Musical Acoustics

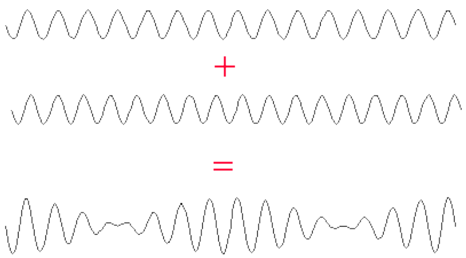
Lecture 10

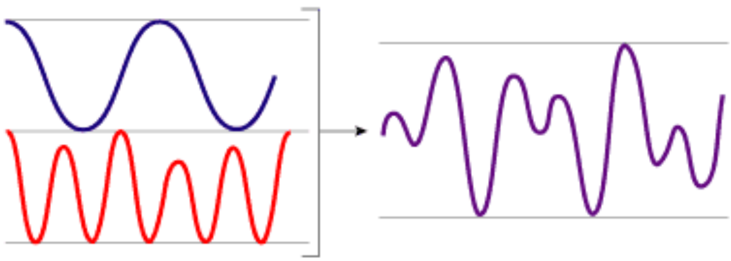
Harmonics in strings, pipes and drums - 2

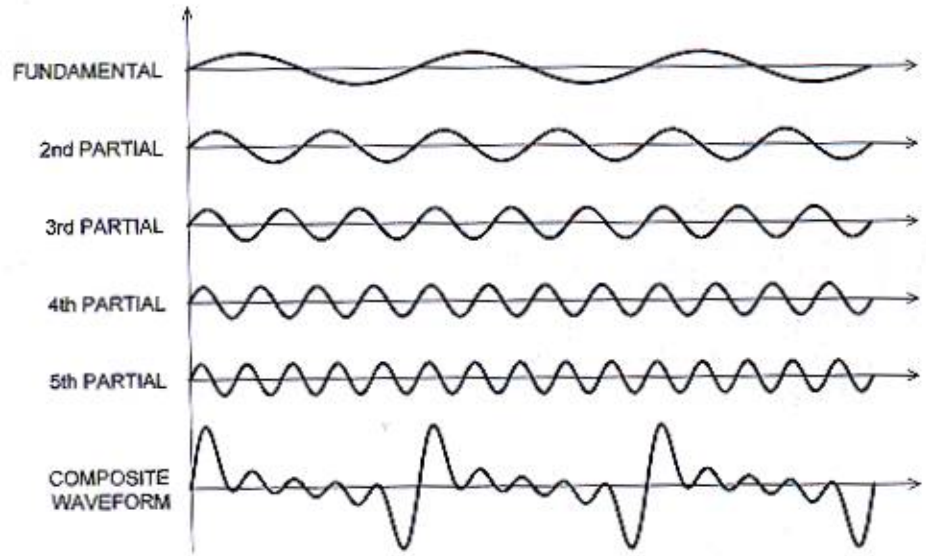
Timbre

Fourier's theorem

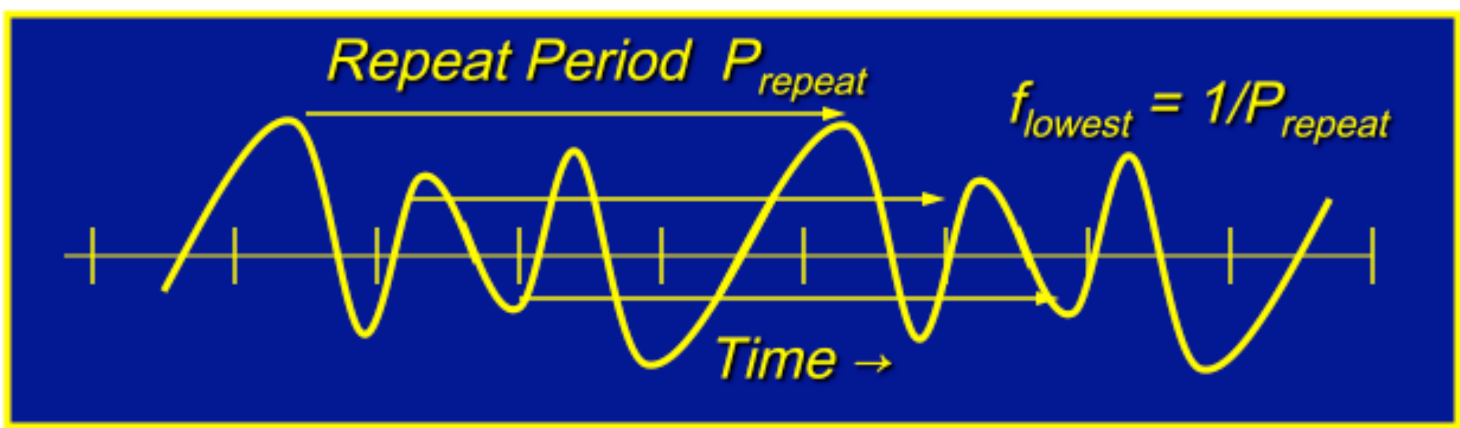
- Fourier's theorem says any complex wave can be made from a sum of single frequency waves.





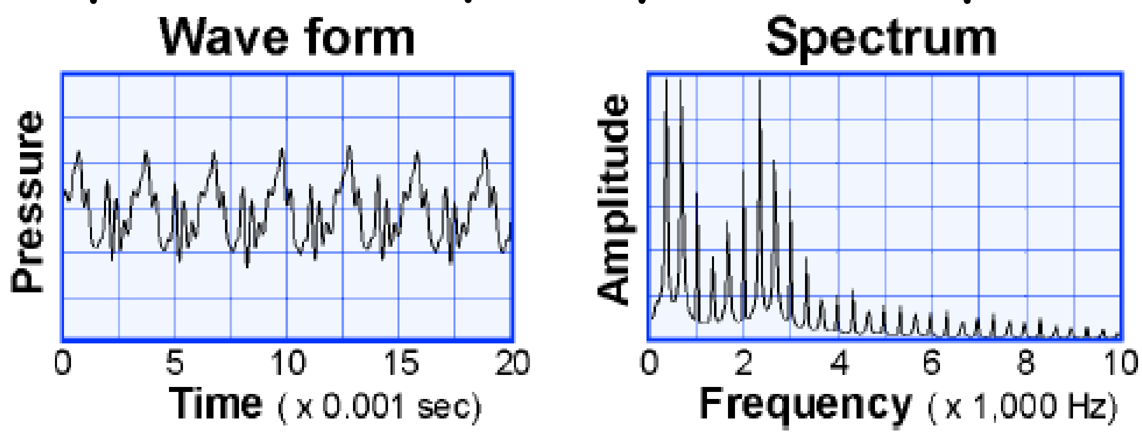


The lowest frequency component of a wave is equal to $1/P_{repeat}$, the reciprocal of the repeat period P_{repeat}

