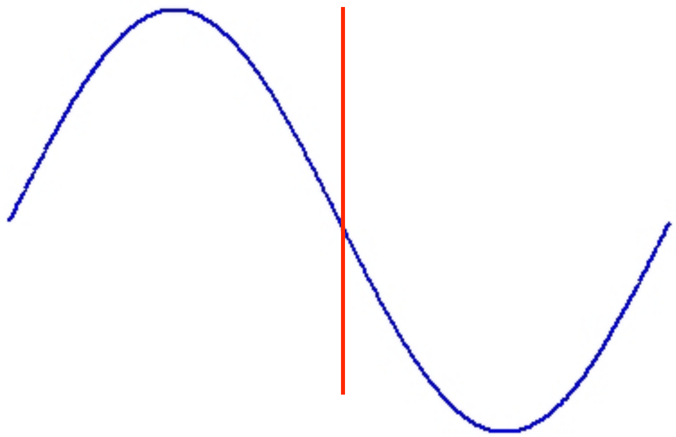
Musical Acoustics

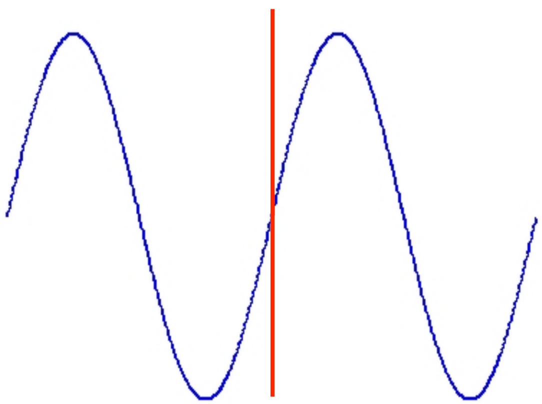
Lecture 14

Timbre / Tone quality II

Odd vs Even Harmonics and Symmetry

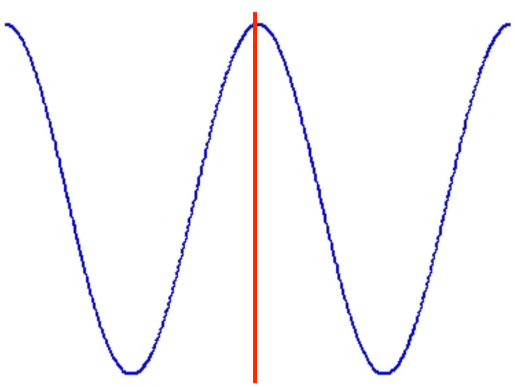


- Sines are Anti-symmetric about mid-point
- If you mirror around the middle you get the same shape but upside down



