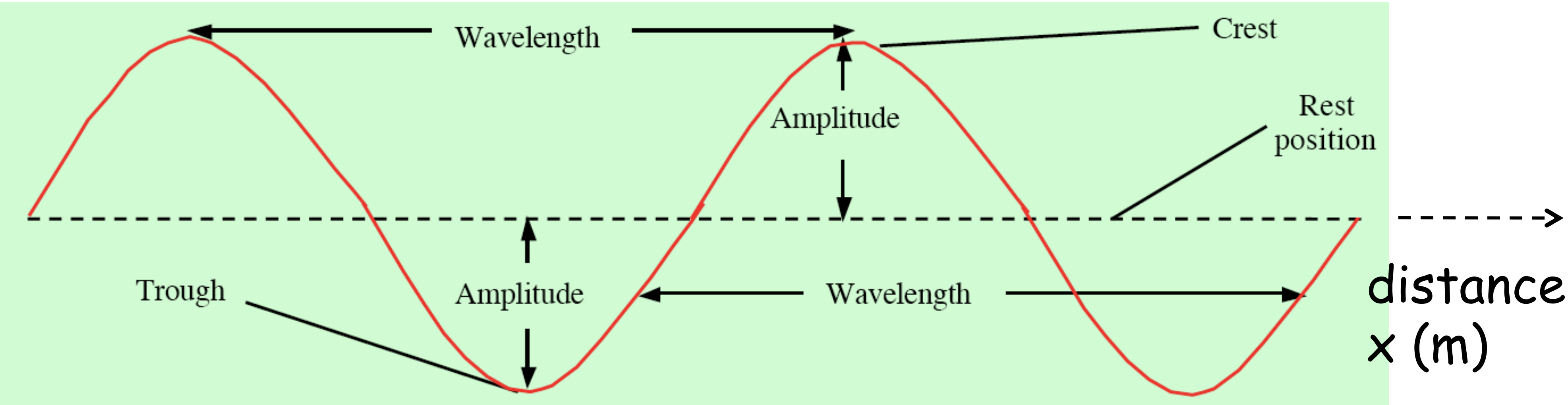
# Musical Acoustics

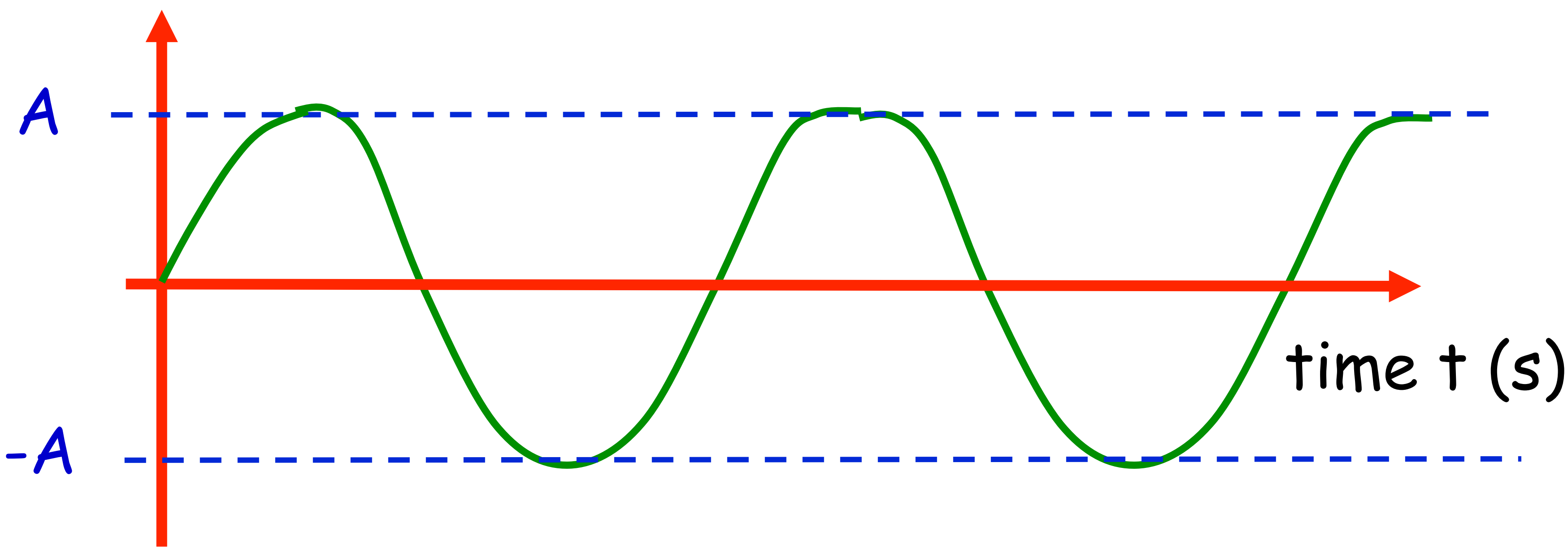
## Lecture 13

## Timbre / Tone quality I

# Waves: review

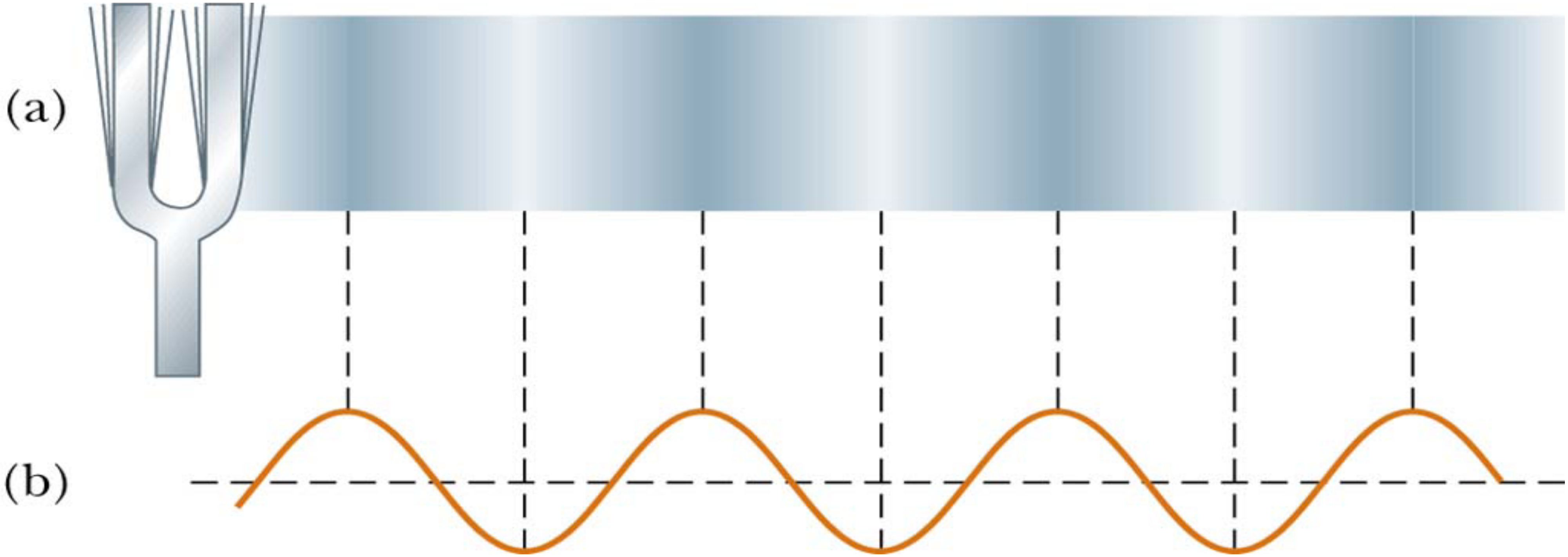



At a given time t: $y = A \sin(2\pi x/\lambda)$



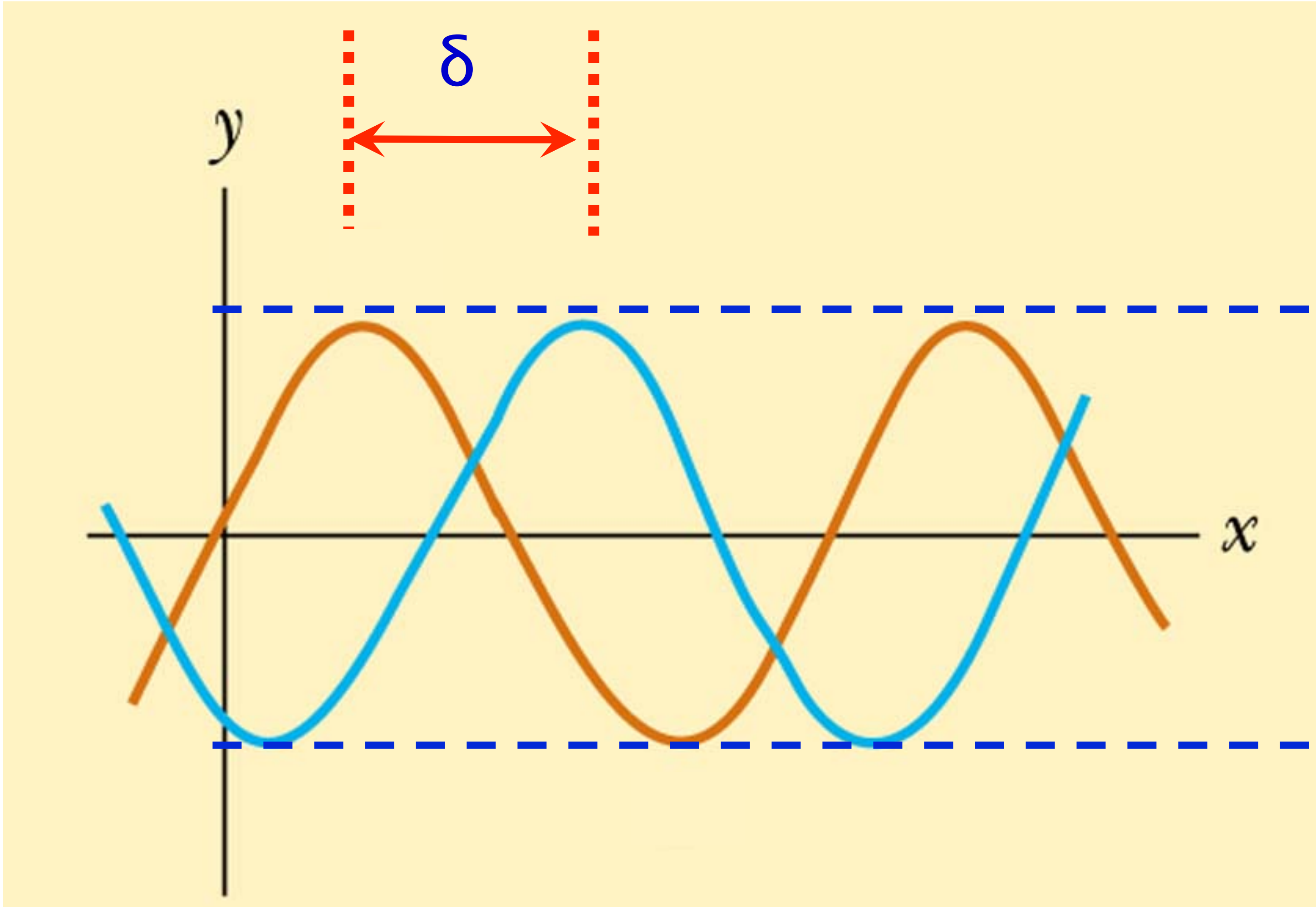

At a given position x: $y = A \sin(2\pi t/T)$

# Perfect Tuning Fork: Pure Tone




- As the tuning fork vibrates, a succession of compressions and rarefactions spread out from the fork
- A **harmonic** (sinusoidal) curve can be used to represent the longitudinal wave
  - Crests correspond to compressions and troughs to rarefactions
  - only **one single harmonic** (pure tone) is needed to describe the wave