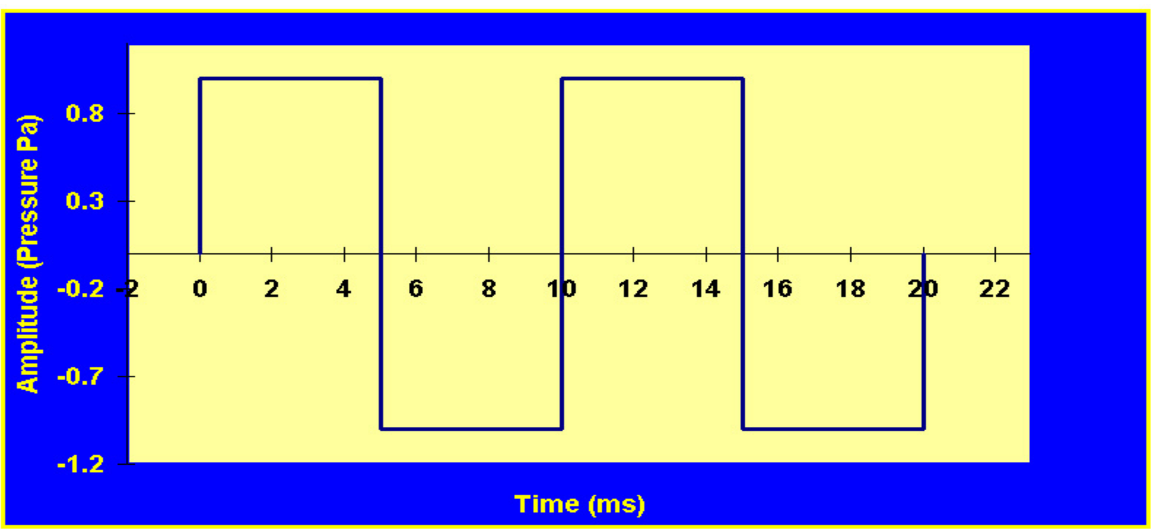


Sound spectrum

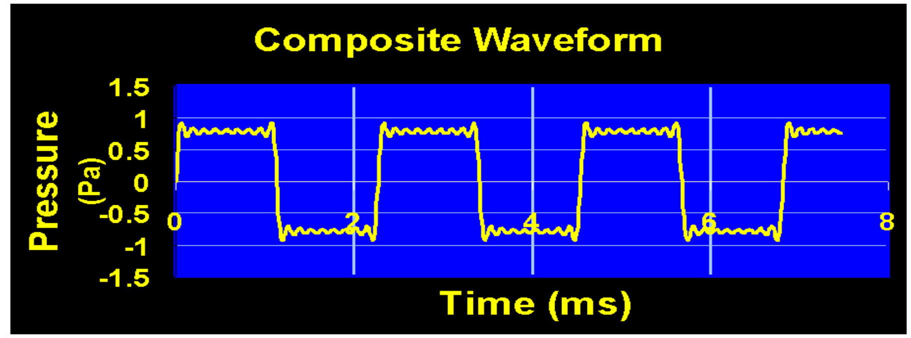
- A complex wave is really a sum of component frequencies.
- A frequency spectrum is a graph that shows the amplitude of each component frequency in a complex wave.

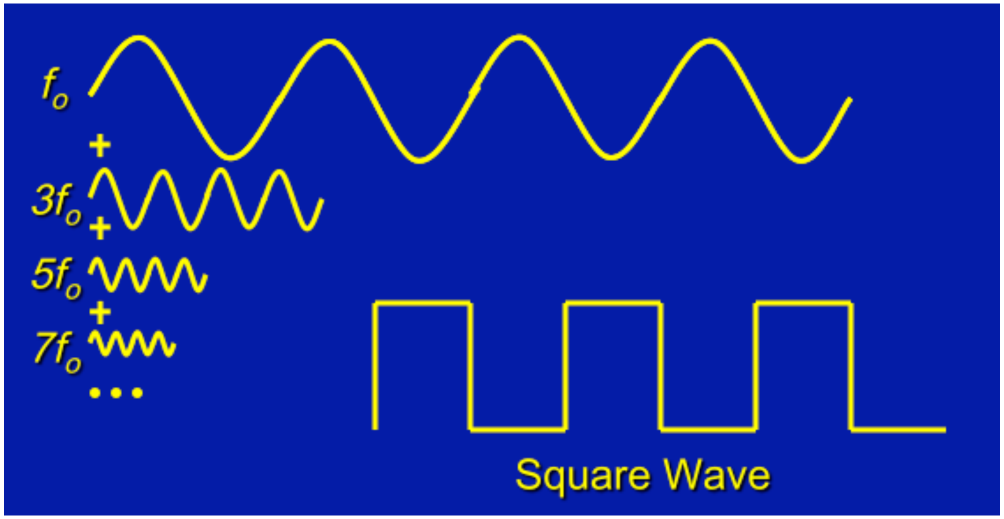


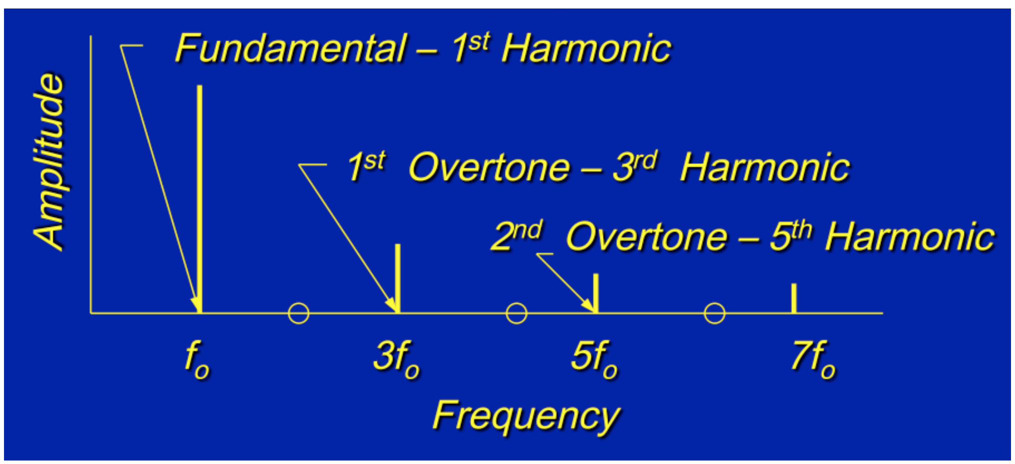
Example: What is the lowest frequency component of the following waveform?