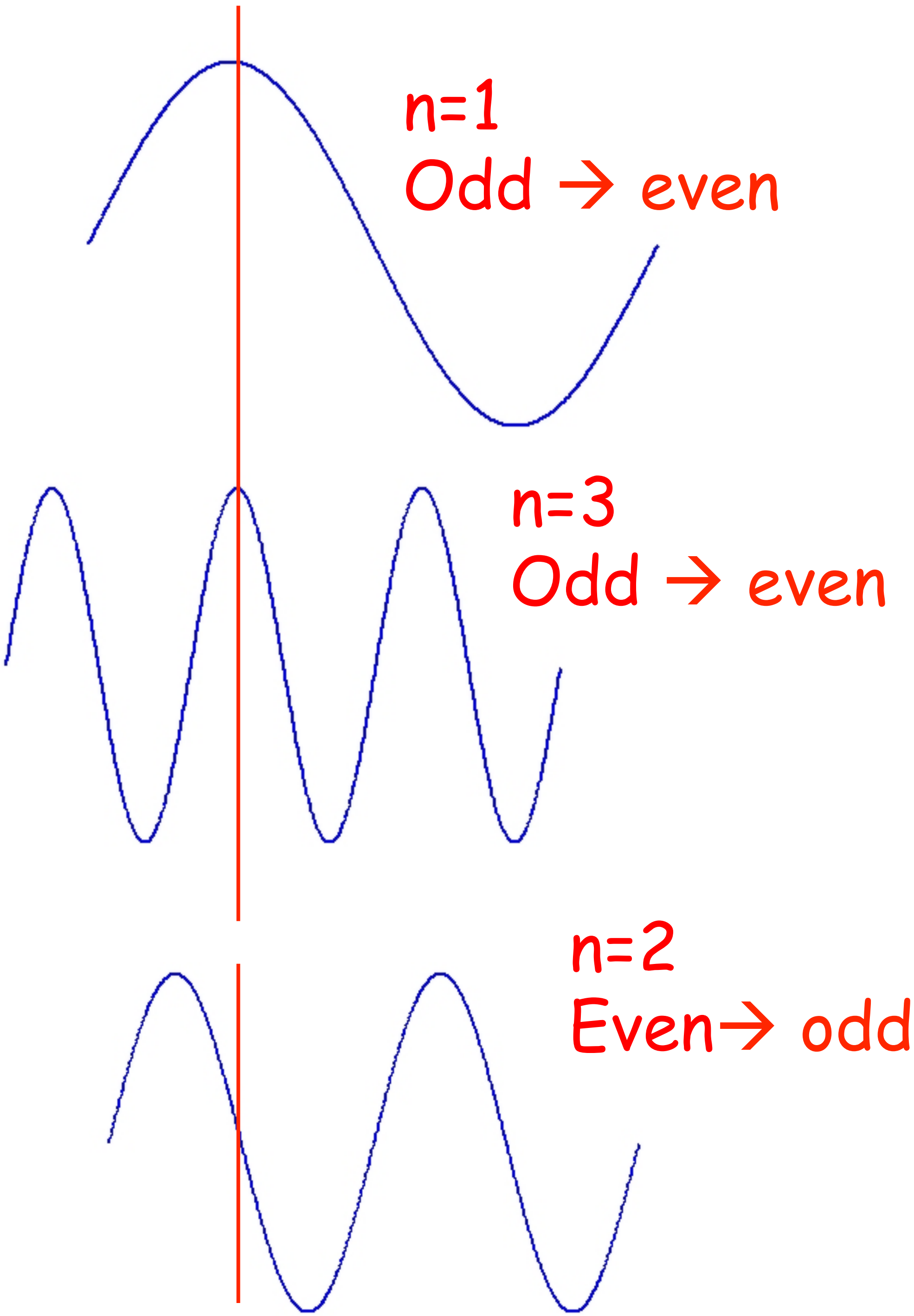
$$f(t) = \sum_N A_N \sin\left(2\pi N f_1 t + \phi_N\right)$$

Fourier series



- Sines are anti-symmetric
- Cosines are symmetric

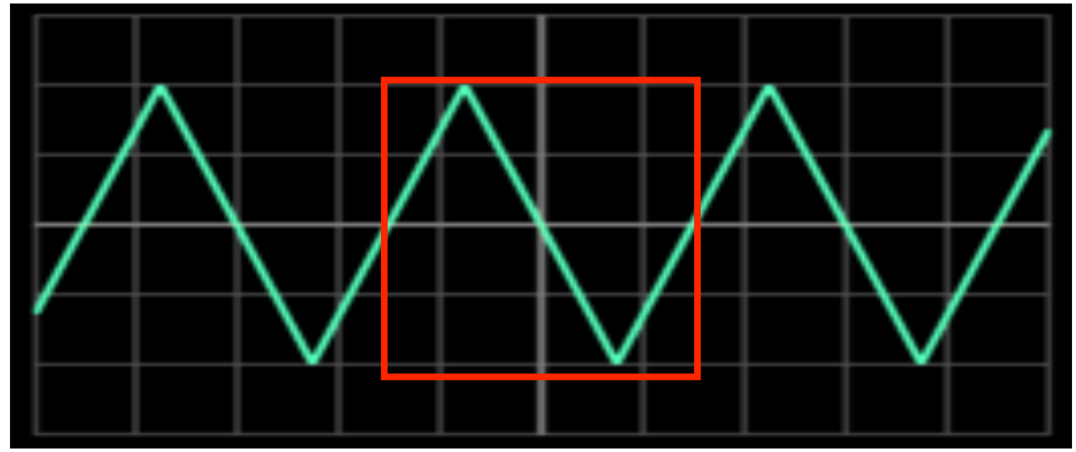
But symmetry depends on choice of origin