# Phase

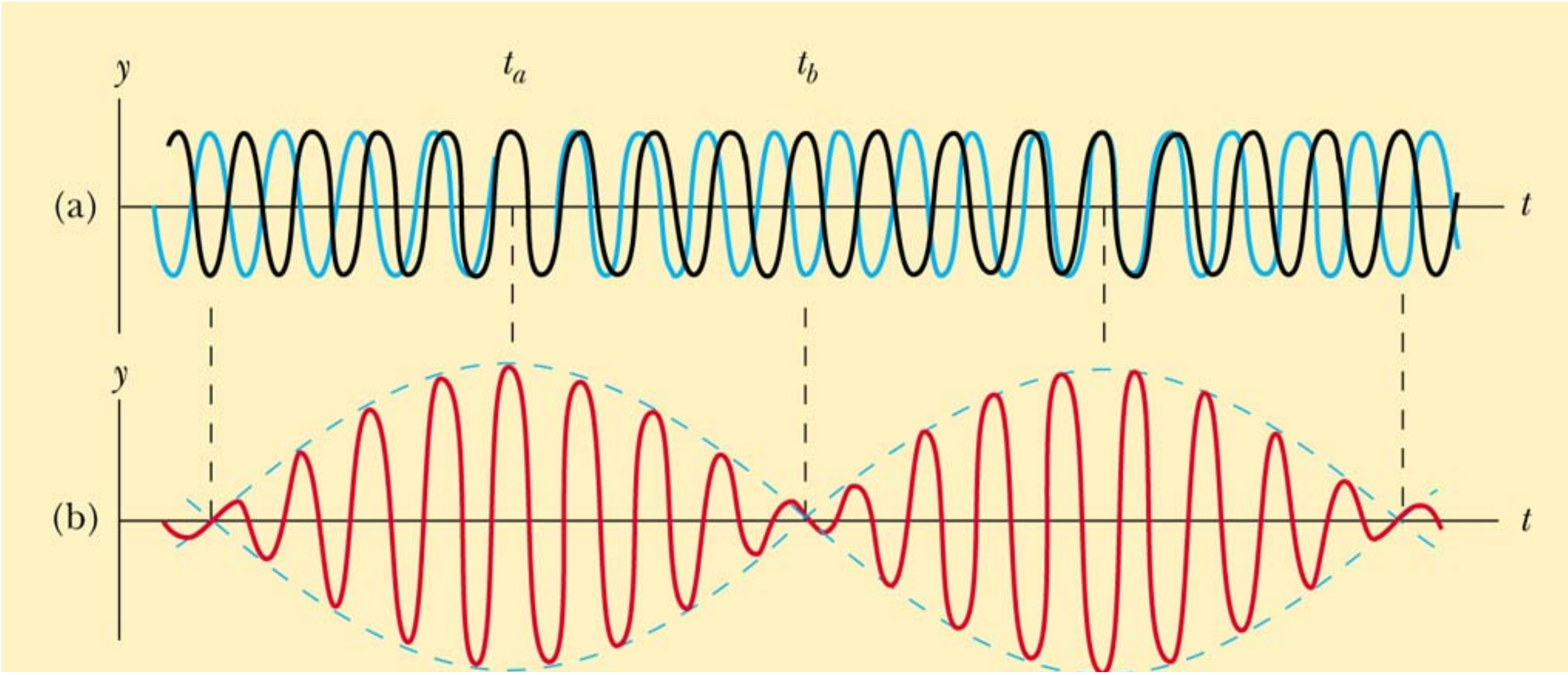



$$y = A\sin\left(2\pi\frac{x}{\lambda}\right)$$

$$y = A\sin\left(2\pi\frac{x}{\lambda} + \delta\right)$$

# *Adding waves: Beats*

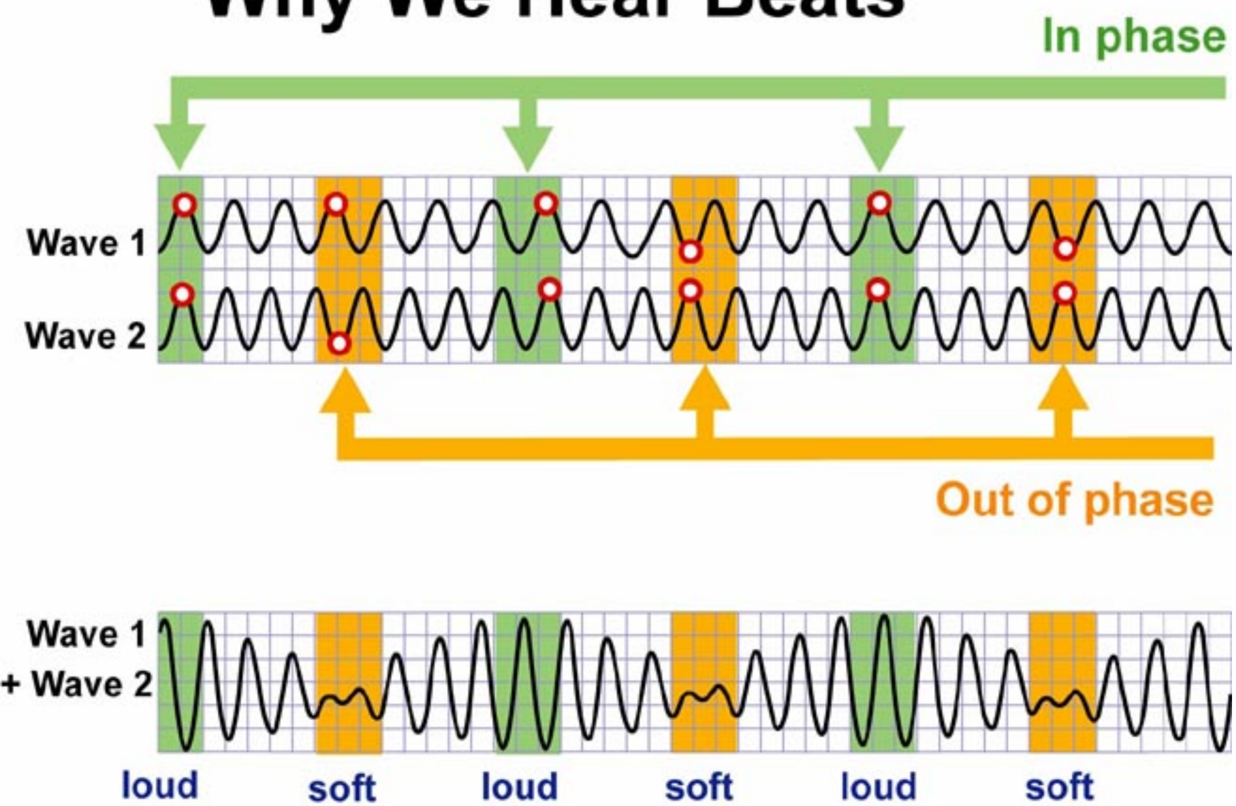
Superposition of 2 waves with slightly different frequency




The amplitude changes as a function of time, so the intensity of sound changes as a function of time.

The beat frequency (number of intensity maxima/minima per second): $f_{beat} = |f_a - f_b|$

# Why We Hear Beats

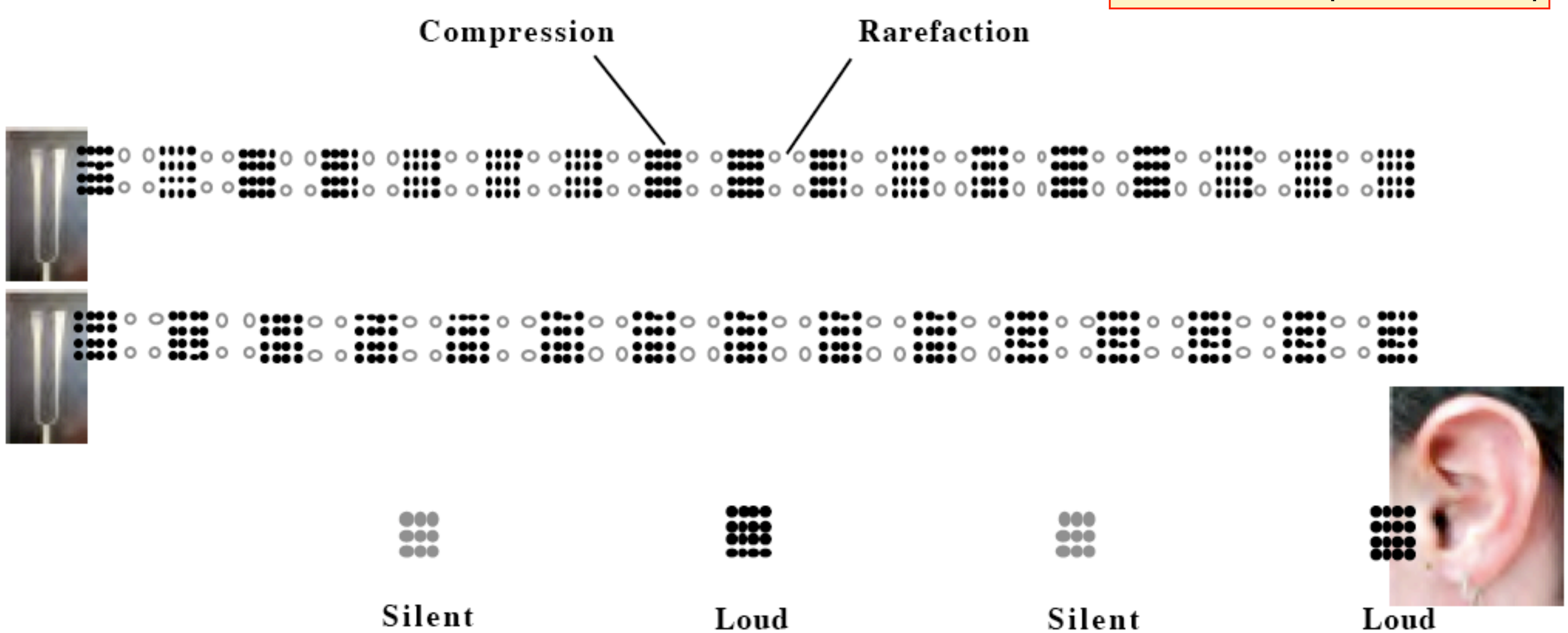



The perceived frequency is the average of the two frequencies:

$$f_{perceived} = \frac{f_1 + f_2}{2}$$

The beat frequency (rate of the throbbing) is the difference of the two frequencies:

$$f_{beats} = |f_1 - f_2|$$



# Factors Affecting Timbre

1. Amplitudes of harmonics
2. Transients (A sudden and brief fluctuation in a sound. The sound of a crack on a record, for example.)
3. Inharmonicities
4. Formants
5. Vibrato
6. Chorus Effect

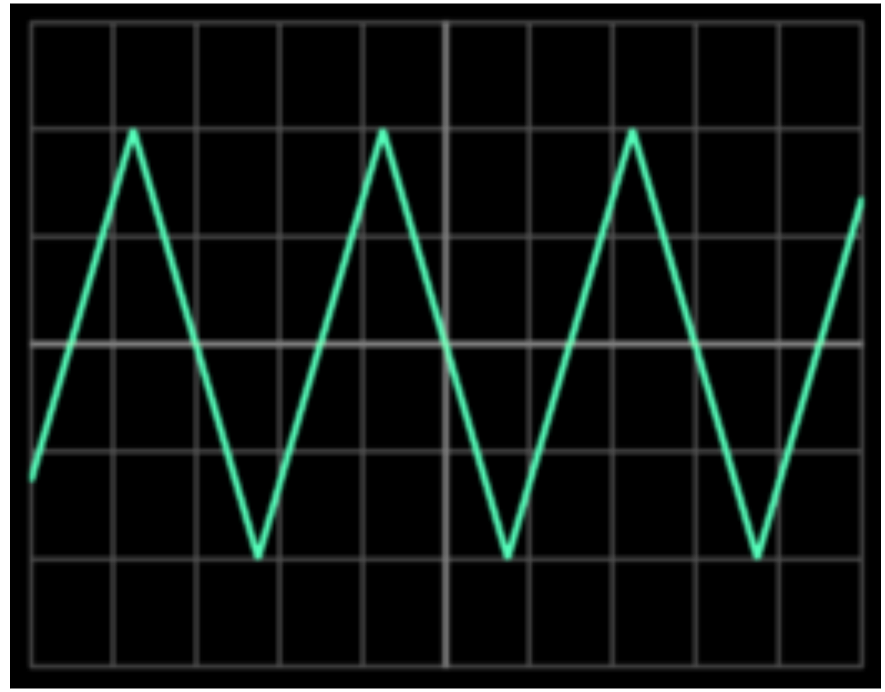
Two or more sounds are said to be in unison when they are at the same pitch, although often an OCTAVE may exist between them.

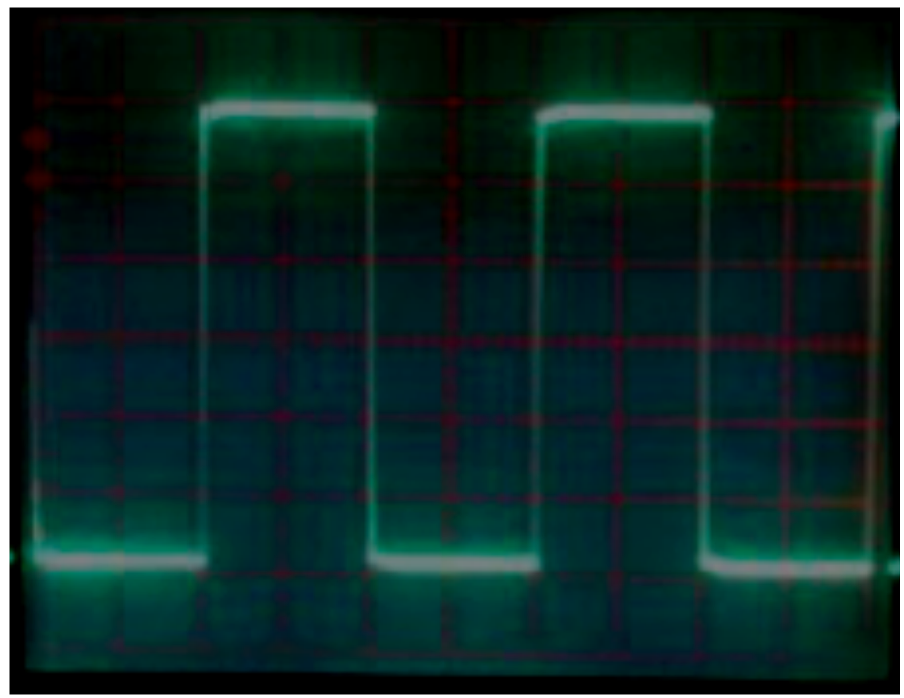
# Complex **Periodic** Waves

- Both the triangle and square wave cross zero at the beginning and end of the interval.

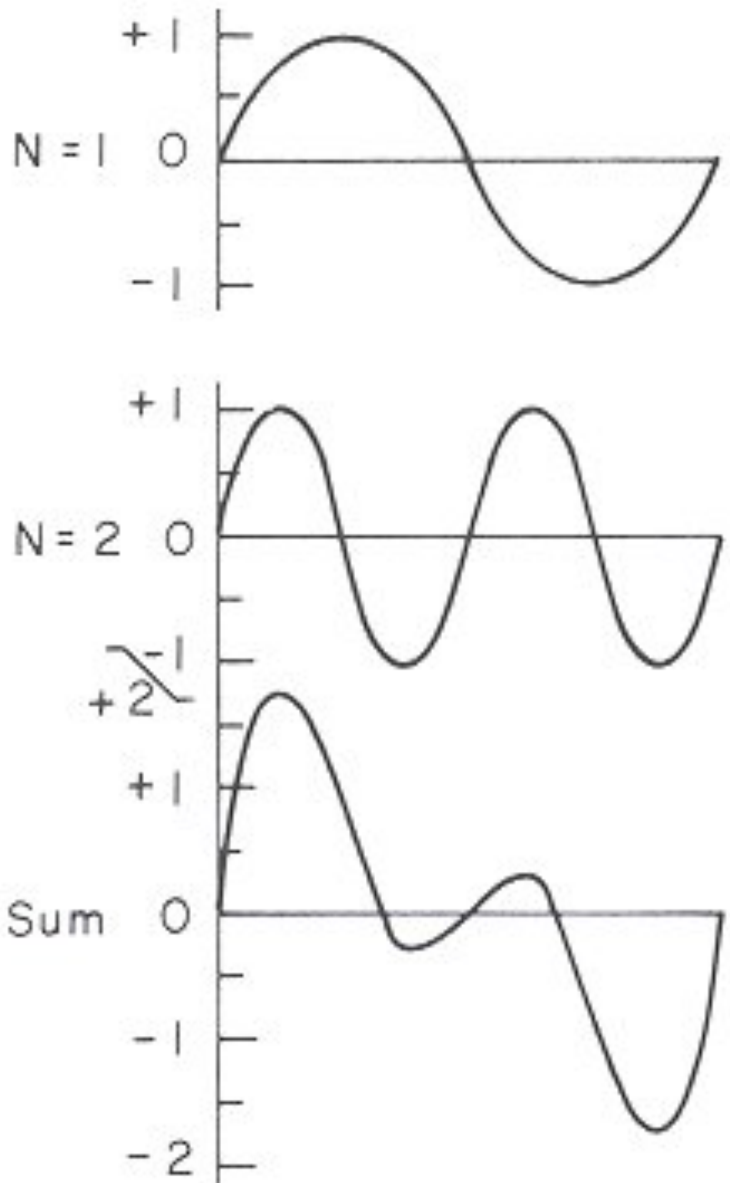
- We can repeat the signal:
  It is "**Periodic**"

- Periodic waves can be decomposed into a sum of harmonics or sine waves with **frequencies** that are **multiples** of the biggest one that fits in the interval.