Example: Synthesis of a square wave



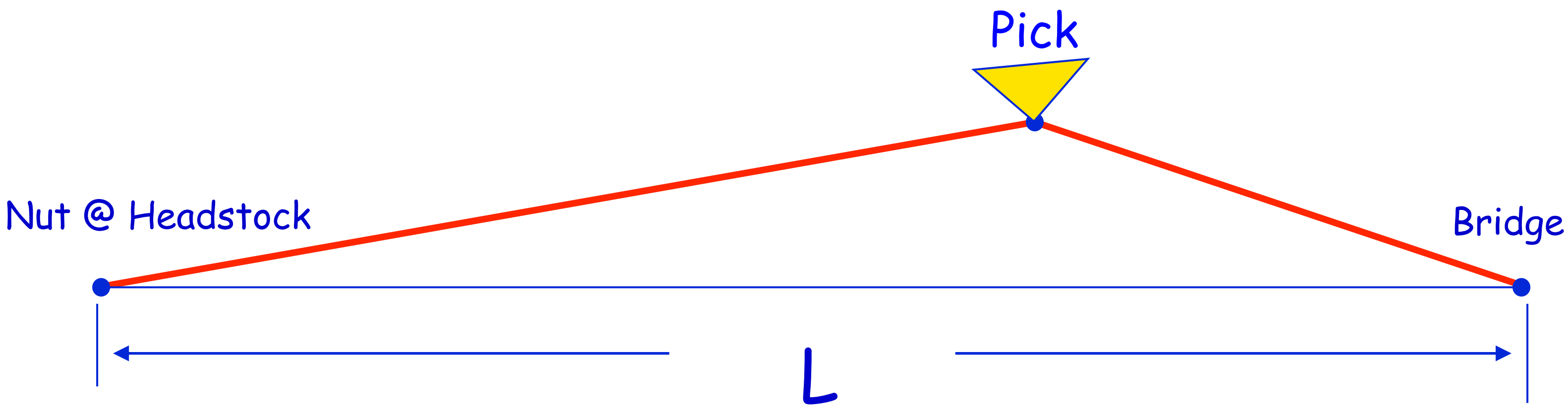




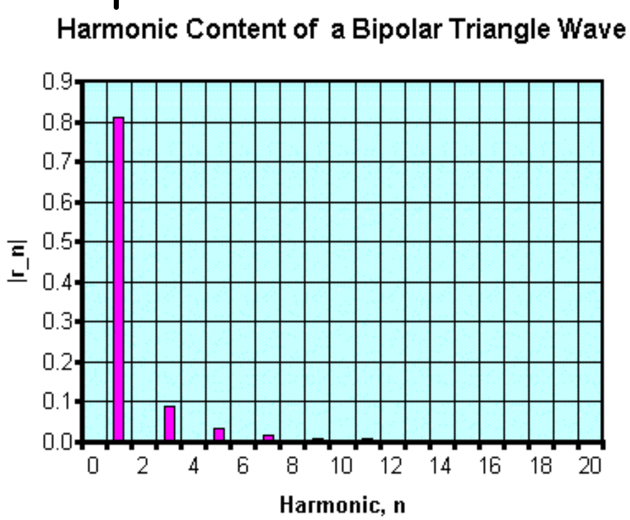
Fourier Analysis *is the decomposition of a wave into the sine wave components from which it can be built up.*

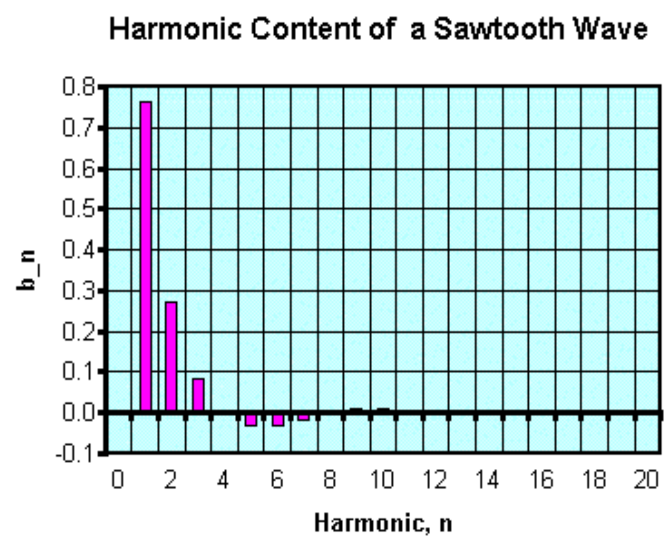
A ***Fourier Analysis*** *is a representation of all the components that comprise a waveform,* ***amplitude*** *versus frequency and* ***phase*** *versus frequency.*

When we e.g. pick (i.e. pluck) the string of a guitar, initial waveform is a triangle wave:



The geometrical shape of the string (a triangle) at the instant the pick releases the string can be shown mathematically (using Fourier Analysis) to be due to a linear superposition of standing waves consisting of the fundamental plus higher harmonics of the fundamental! Depending on where pick along string, harmonic content changes. Pick near the middle, mellower (lower harmonics); pick near the bridge - brighter - higher harmonics emphasized!



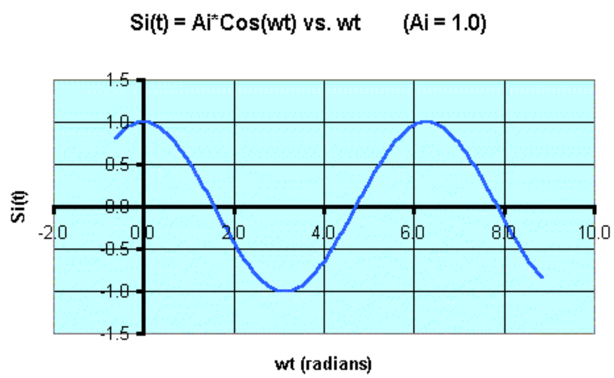


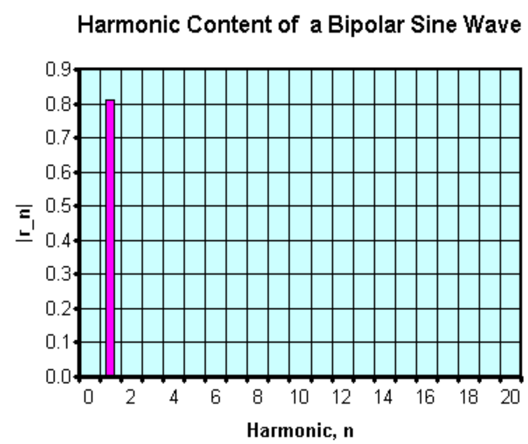
Harmonic Content of Complex Wave Forms

In fact, geometrical/mathematical shape of any *periodic* waveform can be shown to be due to linear combination of fundamental & higher harmonics!

