) Express 4.58×10^{6} in decimal notation.

5) Write $(2.8 \times 10^{3})(5.1 \times 10^{-7})$ in scientific notation.

1. 14.28×10^{-4}
2. 1.428×10^{-3}
3. 14.28×10^{10}
4. 1.428×10^{11}

6) Write 531.42×10^{5} in scientific notation.

1. $.53142 \times 10^{2}$
2. 5.3142×10^{3}
3. 53.142×10^{4}
4. 531.42×10^{5}
5. 53.142×10^{6}
6. 5.3142×10^{7}
7. $.53142 \times 10^{8}$

7) Divide in Scientific Notation

- $(4.6 \times 10^4) / (5.5 \times 10^3) = ?$
- $(3.1 \times 10^3) / (4.2 \times 10^5) = ?$

Significant figures

- There are 2 kinds of numbers:
  - Exact: the amount of money in your account. Known with certainty.
  - Approximate: weight, height—anything MEASURED. No measurement is perfect.

When a measurement is recorded only those digits that are measured are written down.

When to use Significant figures

- If you measured the width of a paper with your ruler you might record 21.7cm.

To a mathematician 21.70, or 21.700 is the same.

But, to a scientist 21.7cm and 21.70cm is NOT the same

If you used an ordinary ruler, the smallest marking is the mm, so your measurement has to be recorded as 21.7cm.

How do I know how many Sig Figs?

- If you used an ordinary ruler, the smallest marking is the mm, so your measurement has to be recorded as 21.7cm.

- **Rule:** All digits are significant starting with the <u>first non-zero</u> digit on the left.

- ***Exception to rule:*** In whole numbers that end in zero, the zeros at the end are not significant.

How many sig figs?

- 7
- 40
- 0.5
- 0.00003
- 7×10^5
- 7,000,000

How do I know how many Sig Figs?

- 2nd Exception to rule: If zeros are sandwiched between non-zero digits, the zeros become significant.

- 3rd Exception to rule: If zeros are at the end of a number that has a decimal, the zeros are significant.

How many sig figs?

- 1.2
- 2100
- 56.76
- 4.00
- 0.0792
- 7,083,000,000

How many sig figs?

- 3401
- 2100
- 2100.0
- 5.00
- 0.00412
- 8,000,050,000

Calculations with sig figs

- Rule: When adding or subtracting measured numbers, the answer can have no more places after the decimal than the LEAST of the measured numbers.

Examples:

- 2.45 cm + 1.2 cm = 3.65 cm,
- Round off to = 3.7cm

- 7.432 cm + 2 cm = 9.432 round to → 9 cm

Multiplication and Division

- Rule: When multiplying or dividing, the result can have no more significant figures than the least reliable measurement.

Examples

- 56.78 cm x 2.45 cm = 139.111 cm^2
- Round to → 139 cm^2

- 75.8cm x 9.cm = ?