# *Synthesis of complex waves*



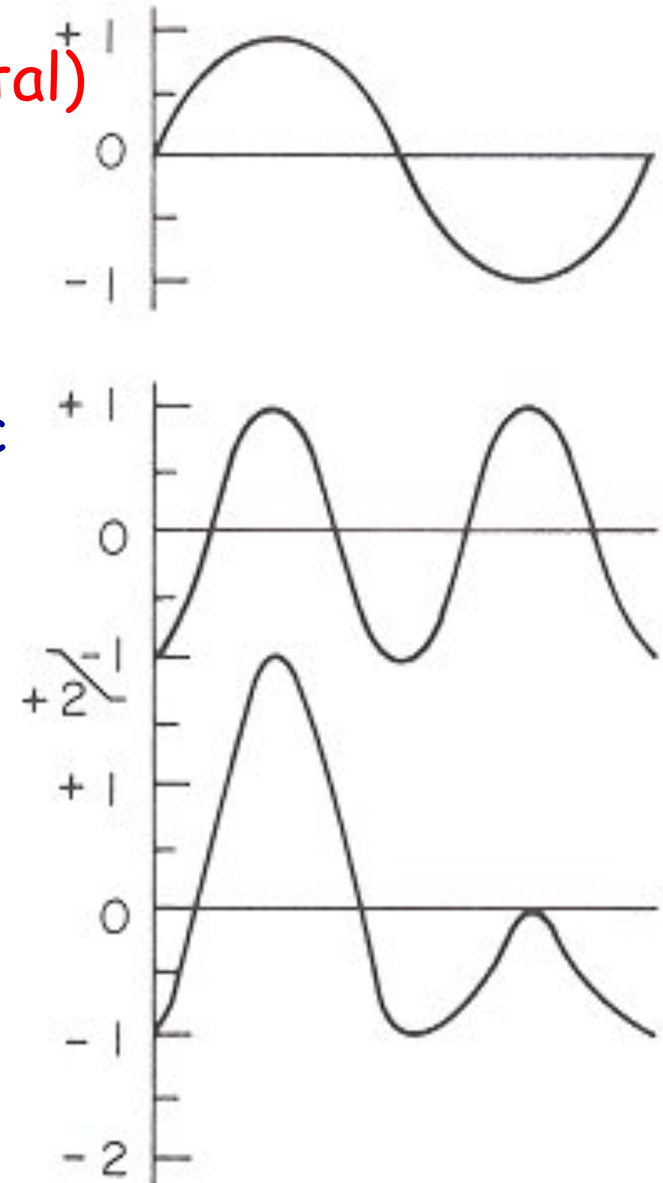

first (or fundamental) harmonic

$\lambda_1$

second harmonic

$\lambda_2 = \lambda_1/2$

resulting wave

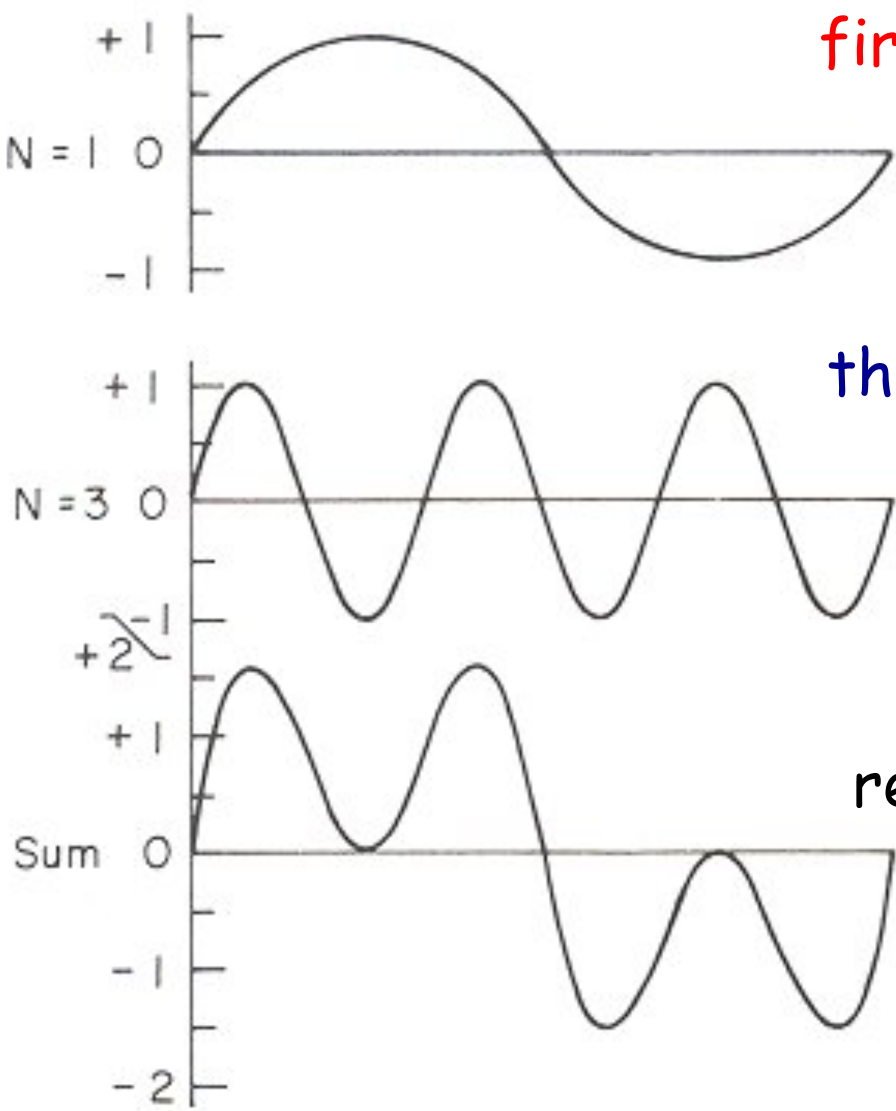



same amplitude
same phases

same amplitude
different phases

Harmonics higher than the fundamental are often called **overtones**.