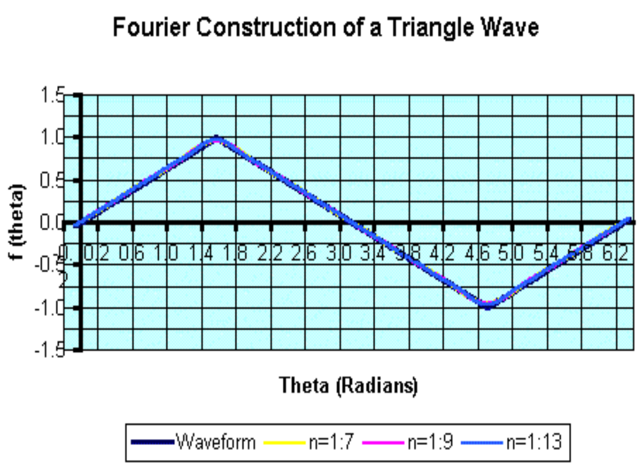
Sound Tonal Quality - *Timbre* - harmonic content of sound wave

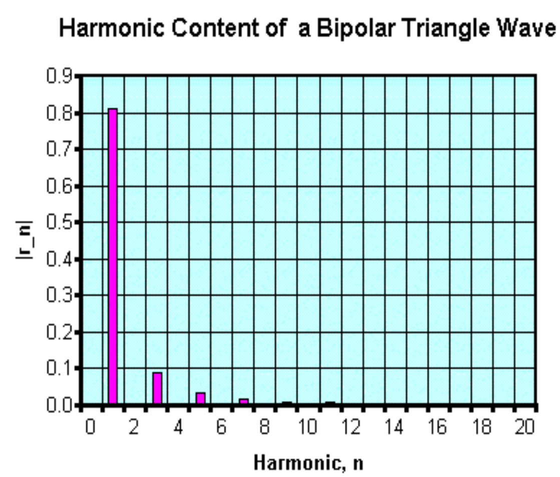
Sine/Cosine Wave: Mellow Sounding – fundamental, no higher harmonics



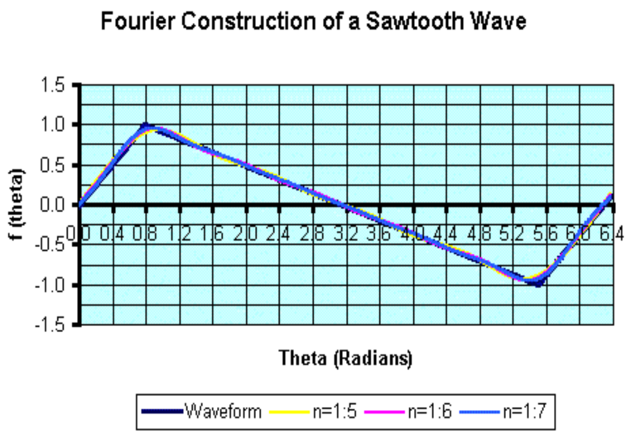


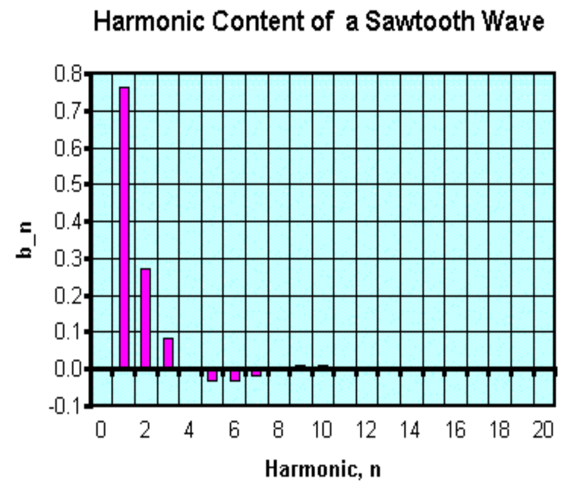
Triangle Wave: A Bit Brighter Sounding – has higher harmonics!



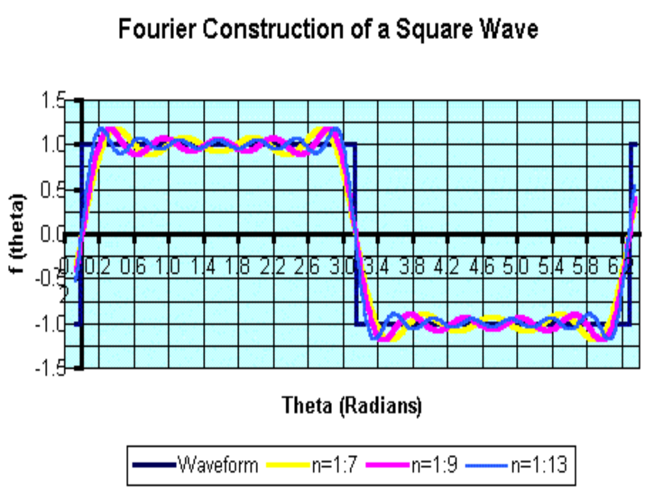


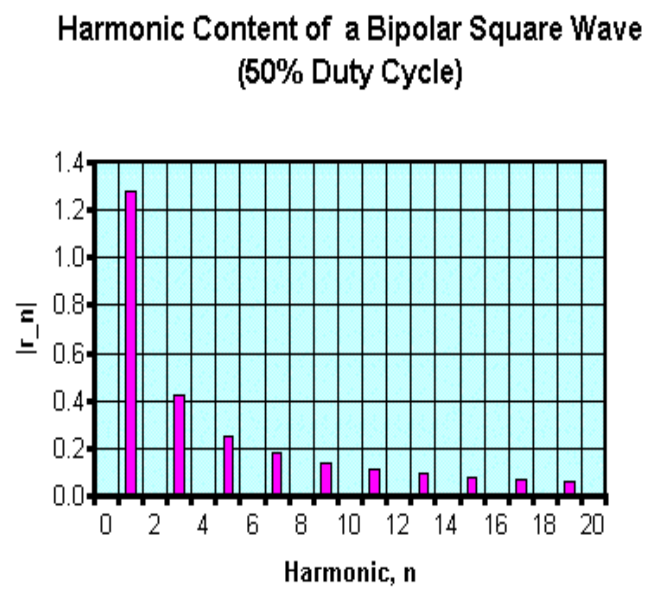
Asymmetrical Sawtooth Wave: Even Brighter Sounding – even more harmonics!