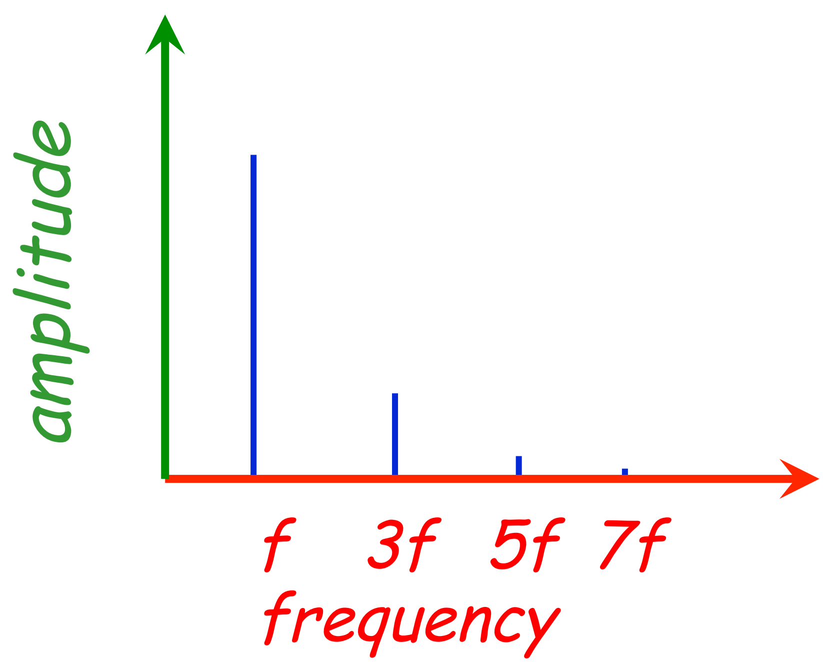
- Additional symmetry of odd sines if you consider reflection at the red line.
- About this line, odd harmonics are symmetric but even ones are anti-symmetric