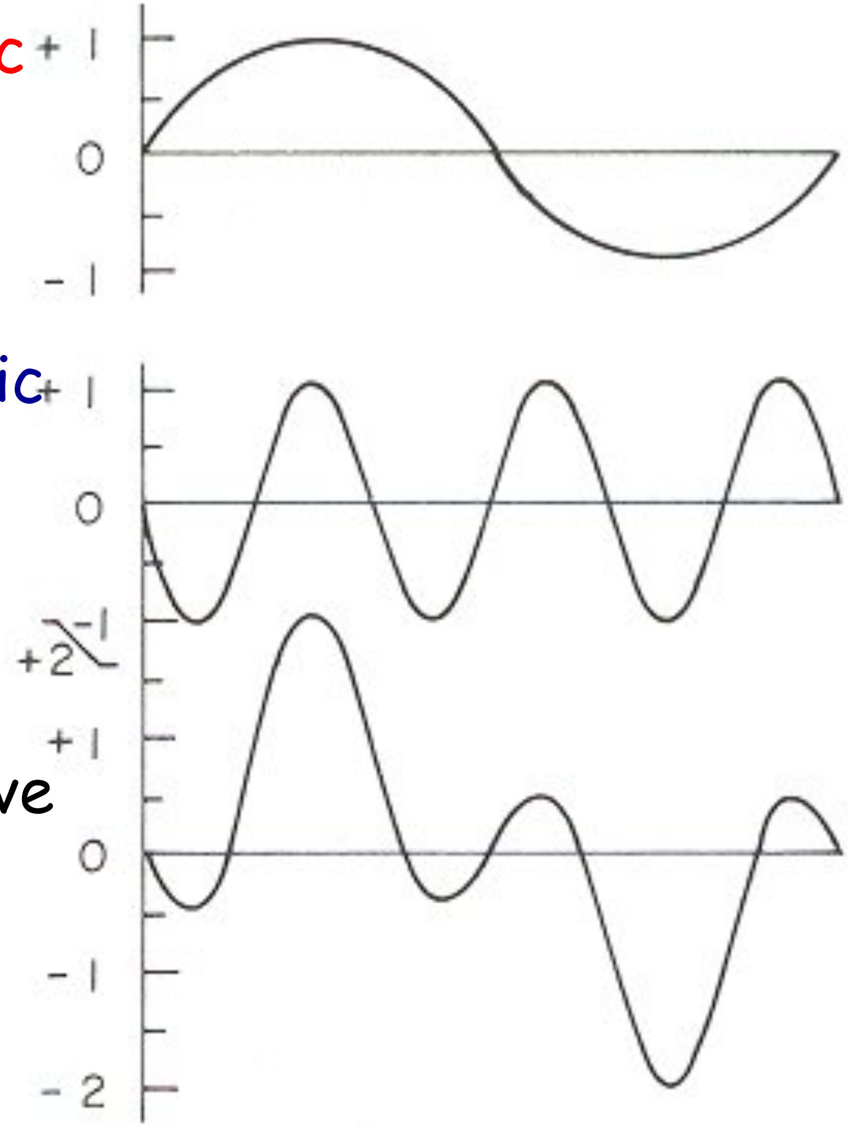
# *Synthesis of complex waves*




first harmonic
$\lambda_1$

third harmonic
$\lambda_2 = \lambda_1/3$

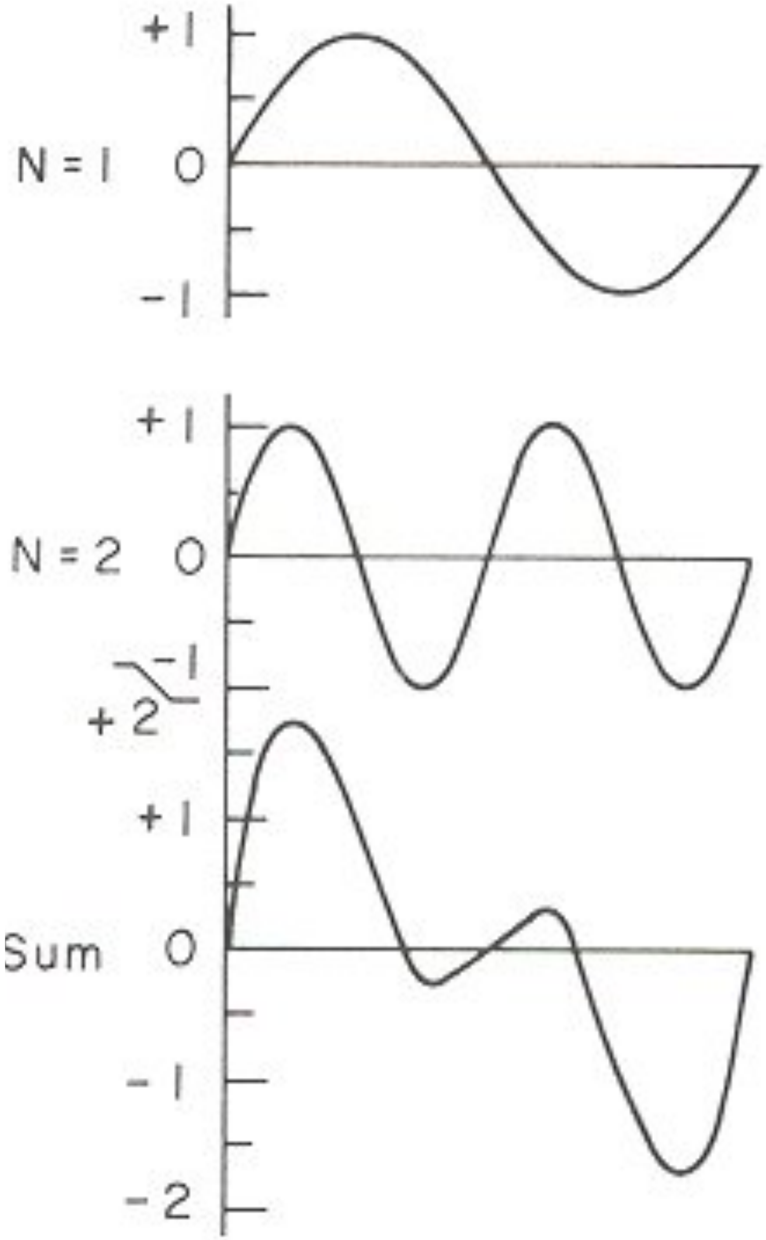
resulting wave




same amplitudes
same phases

same amplitudes
different phases

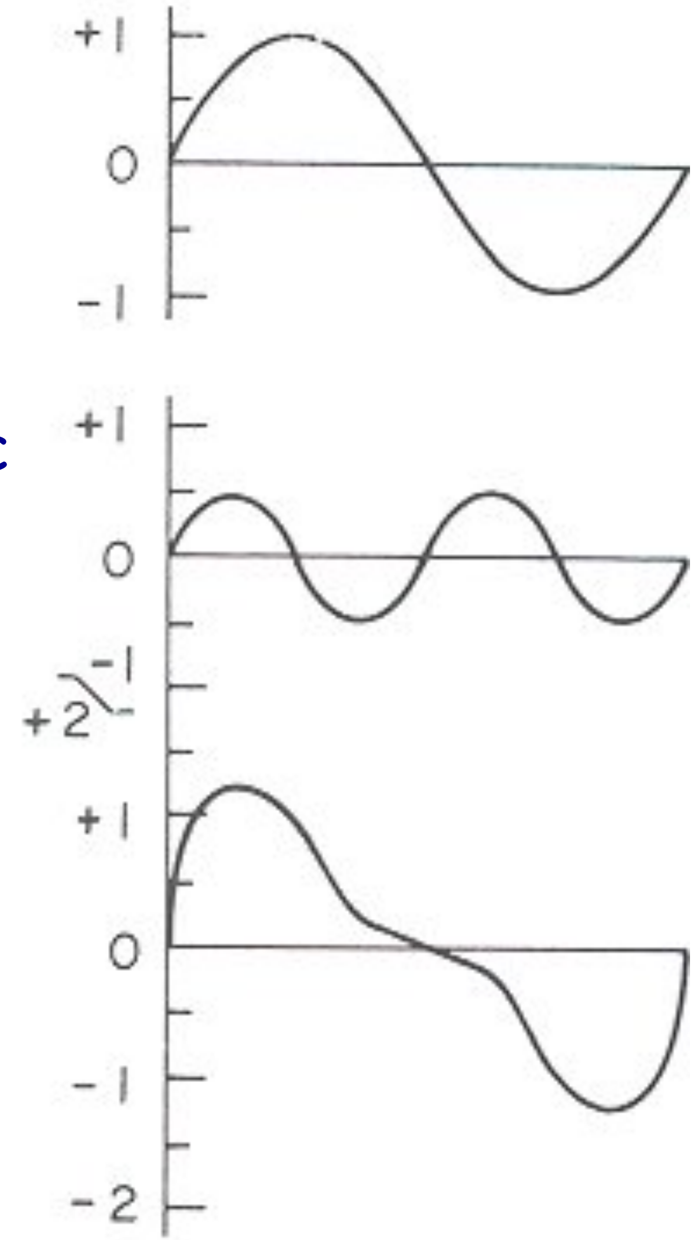
# *Synthesis of complex waves*




first harmonic
$\lambda_1$

second harmonic
$\lambda_2 = \lambda_1/2$

resulting wave




same amplitudes
same phases

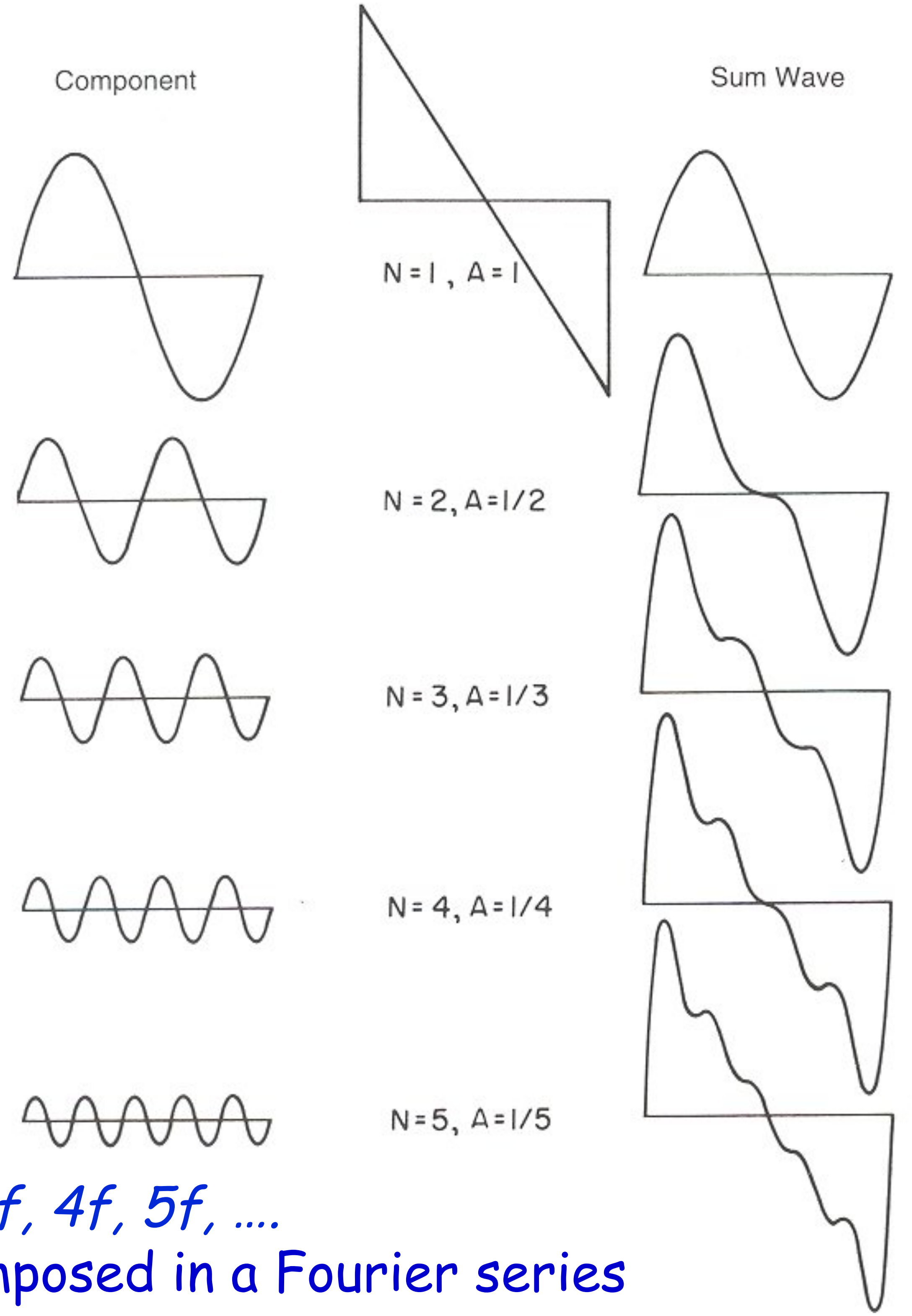
different amplitudes
same phases

# Synthesis of complex waves

## building a triangle wave (Fourier decomposition)

| Mode of vibration | Frequency name (for any type of overtone) | Frequency name (for harmonic overtones) |
|---|---|---|
| First | Fundamental | First harmonic |
| Second | First overtone | Second harmonic |
| Third | Second overtone | Third harmonic |
| Fourth | Third overtone | Fourth harmonic |

Adding in higher frequencies makes the triangle tips sharper and sharper



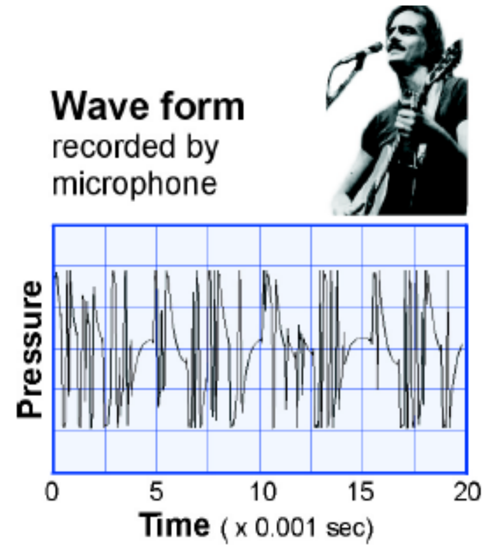

→ Harmonic series

a sum of sine and cosine waves which have frequencies *f, 2f, 3f, 4f, 5f, ….*

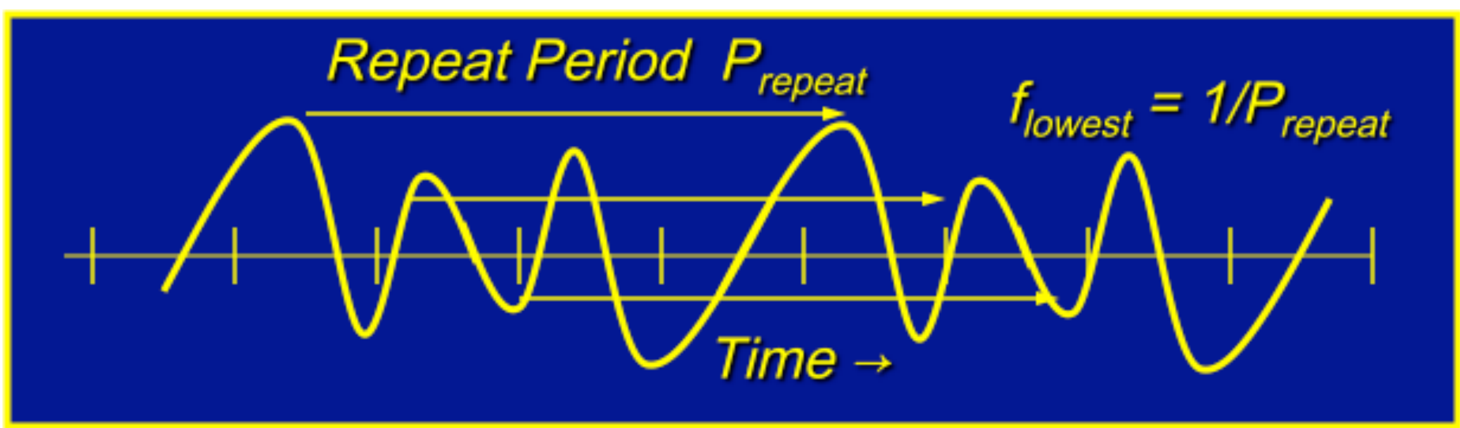
Any **periodic** wave can be decomposed in a Fourier series

# Fourier's theorem

- Fourier's theorem says any complex wave can be made from a sum of single frequency waves.



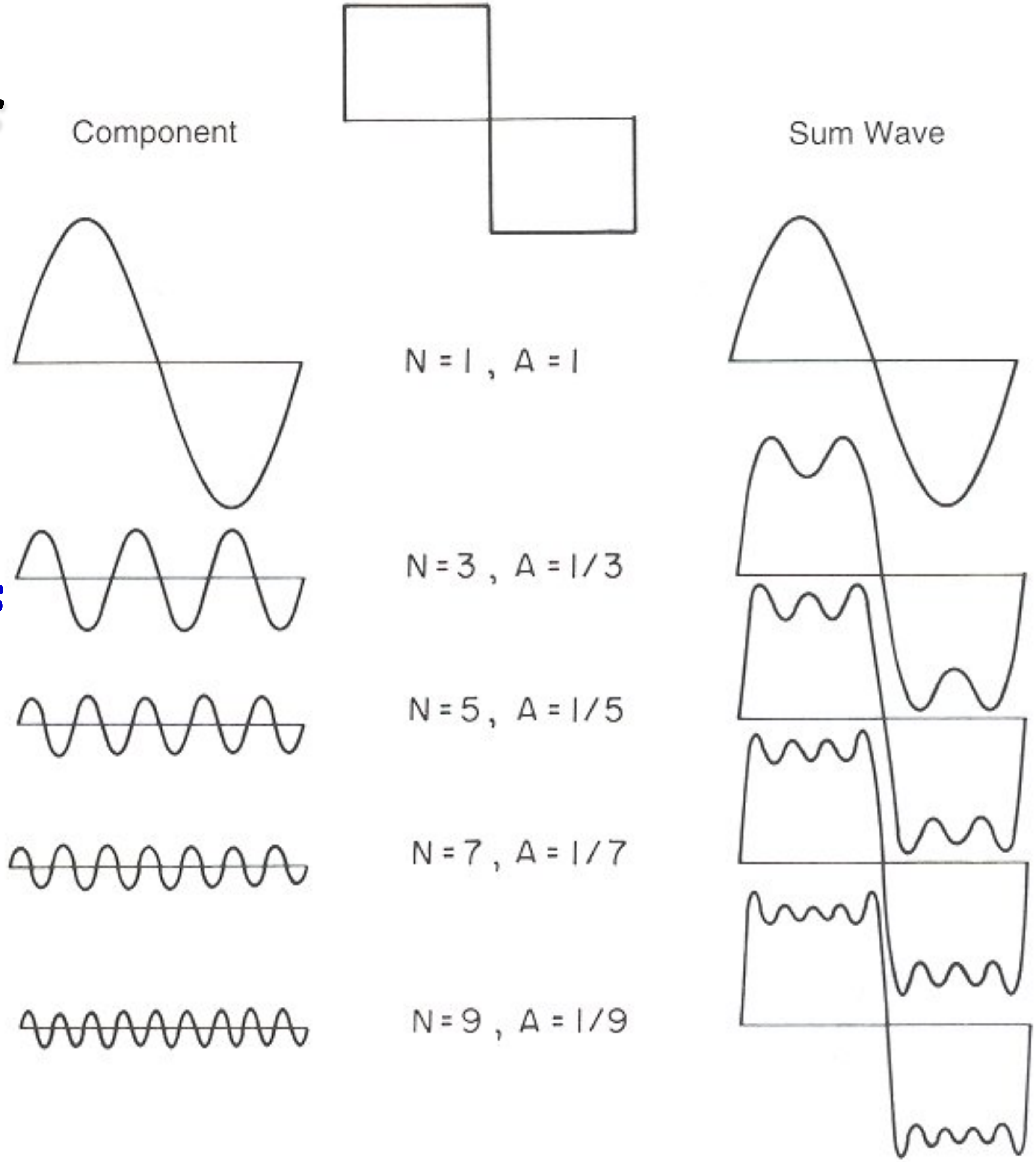

The lowest frequency component of a wave is equal to $1/P_{repeat}$ , the reciprocal of the repeat period $P_{repeat}$



# *Synthesis of complex waves*

building a square wave

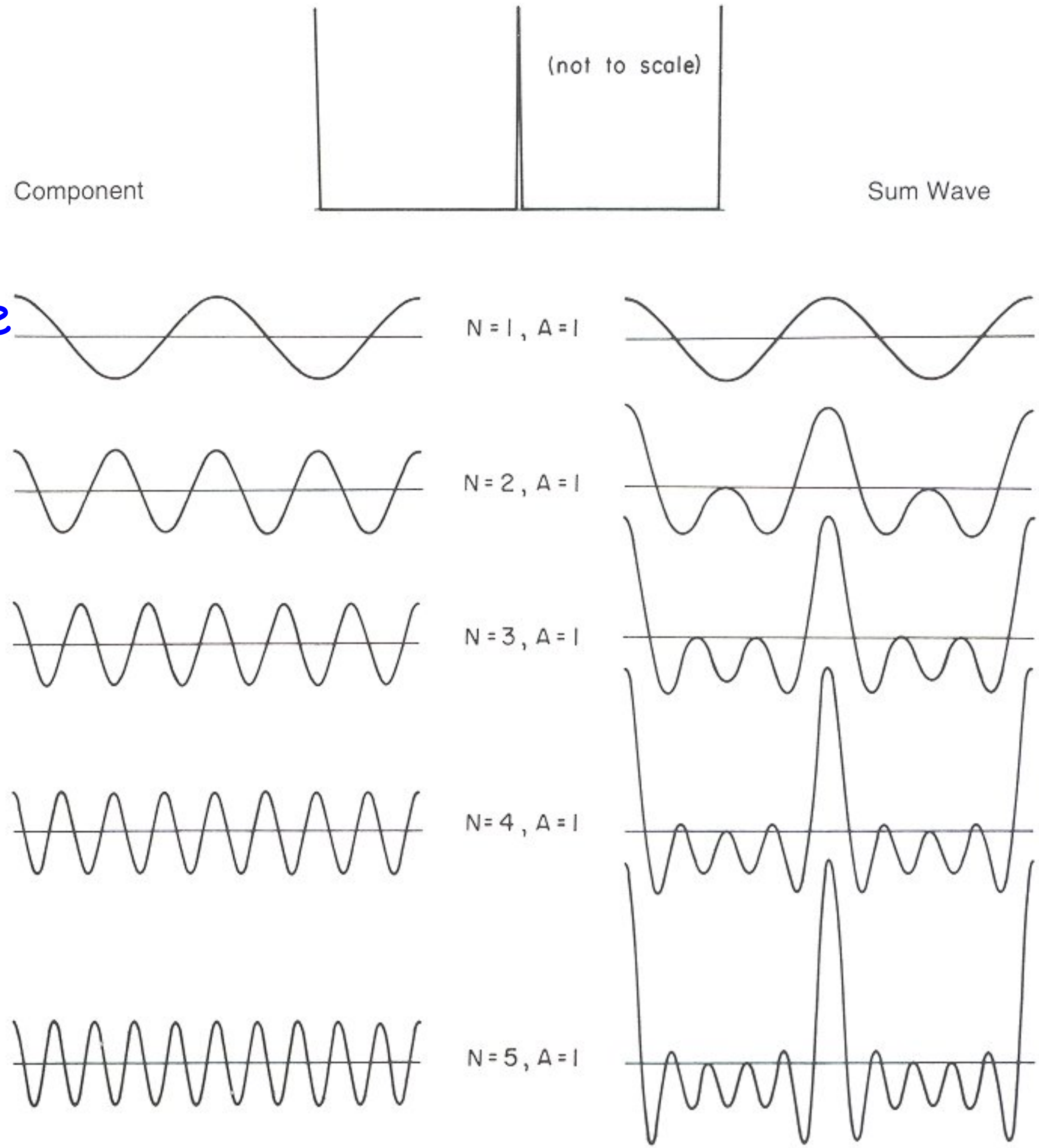
- Sharp bends imply high frequencies
- Leaving out the high frequency components smoothes the curves
- **Low pass filters** remove high frequencies and makes the sound less shrill or bright