Square Wave: Brighter Sounding – has the most harmonics!





What is Music?

Music

- The **pitch** of a sound is how high or low we hear its frequency. Though pitch and frequency usually mean the same thing, the way we hear a pitch can be affected by the sounds we heard before and after.
- **Rhythm** is a regular time pattern in a sound.
- **Music** is a combination of sound and rhythm that we find pleasant*.
- Most of the music you listen to is created from a pattern of frequencies called a **musical scale.**

* Remember: "**pleasant**" is a subjective concept.

Correlation of physical with musical characteristics

- Amplitude—Intensity (A^2) ♫ **Loudness**
- Frequency ♫ **Pitch**
- Waveform (phase + harmonic series) ♫ **Timbre**

- **Pitch** is based on the ratio of frequencies.
- A given frequency ratio is called an **interval**.
- An **octave** is the pitch interval that corresponds to a ratio of $f_2 / f_1 = 2 / 1$.

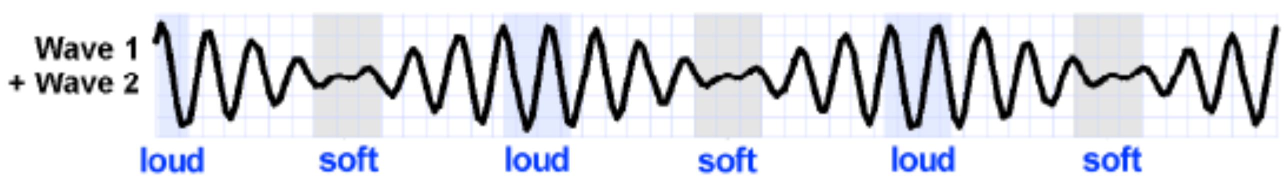
Consonance & Dissonance

Ancient Greeks - Aristotle and his followers - discovered using a *Monochord* that certain combinations of sound were pleasing to the human ear, for example:

- **Unison** - 2 sounds of *same* frequency, i.e. $f_2 = 1\ f_1 = f_1$ (= e.g. 300 Hz)
- **Minor Third** - 2 sounds with $f_2 = (6/5)\ f_1 = 1.20\ f_1$ (= e.g. 360 Hz)
- **Major Third** - 2 sounds with $f_2 = (5/4)\ f_1 = 1.25\ f_1$ (= e.g. 375 Hz)
- **Fourth** - 2 sounds with $f_2 = (4/3)\ f_1 = 1.333\ f_1$ (= e.g. 400 Hz)
- **Fifth** - 2 sounds with $f_2 = (3/2)\ f_1 = 1.50\ f_1$ (= e.g. 450 Hz)
- **Octave** - one sound is 2nd harmonic of the first - i.e. $f_2 = (2/1)\ f_1 = 2\ f_1$ (= e.g. 600 Hz)
- These 2-sound combinations are indeed very special!
- The resulting, overall waveform(s) are *time-independent* – they create standing waves on basilar membrane in cochlea of our inner ears!!!
- The human brain's signal processing for these special 2-sound consonant combinations is especially easy!!!

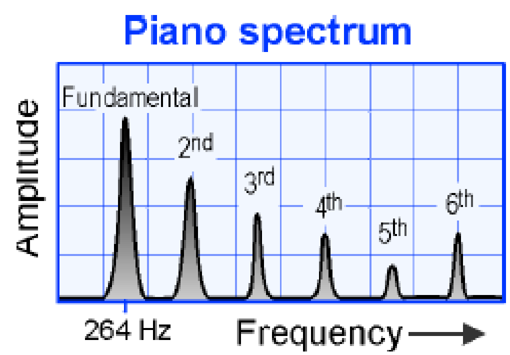
Consonance, dissonance, and beats

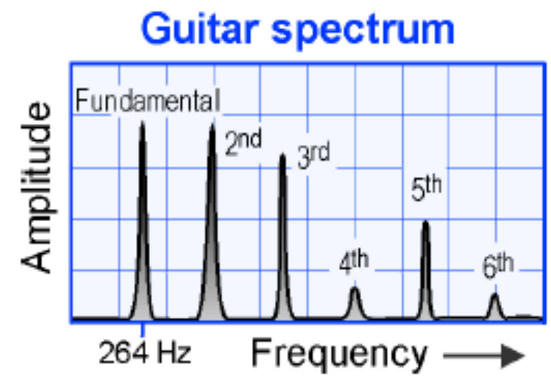
- **Harmony** is the study of how sounds work together to create effects desired by the composer.
- When we hear more than one frequency of sound and the combination sounds good, we call it **consonance**.
- When the combination sounds bad or unsettling, we call it **dissonance**.
- Consonance and dissonance are related to **beats**.
- When frequencies are far enough apart that there are no beats, we get consonance.
- When frequencies are too close together, we hear beats that are the cause of dissonance.
- **Beats** occur when two frequencies are close, but not exactly the same.



Musical Instruments

- Each musical instrument has its own characteristic sounds - quite complex!
- Any note played on an instrument has fundamental + harmonics of fundamental.
- Higher harmonics - brighter sound
- Less harmonics - mellower sound