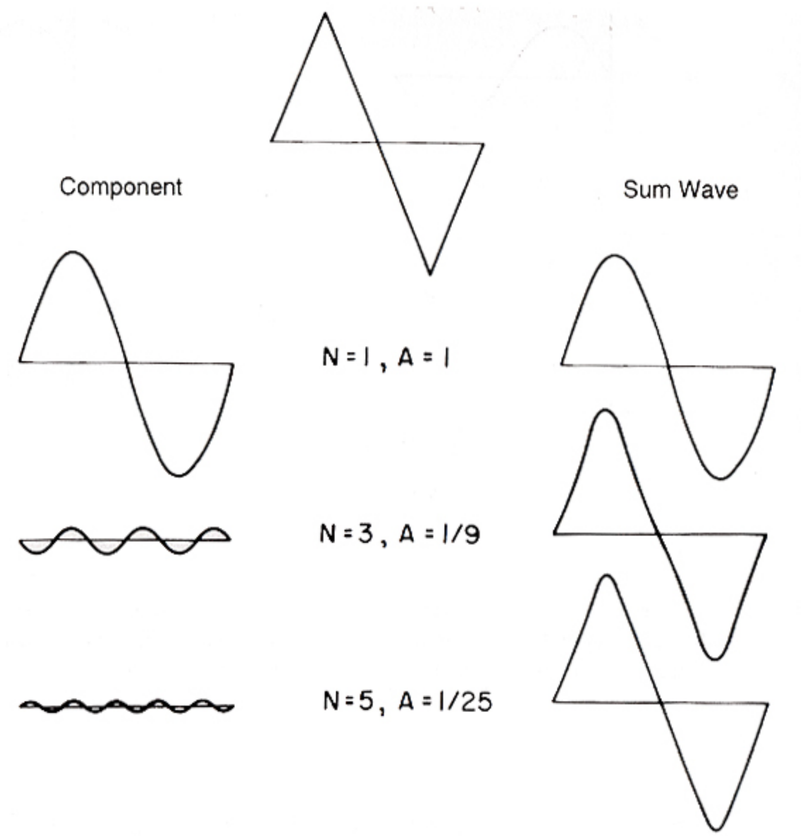
Symmetry of the triangle wave

Obeys same symmetry as the odd harmonics so cannot contain even harmonic components

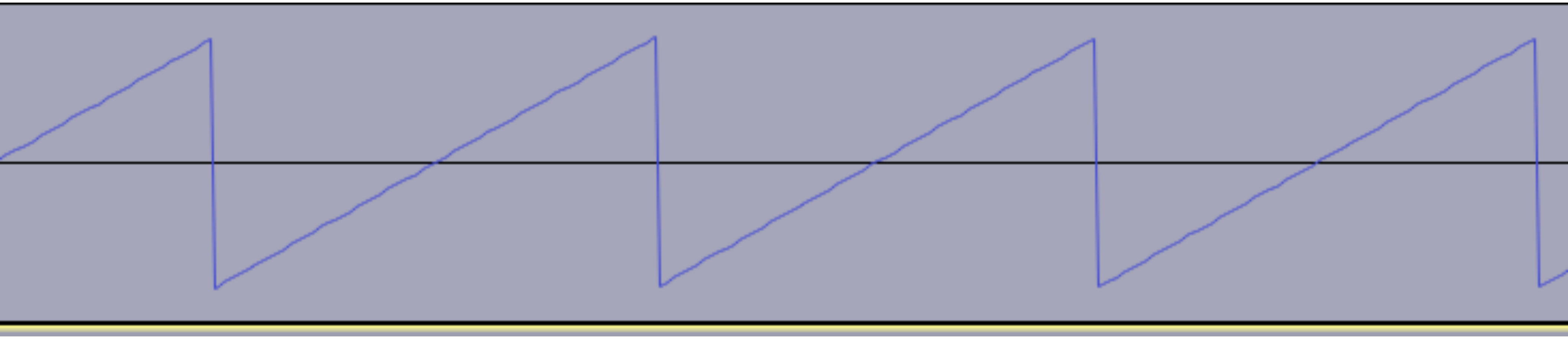


Both triangle waves and square waves contain **odd** Fourier components.