# *Synthesis of complex waves*

building a strange, spike wave

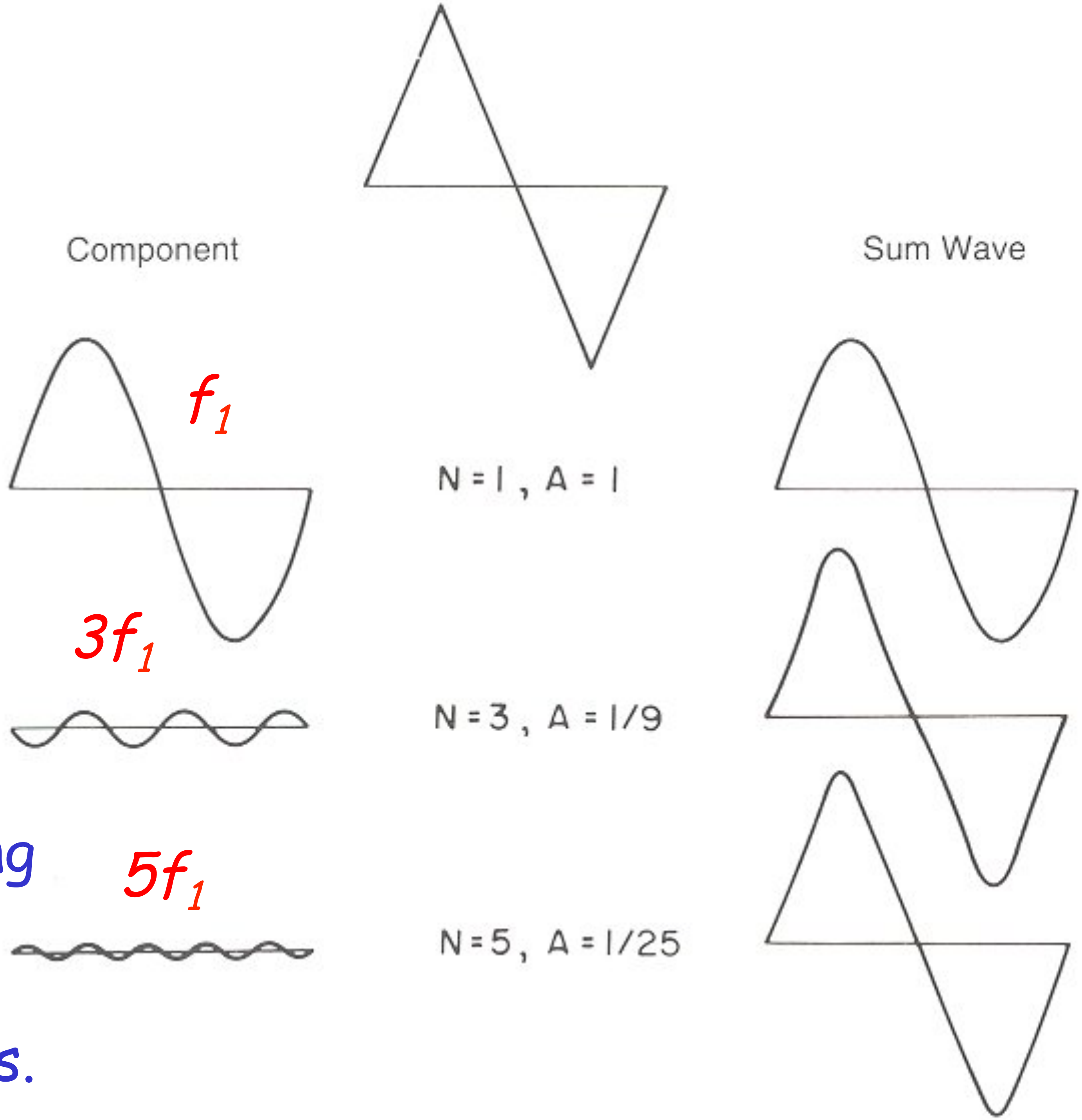
Needs many harmonics



# *Synthesis of complex waves*

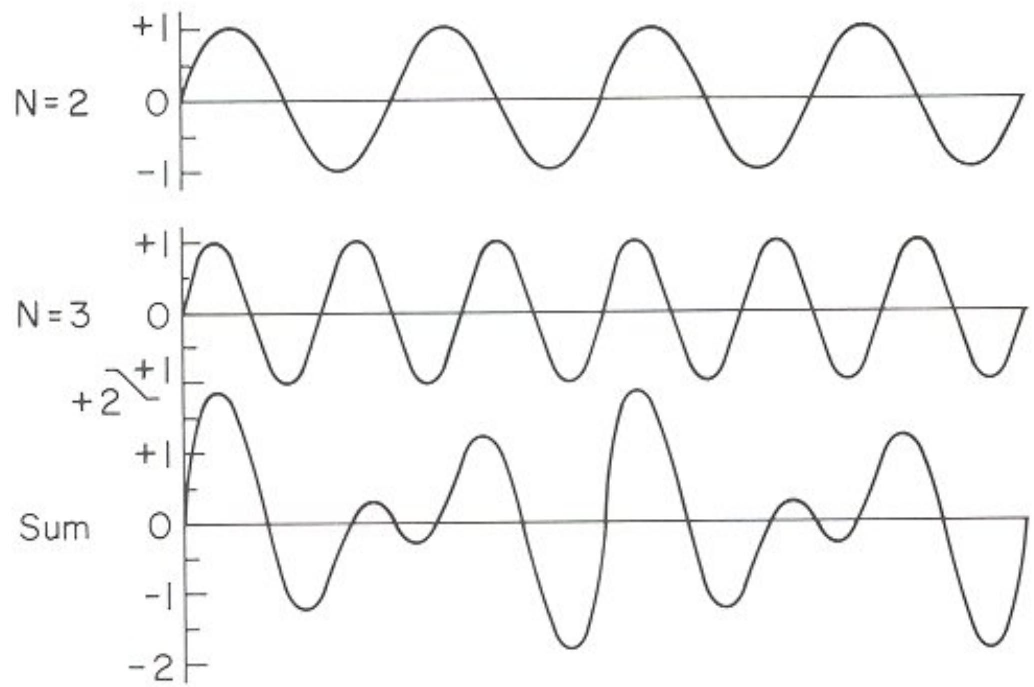
building a tringle wave with phase shifts

To get a triangle or square wave we only add sine waves that fit exactly in one period. They cross zero at the beginning and end of the interval. These are harmonics.



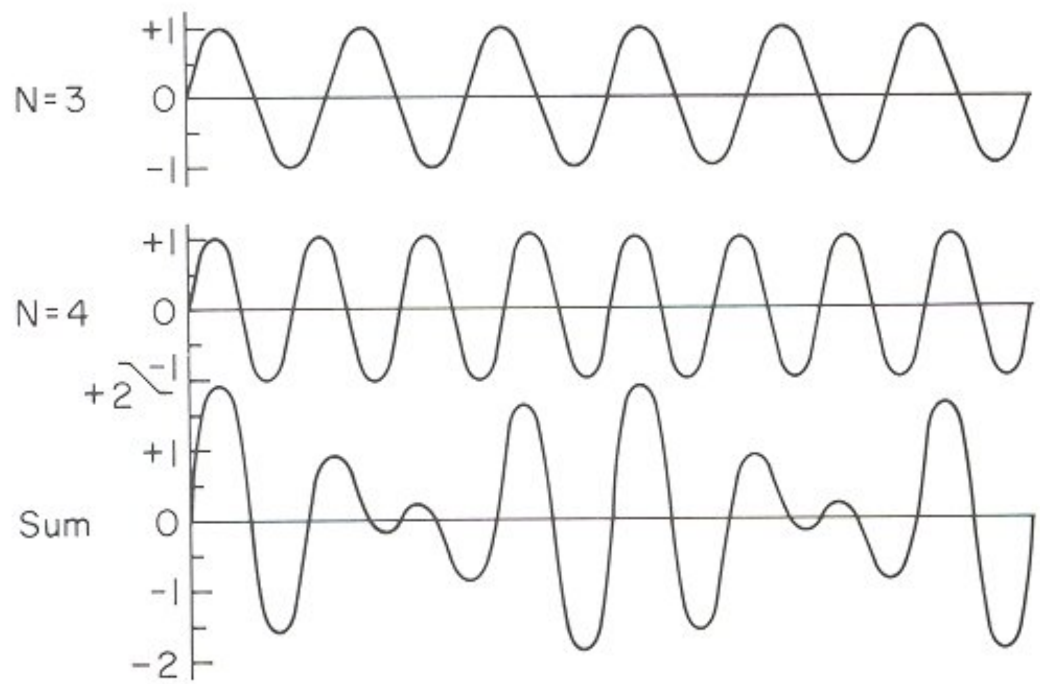
# *Synthesis with missing fundamental*

2nd and 3rd harmonics




(a)

3rd and 4th harmonics




(b)

# What happens if we vary the phase of the components we used to make the triangle wave?

$y_1 = \sin(2\pi f_1 t)$

$+$

$y_2 = \sin(2\pi f_2 t - 1.6)$

$+$

$y_3 = \sin(2\pi f_3 t - 0.1)$

$+$

$y_4 = \sin(2\pi f_4 t + 1.3)$

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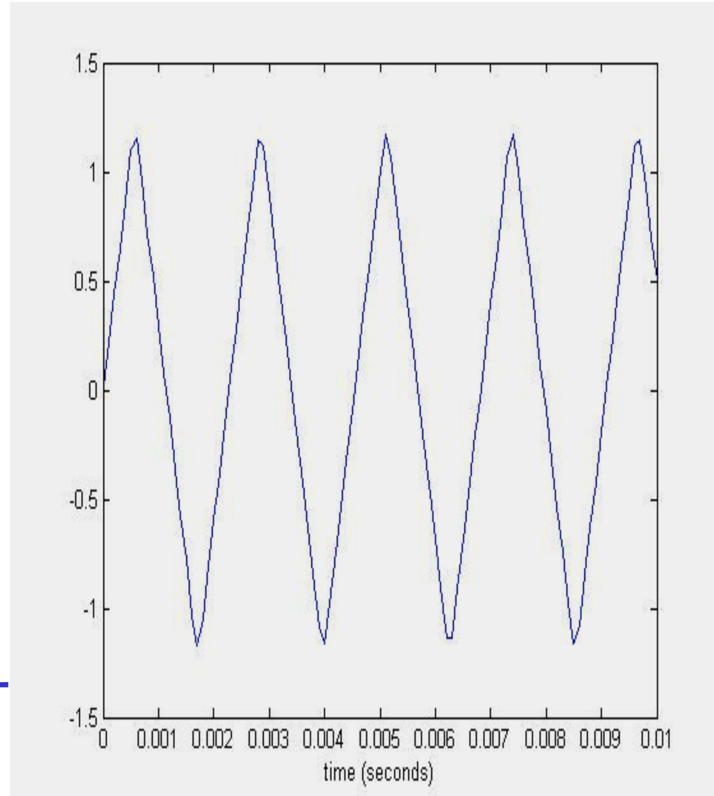
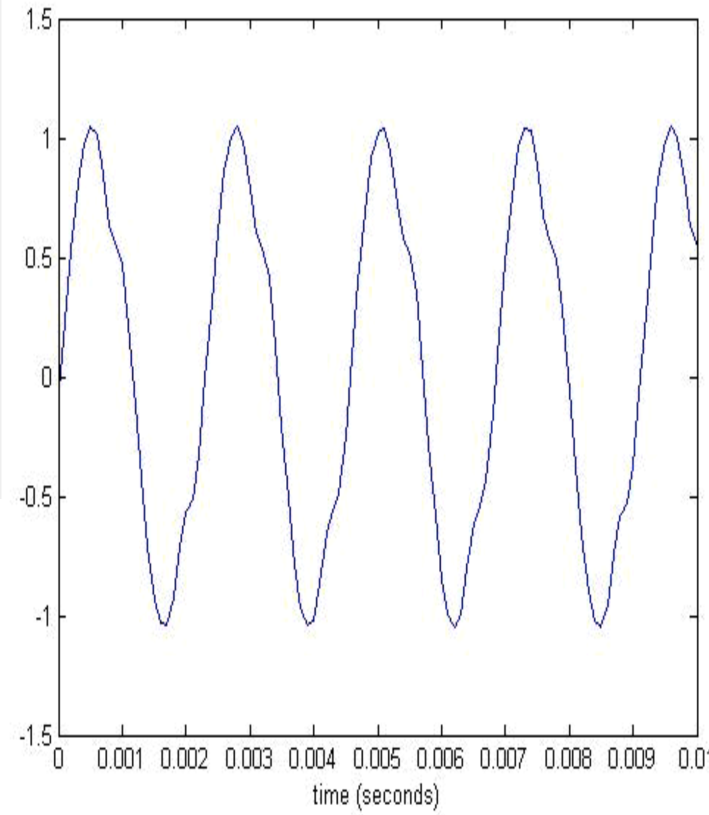
$y = y_1 + y_2 + y_3 + y_4$




Shape of wave is changed even though frequency spectrum is the same

# Do we hear phase?

- Helmholtz and Ohm argued that our ear and brain are **only sensitive to the frequencies of sounds**. Timbre is a result of frequency mix.