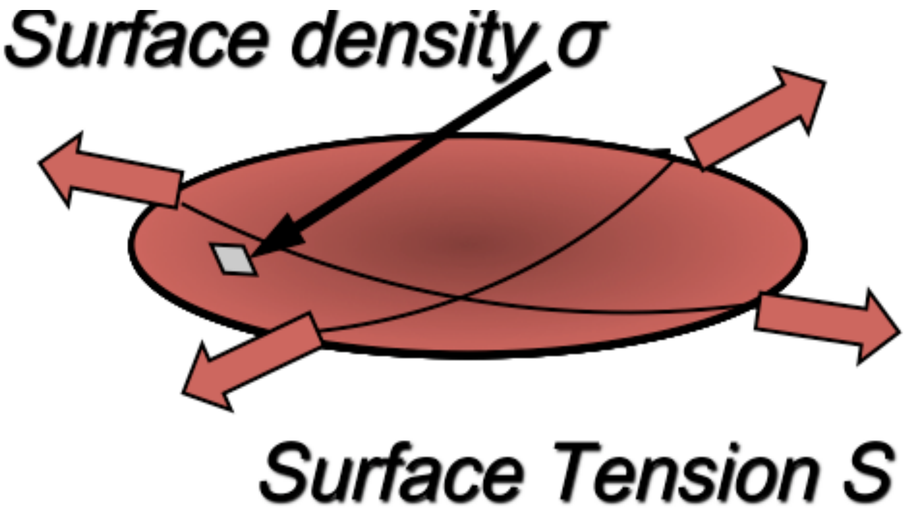


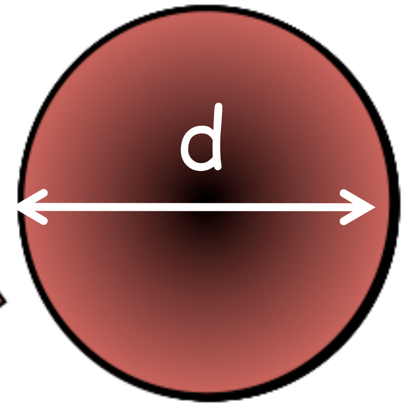


- Harmonic content of note can/does change with time:
  - Takes time for harmonics to develop - "attack" (leading edge of sound)
  - Harmonics don't decay away at same rate (trailing edge of sound)
  - Higher harmonics tend to decay more quickly
- Sound output of musical instrument is not uniform with frequency
  - Details of construction, choice of materials, finish, etc. determine *resonant structure* (formants) associated with instrument - mechanical vibrations!

DRUMS

The Oscillation of a Clamped Membrane





Mode: (0,1)

$f_{0\,1} = v/\lambda; \;\; v = \sqrt{(S/\sigma)}$

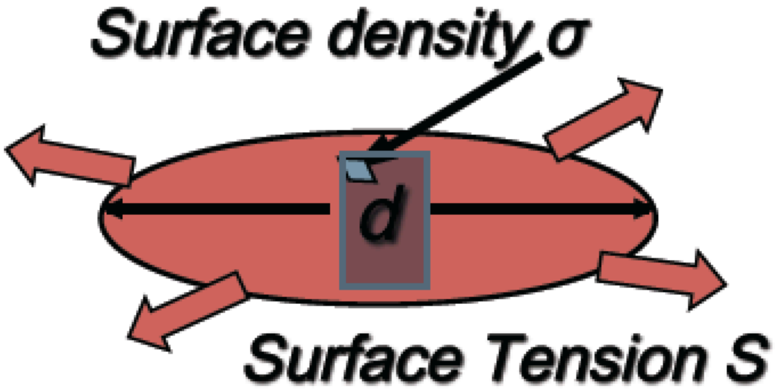
$f_{0\,1} = x_{0\,1}/(\pi\, d) \cdot \sqrt{(S/\sigma)}$

$x_{0\,1} = 2.405$

Surface density σ = mass/area
σ = density · thickness

Surface Tension S = force/length

Clamped Membrane vs String



- Surface density
σ = mass/area
- Surface Tension
S = force/length

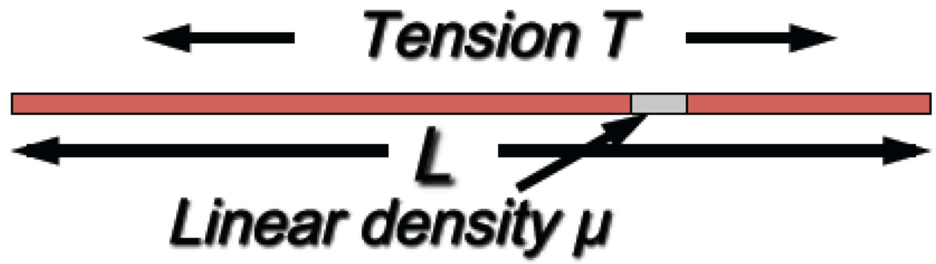
$$f_{nm} = c_{nm} \frac{1}{\pi d} \sqrt{\frac{S}{\sigma}}$$

$$c_{01} = 2.405$$

$$c_{11} / c_{01} = 1.594$$

$$c_{21} / c_{01} = 2.136$$

$$\ldots$$

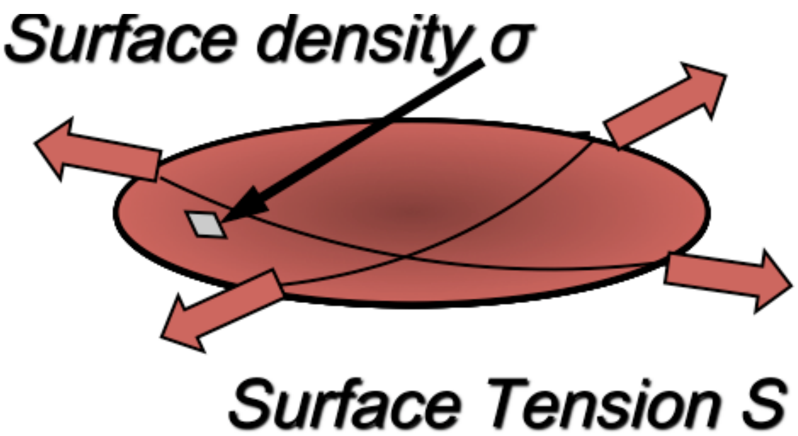


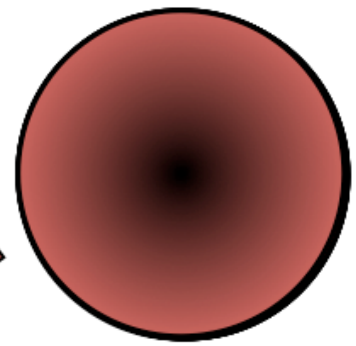
- Linear density
μ = mass/length
- Tension
T = force

$$f_n = n \frac{1}{2L} \sqrt{\frac{T}{\mu}}$$

$$n = 1,2,3,4,5,6,7,\ldots$$

The Modes of Oscillation of an (Ideal) Clamped Membrane



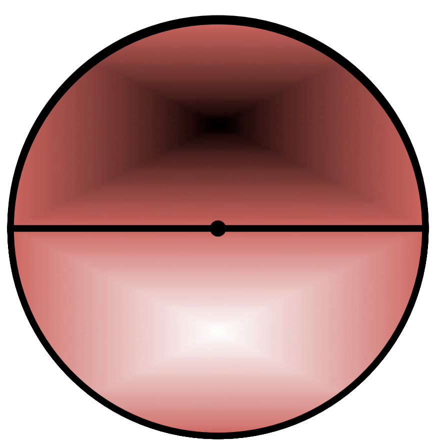


Mode: (0,1)

$$f_{01} = \frac{v}{\lambda}; \qquad v = \sqrt{\frac{S}{\sigma}}$$

$$f_{01} = c_{01} \frac{1}{\pi d} \sqrt{\frac{S}{\sigma}}$$

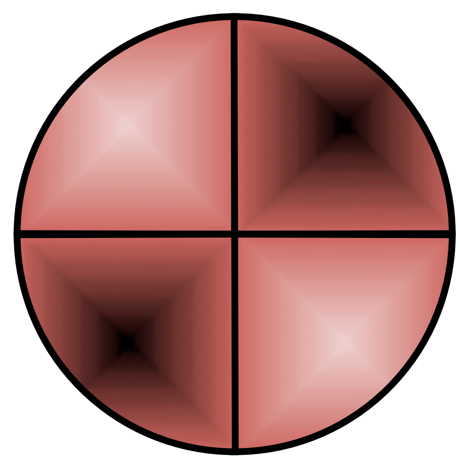
$$c_{01} = 2.405$$



Mode: (1,1)