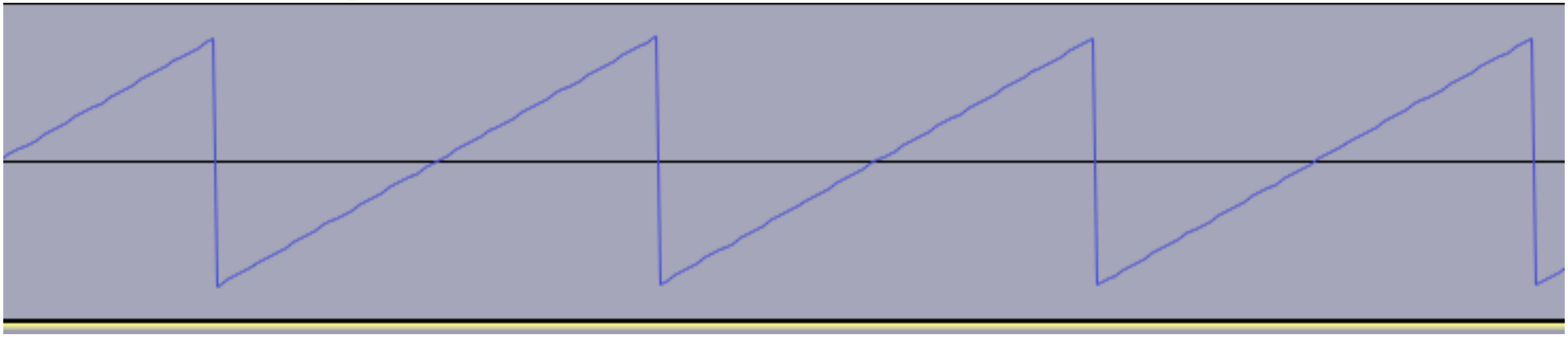


Sawtooth wave

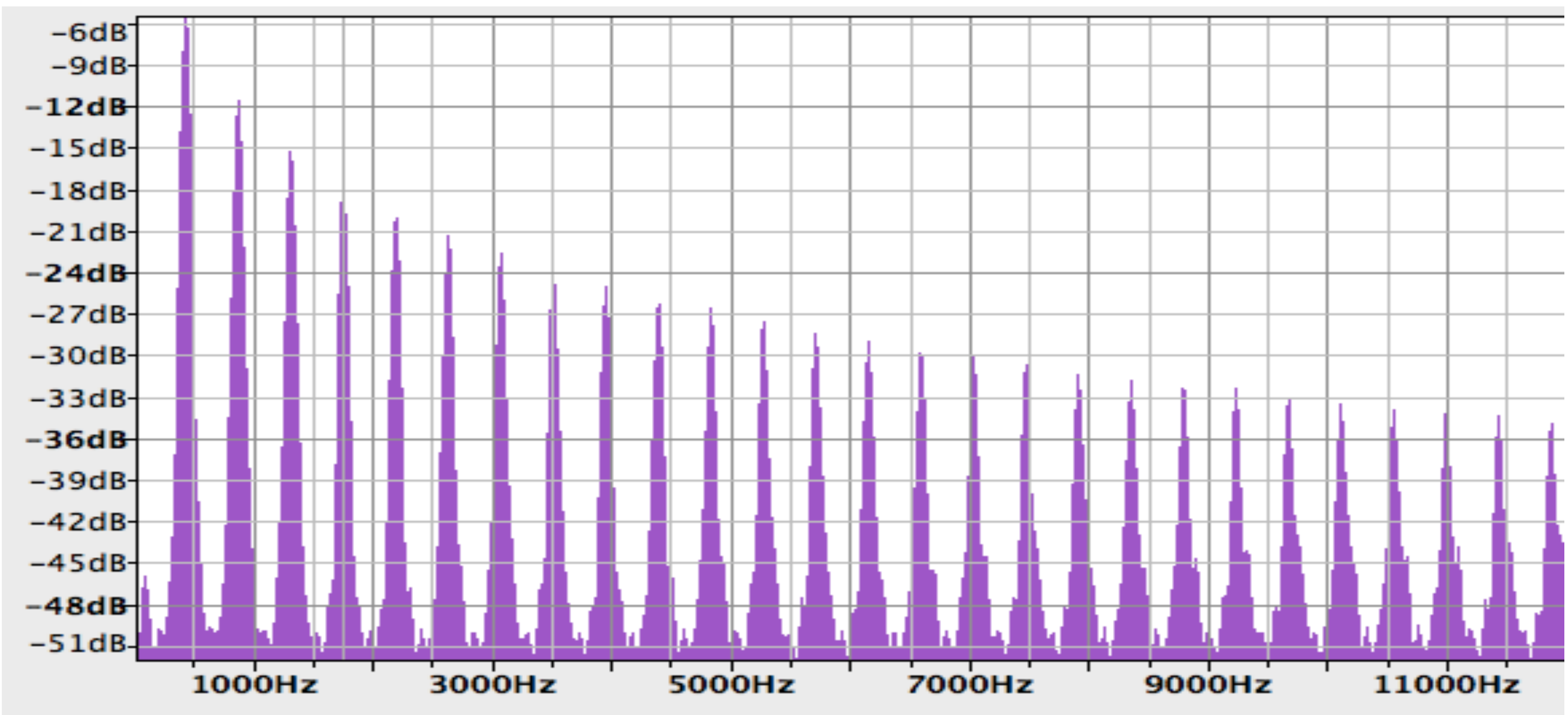


- What overtones are present in this wave? Use its symmetry to guess the answer.

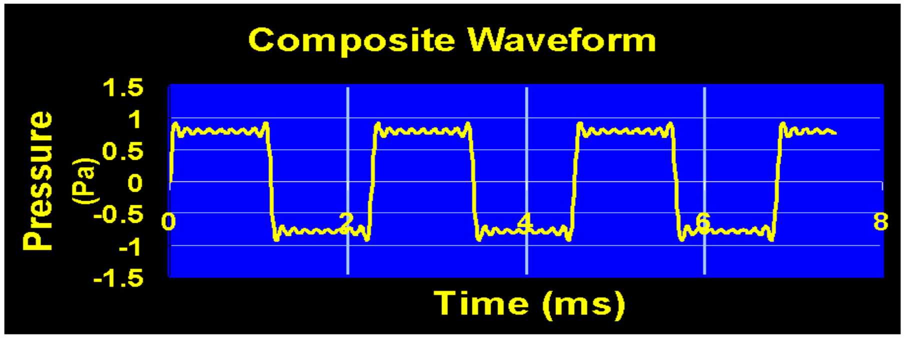