- There are exceptions to this (e.g., low frequencies)

- There are two major psycho-acoustic models

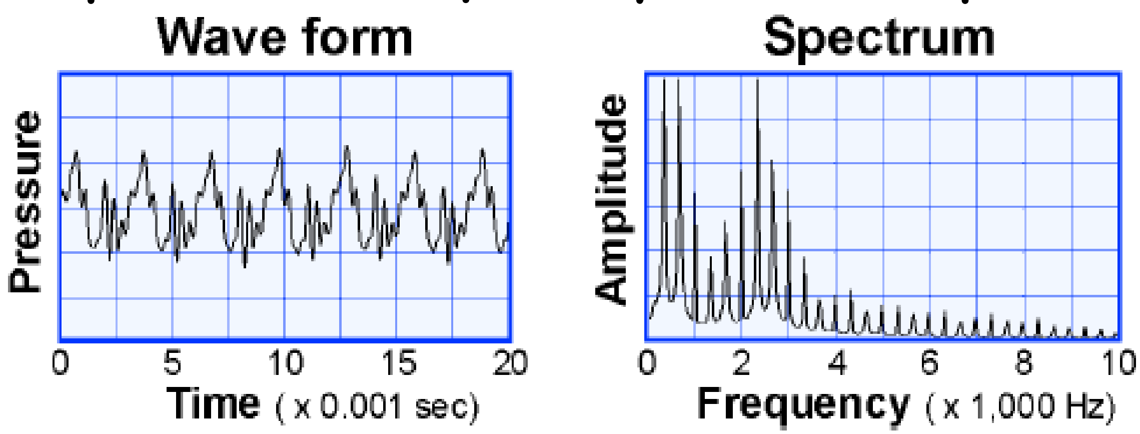
- Place theory – each spot in basal membrane is sensitive to a different frequency

- Timing – rate of firing of neurons is important and gives us phase information

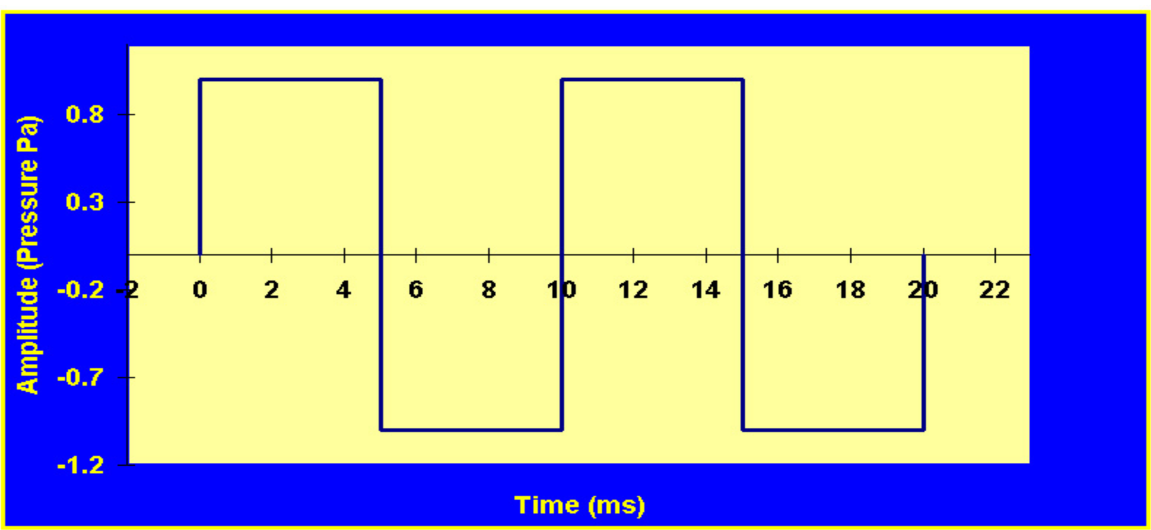
- What is the role of each in how our ear and brains process information? ***Open questions remain on this.***

# Sound spectrum

- A complex wave is really a sum of component frequencies.
- A frequency spectrum is a graph that shows the amplitude of each component frequency in a complex wave.



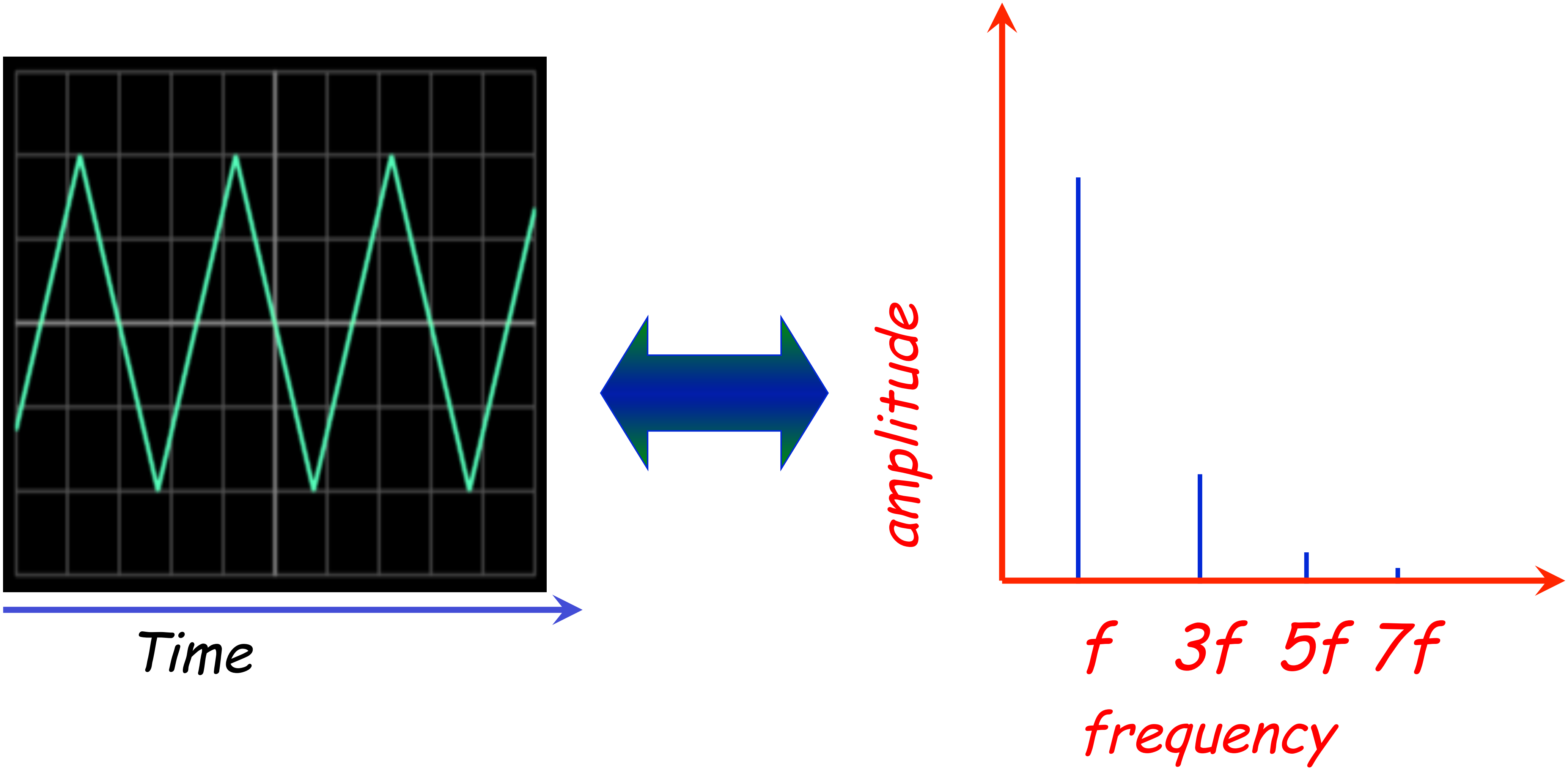

**Example:** What is the lowest frequency component of the following waveform?




Repeat period
P =10 ms =0.010 sec

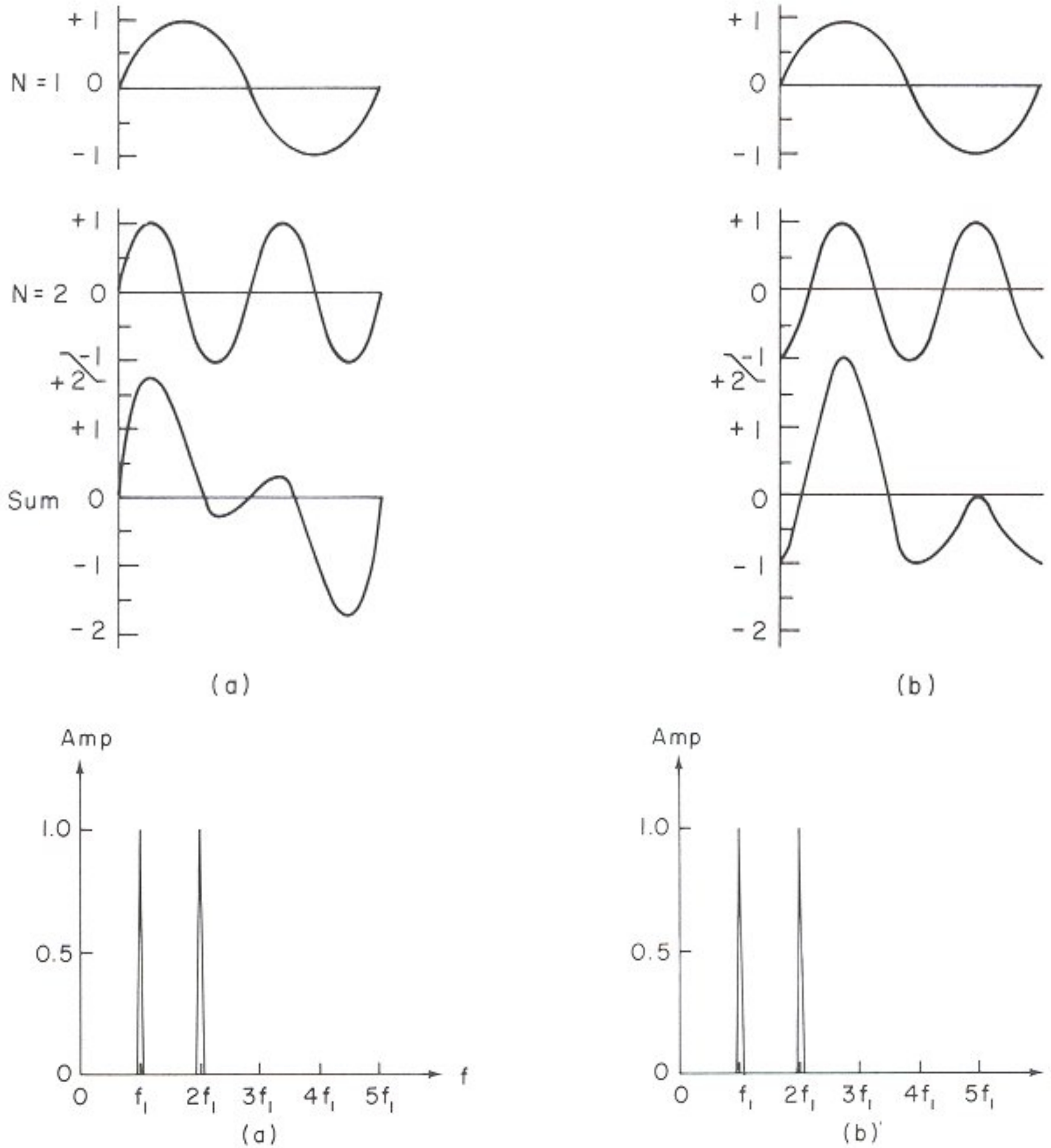
f = 1/P = 100. Hz

# Sound spectrum

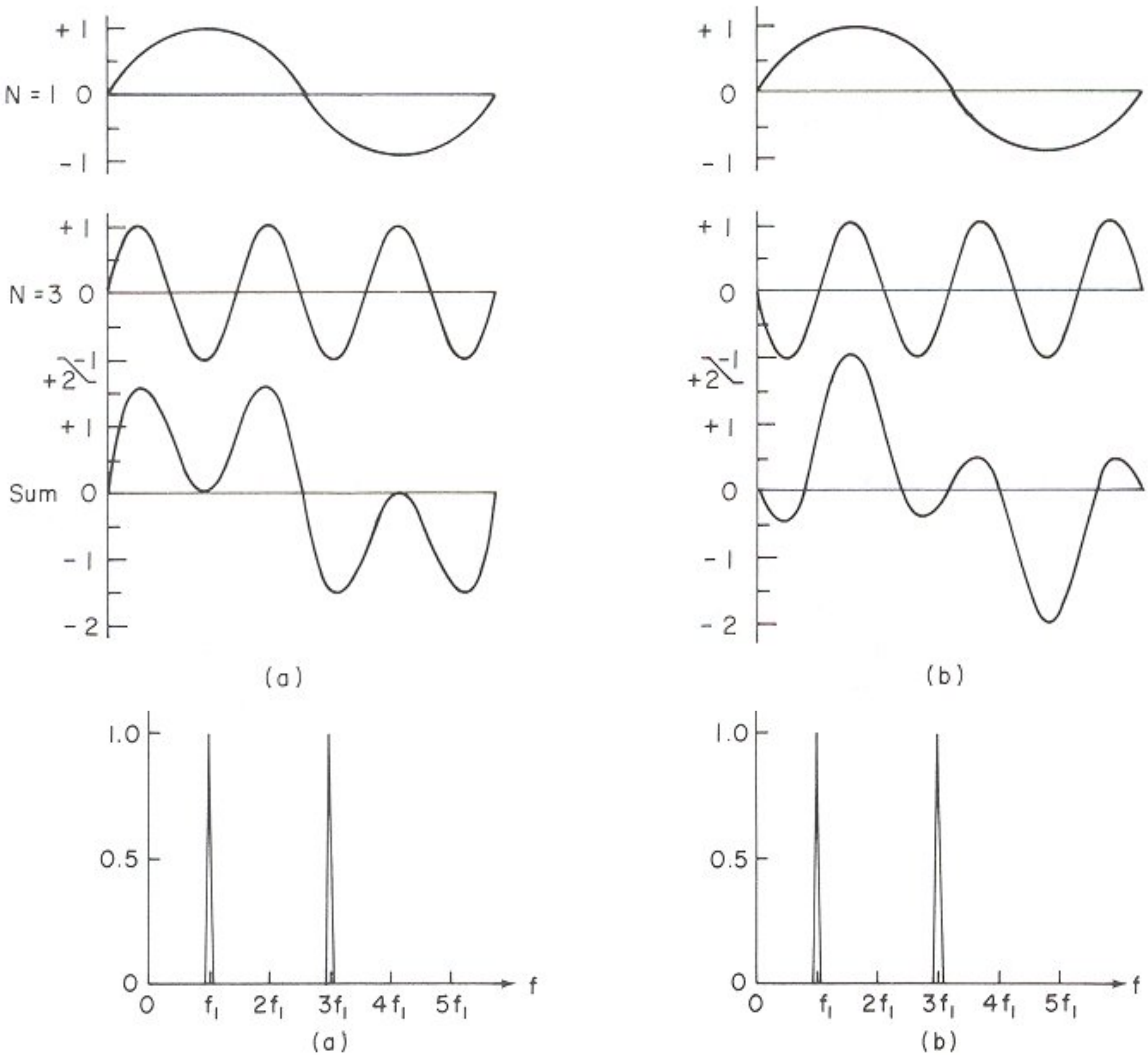



**Partial** is a frequency component in a spectrum which is not an integer multiple of the fundamental, that is, it is an **inharmonic** overtone. However, in some texts, partial refers to both harmonic and inharmonic overtones. Bells, and other percussion instruments, have rich partials in their spectra.