$f_{1\,1} = (c_{1\,1} / c_{0\,1})\, f_{0\,1}$

$c_{1\,1} / c_{0\,1} = 1.594$

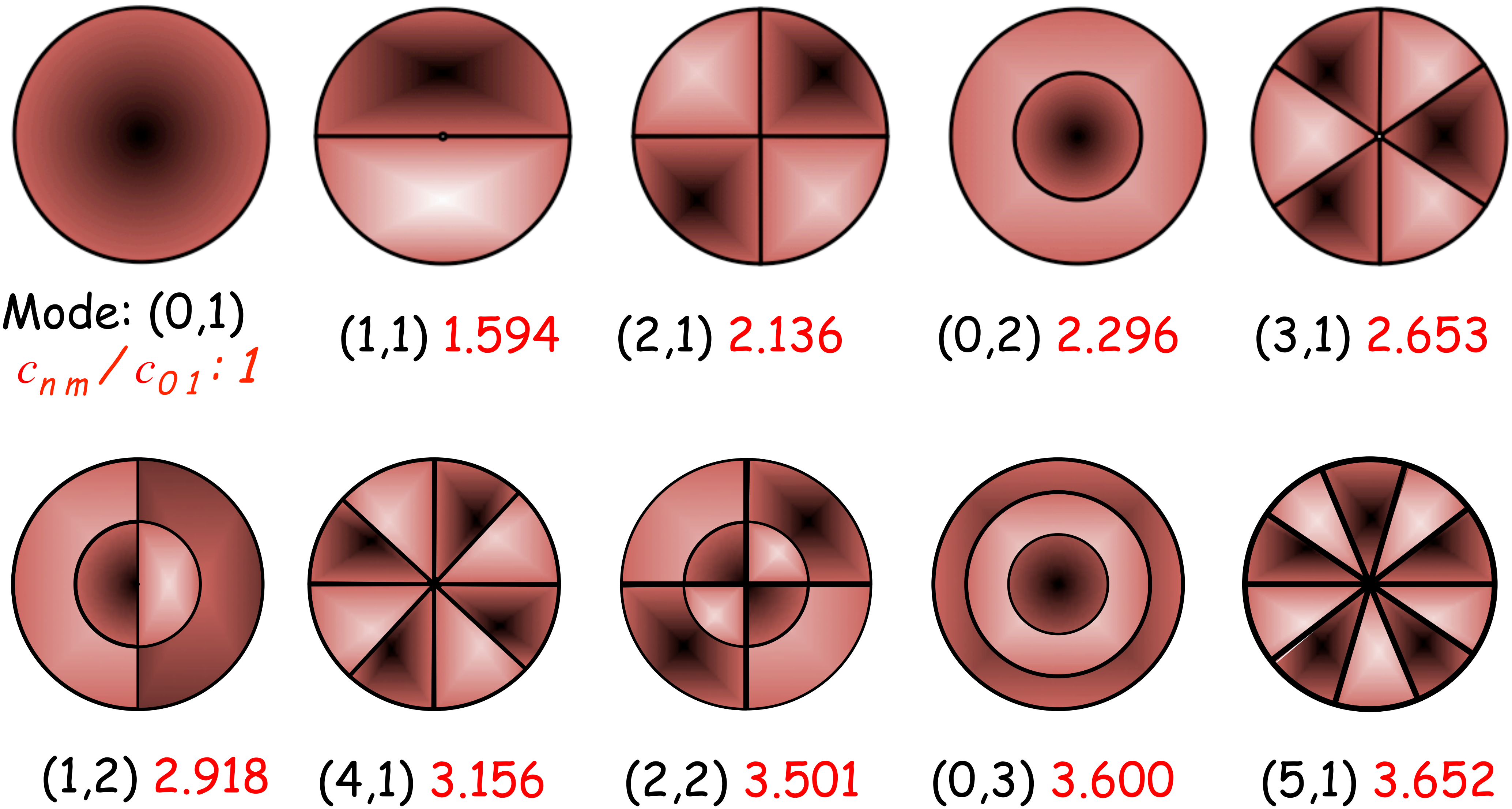


Mode: (2,1)

$f_{2\,1} = (c_{2\,1} / c_{0\,1})\, f_{0\,1}$

$c_{2\,1} / c_{0\,1} = 2.136$

The Modes of Oscillation of a Clamped Membrane



Membrane Acoustics

- The overtones of a circular membrane clamped at the edge are not harmonic and, therefore, they have no pitch.

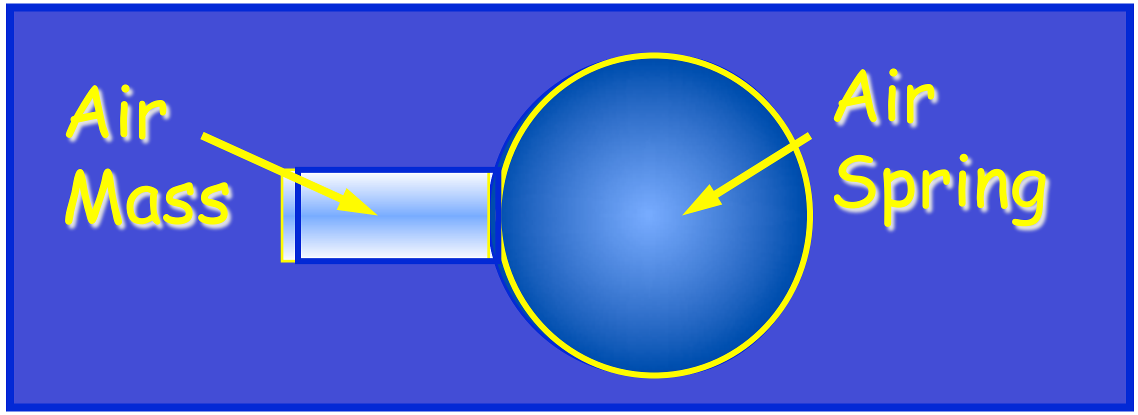
$$f_{n\,m} = (c_{n\,m} / c_{01}) f_{01}$$

- The frequencies f_{nm} of a membrane are
  (1) proportional to the square root of the ratio of surface tension of the head to the surface density $\sim \sqrt{(S / \sigma)}$ and
  (2) inversely proportional to its diameter $\sim 1/d$.