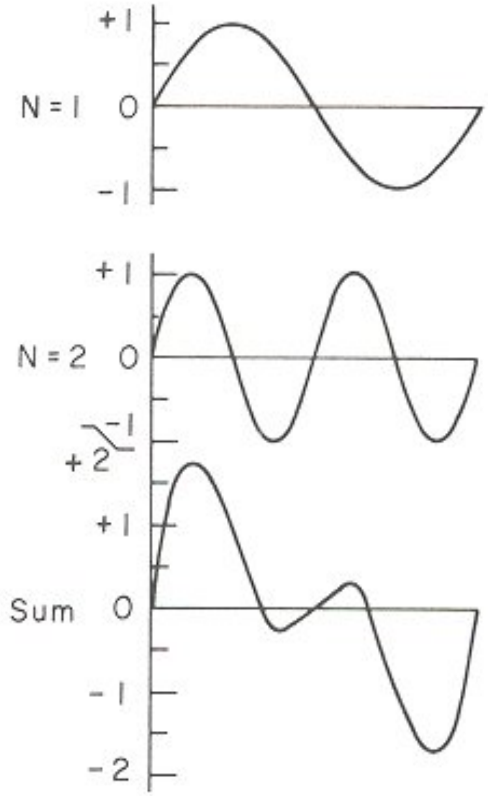
# Examples:
# First and Second Harmonics



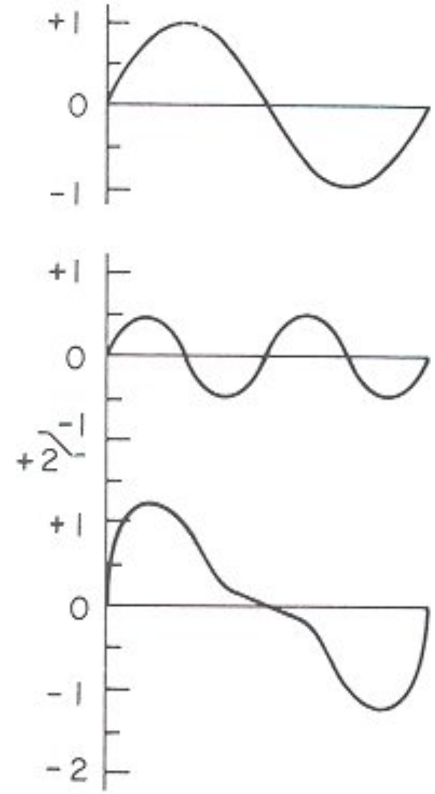
# First and Third Harmonics



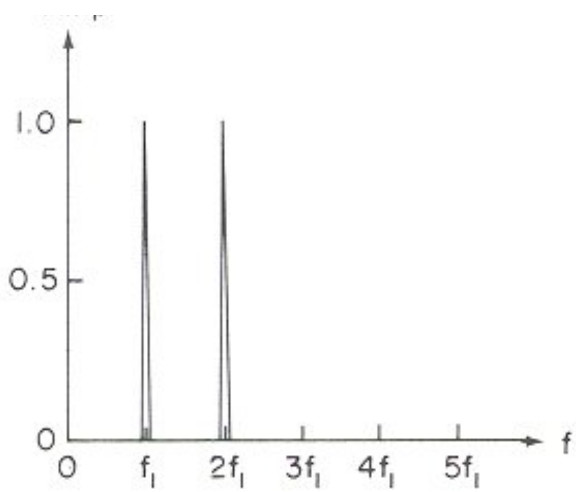
# First and Second Harmonics



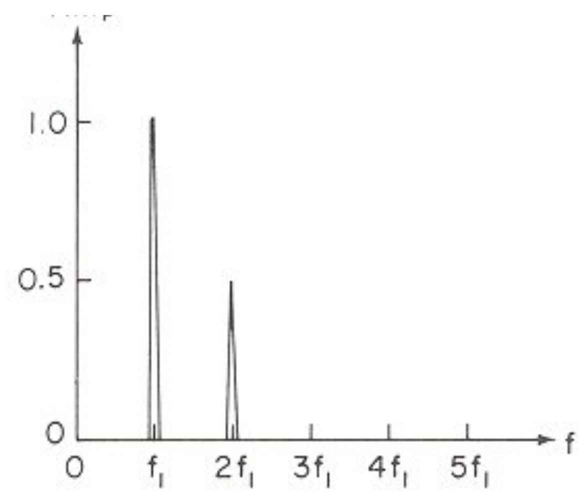

(a)




(b)




(a)

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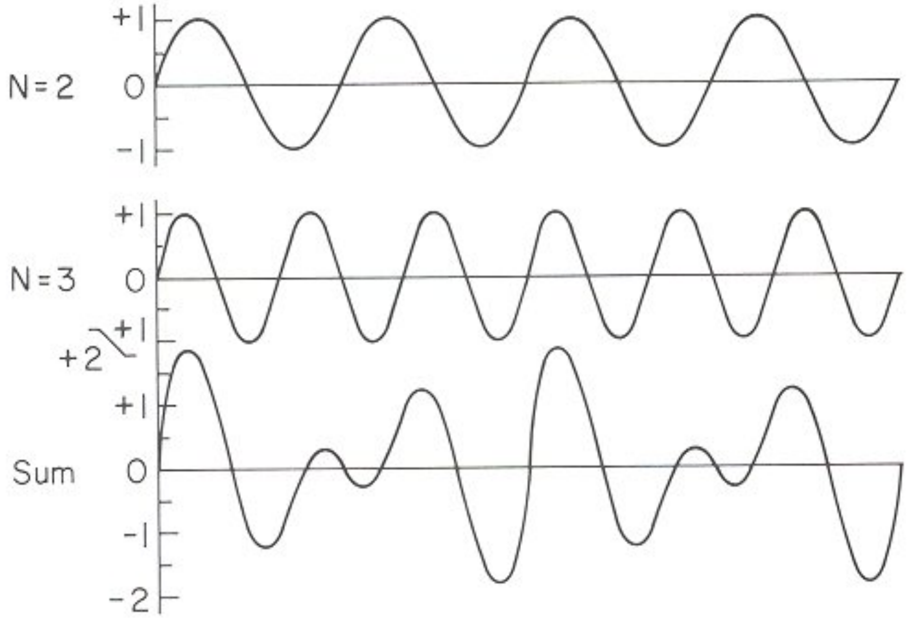
# First and Third Harmonics

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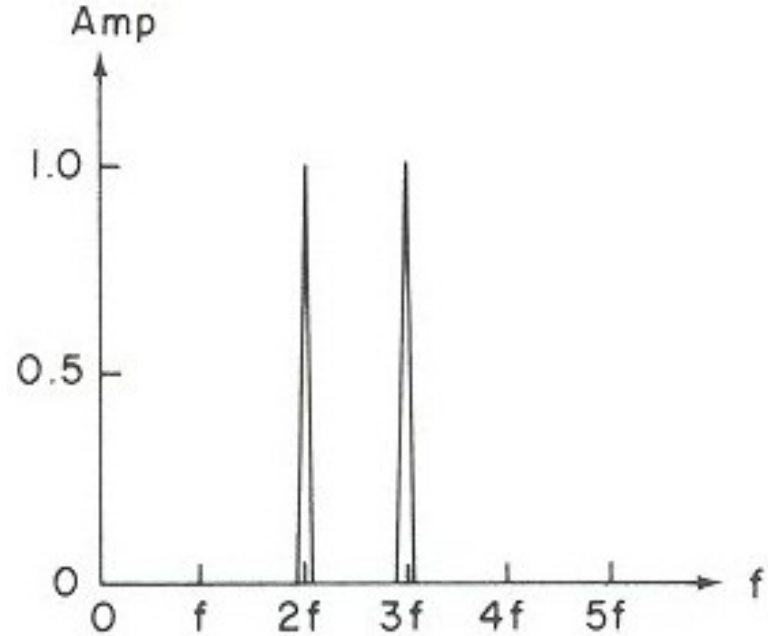


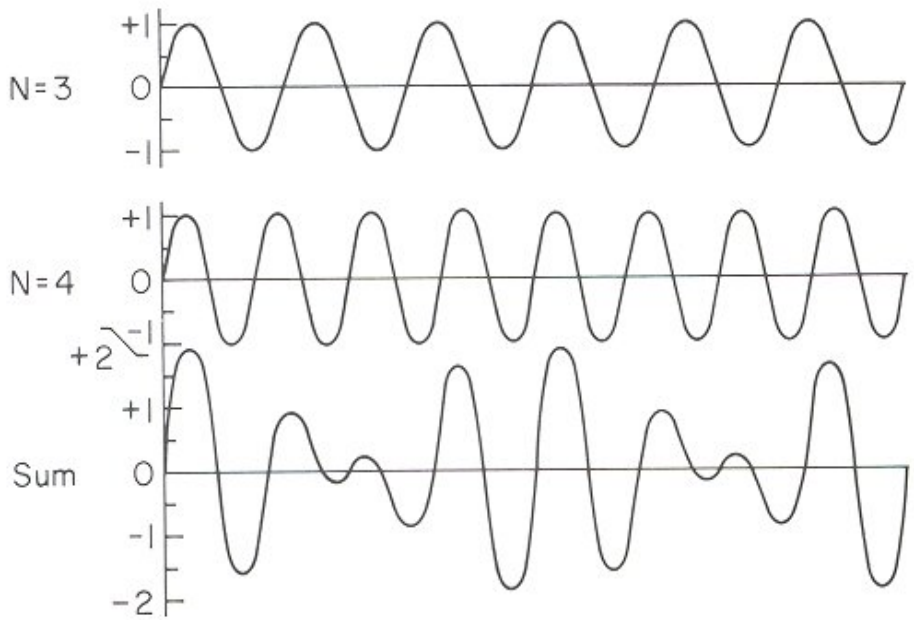


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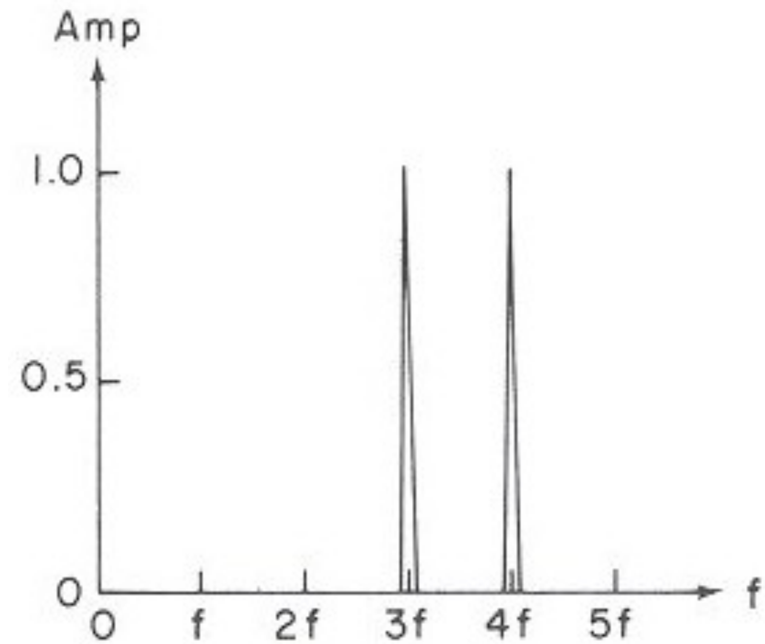


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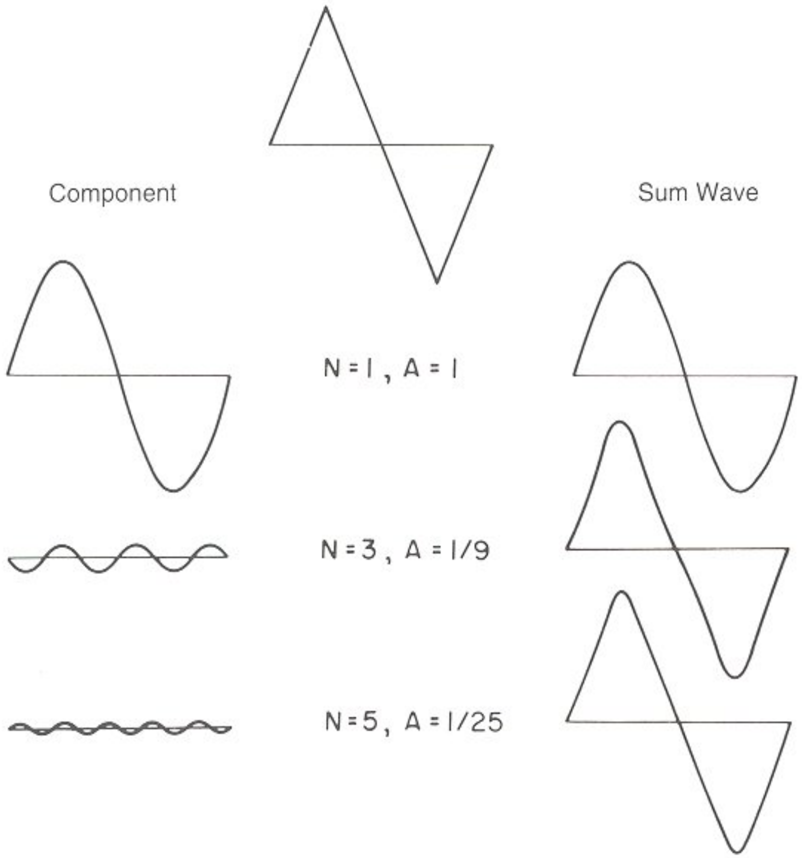
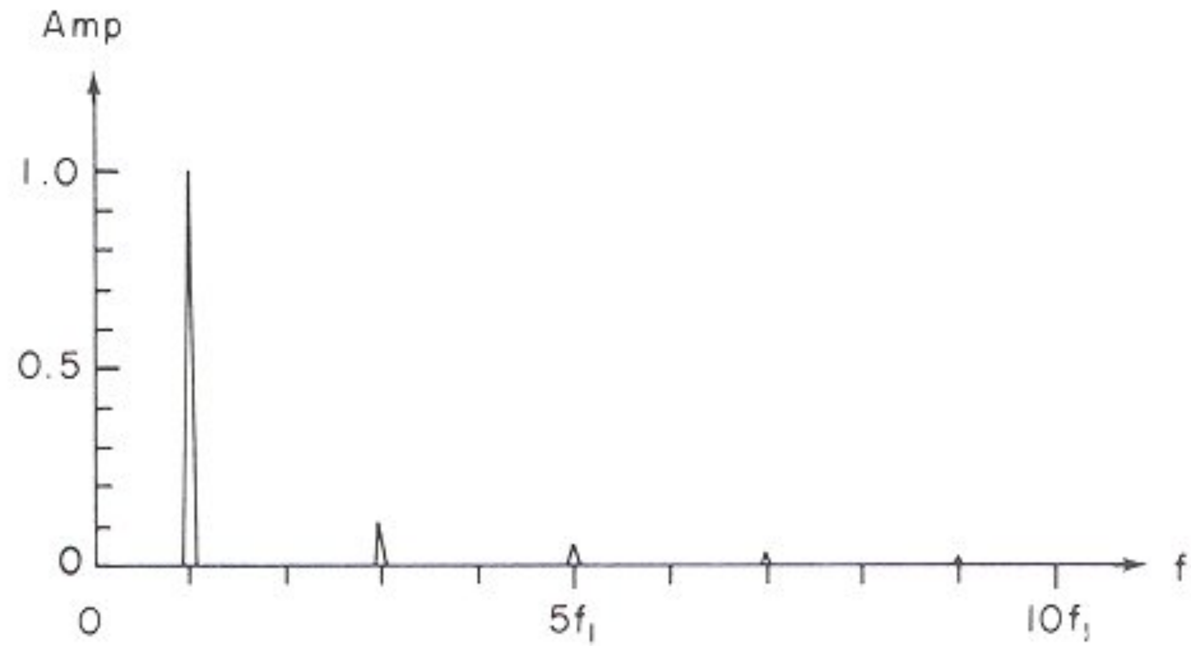
# Triangle Wave



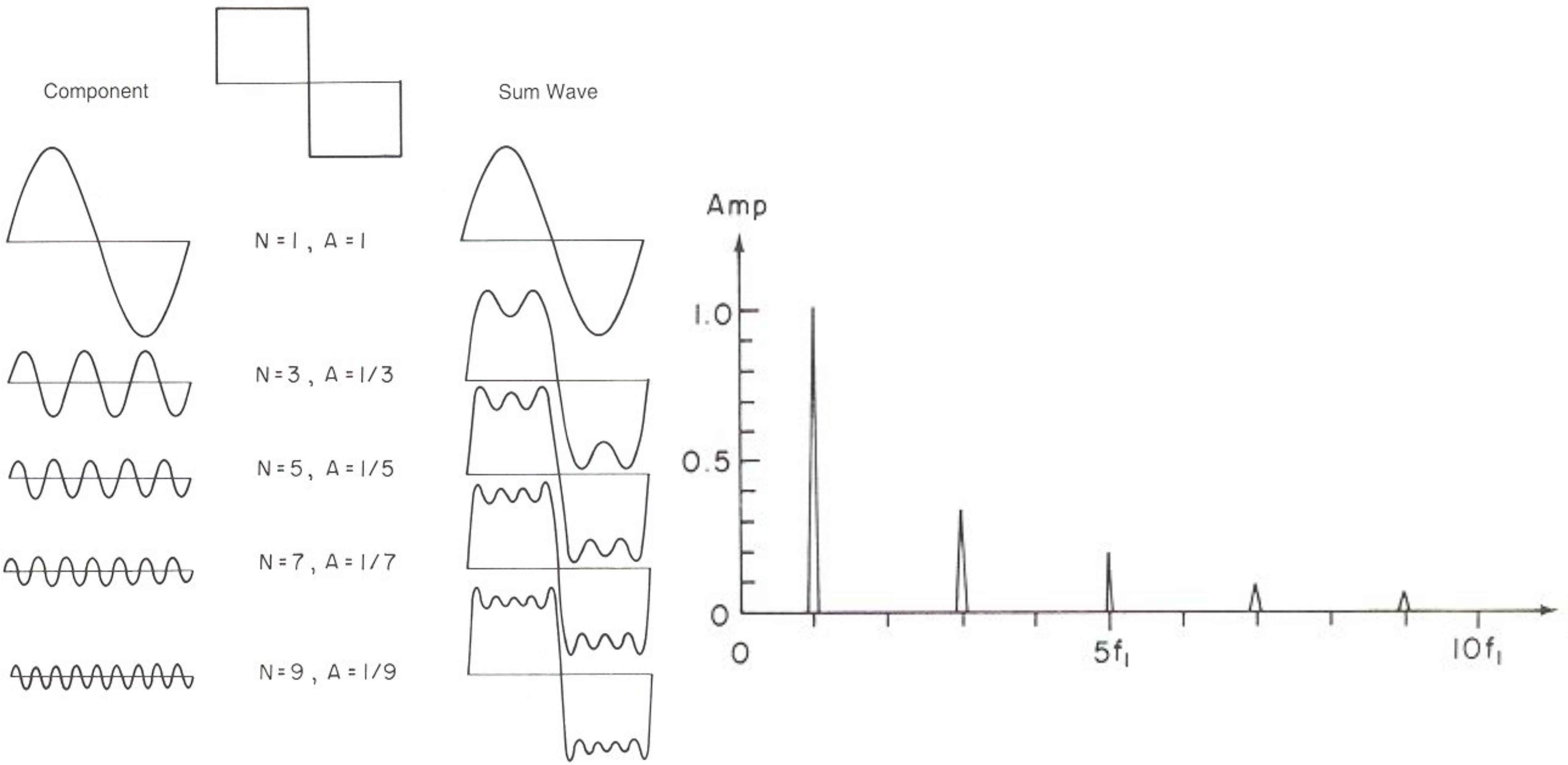




$$f(t) = \sum_N A_N \sin\left(2\pi N f_1 t + \phi_N\right)$$

**Fourier series**

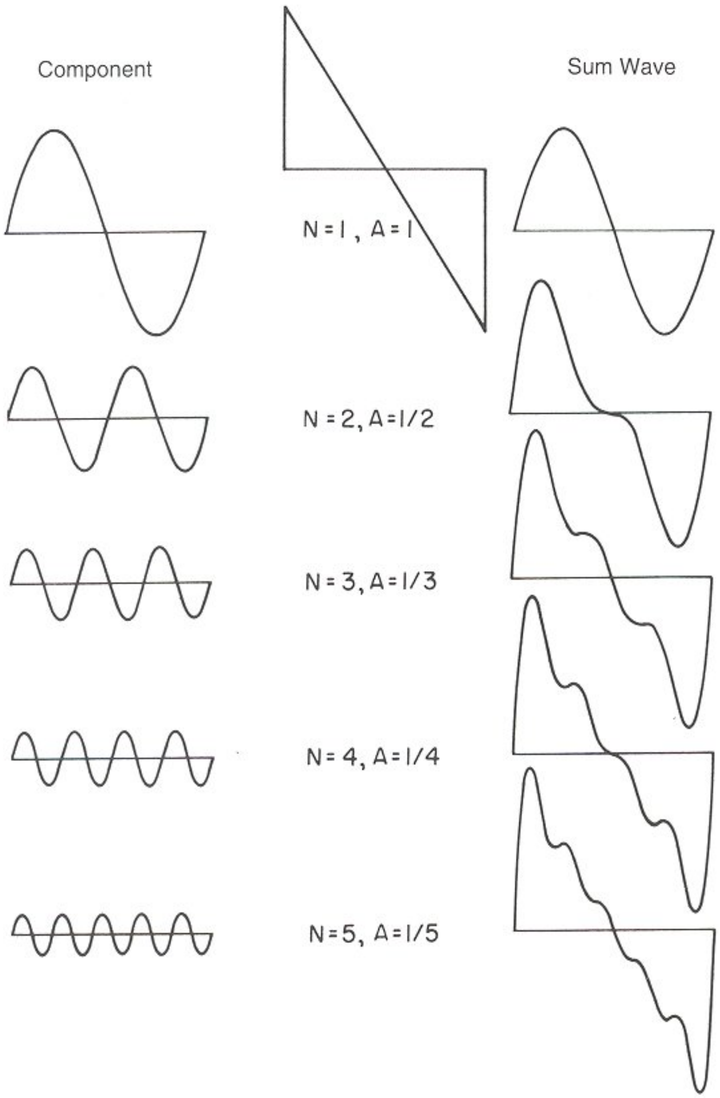
$A_N$ = 1, 0, 1/9, 0, 1/25, 0, 1/49, …. odd N only

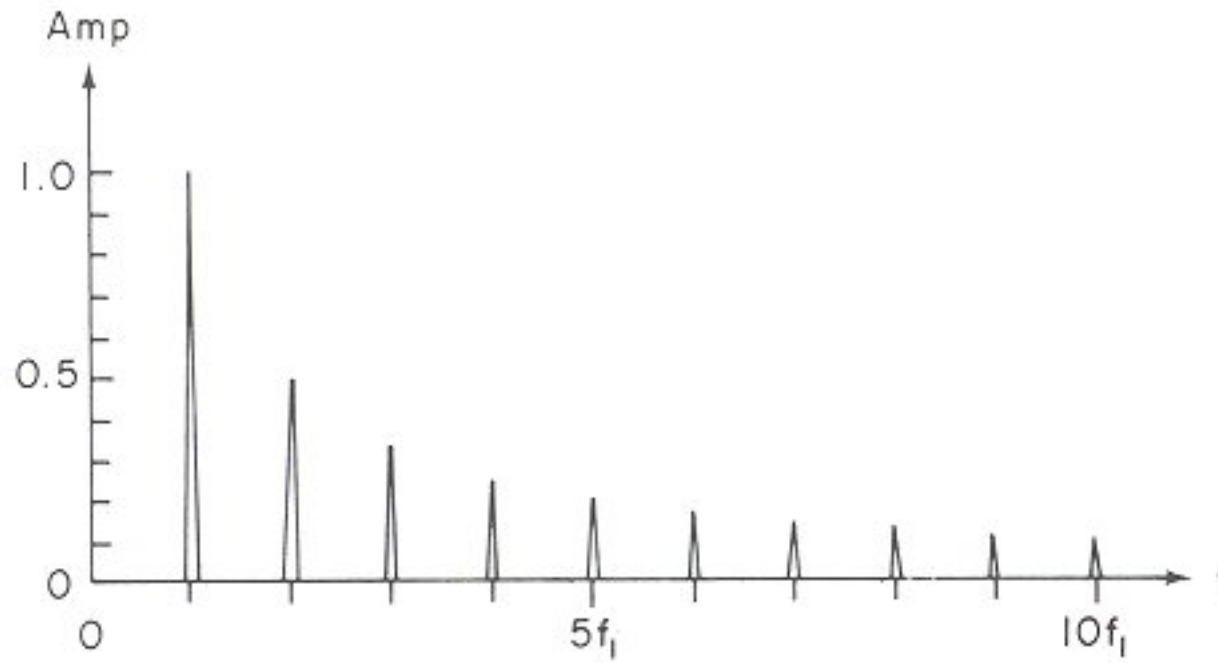
# *Square Wave*




$A_N$ = 1, 0, 1/3, 0, 1/5, 0, 1/7, .... odd N only

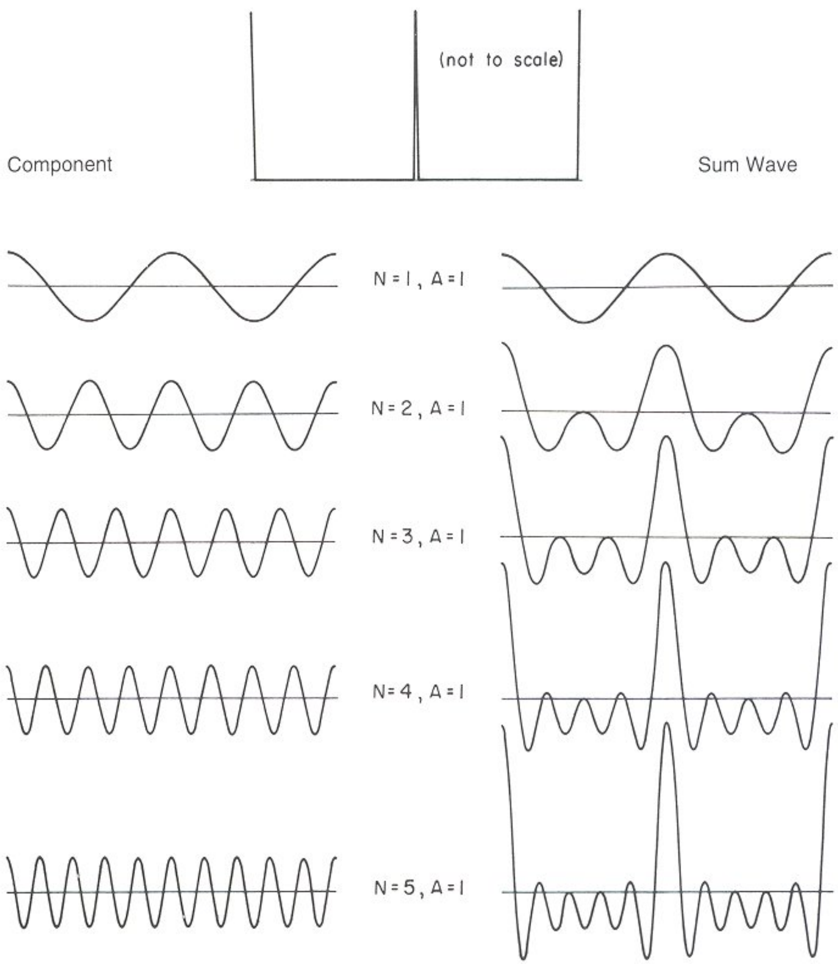
# Sawtooth Wave

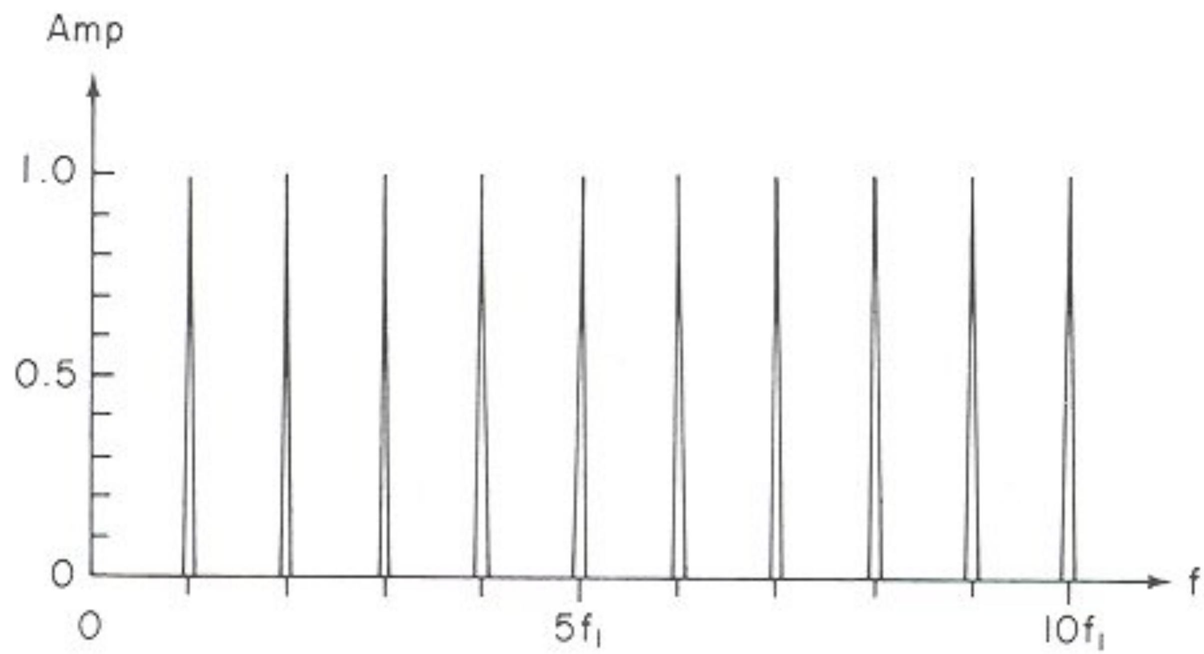



$A_N$ = 1, 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, .... all N

# *Pulse Train (Series of Pulses)*




$A_N$ = 1, 1, 1, 1, 1, 1, 1, .... all N