Ideal vs Real Membranes: Real membranes have lower frequencies than predicted for ideal membranes because of air loading; the lowest frequencies are lowered the most.

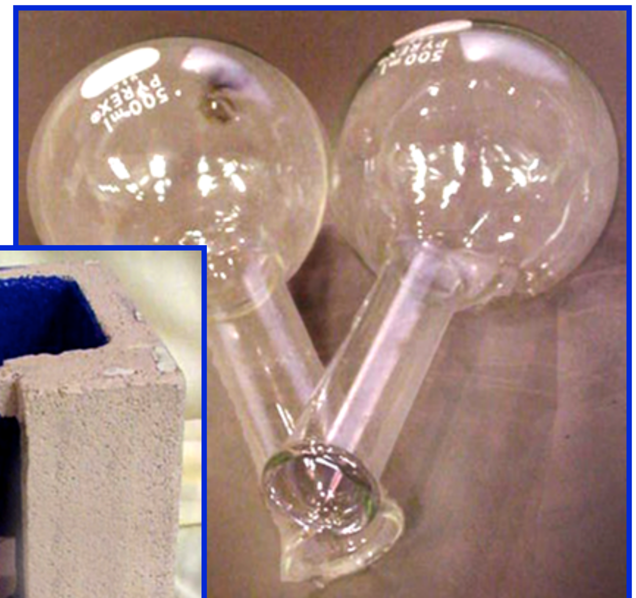
Cavity Resonators

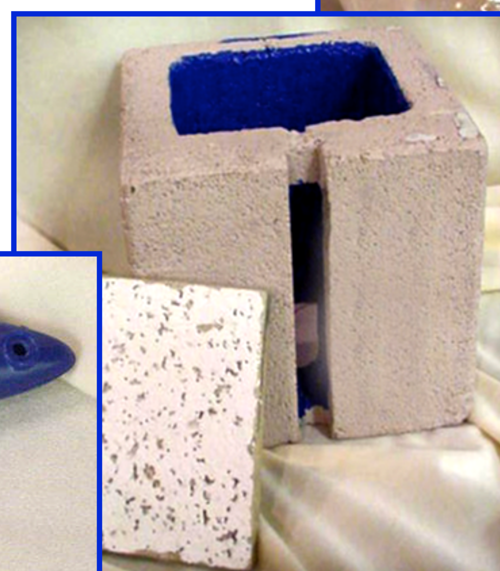
A ***Helmholtz Resonator*** is a simple harmonic oscillator that uses air in a narrow neck as a mass and air trapped in a volume as a spring.



Examples:

- ***Bottle***
- ***Acoustic Tile***
- ***Cinder Block***
- ***Ocarina***





- A **Helmholtz resonator** is a simple harmonic oscillator where the mass is provided by the air in a narrow neck while the spring is provided by a volume of trapped air.
- The natural frequency of a Helmholtz Resonator is given by the formula:

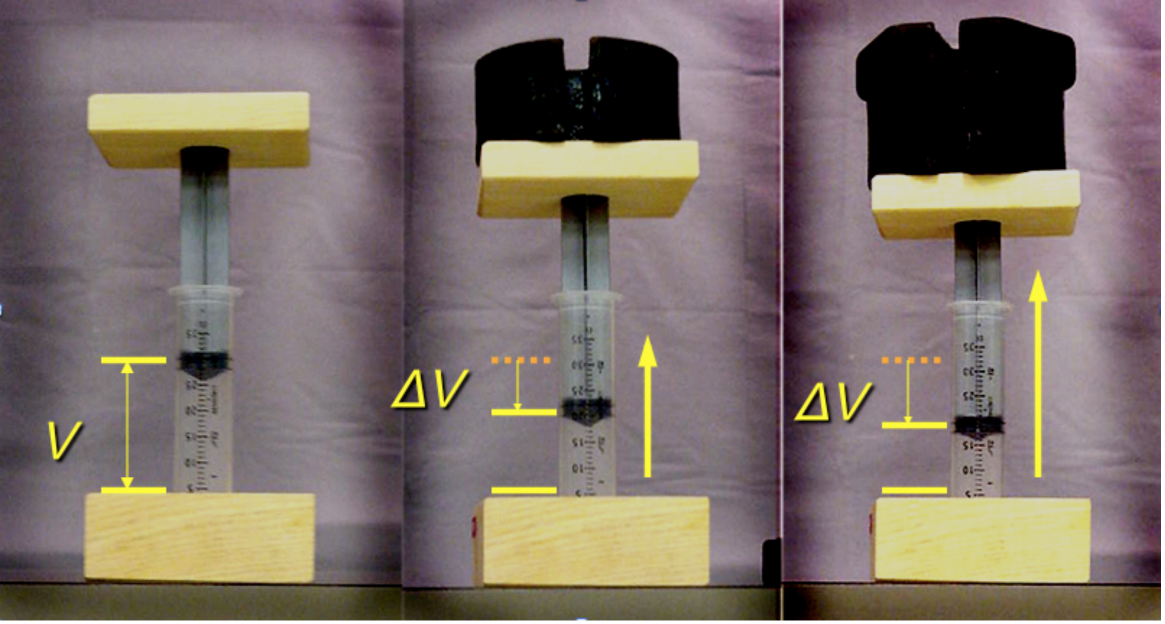
A: area of neck
V: volume of Bottle
v: velocity of sound in air
L: length of neck

$$f = \frac{v}{2\pi}\sqrt{\frac{A}{VL}}$$

The oscillations are due to the "**Bulk Modulus**" **B**: the springiness of a gas.

B is equal to the change in pressure (in Pa) for a fractional change in volume.

$$B = \frac{\Delta p}{\Delta V / V}$$



Air has "Springiness"

$\Delta V/V$:	0	0.33	0.50
Force:	0	20. N	30. N

$k \propto 1/V$

$\rightarrow$ $$\boxed{f \propto \frac{1}{\sqrt{V}}}$$

Example: bottle whistle

Largest Volume

Smallest Volume

Helmholtz Resonator

- Examples: (a) Ocarina

Open holes increase area of "neck."

(b) Ported speaker cabinet