Spectrum of a sawtooth wave



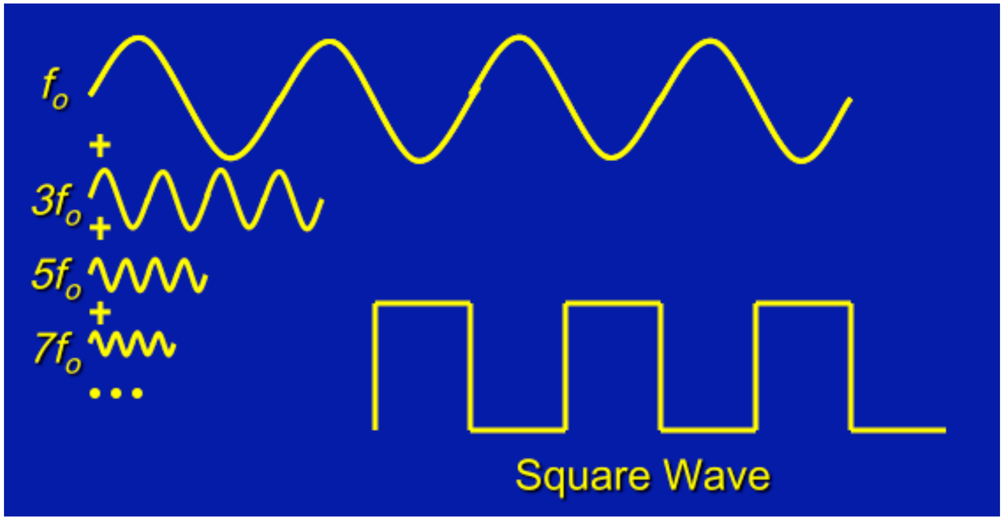
All integer harmonics are present. The additional symmetry about the $\frac{1}{4}$ wave that both triangle and square wave have is not present in the sawtooth.



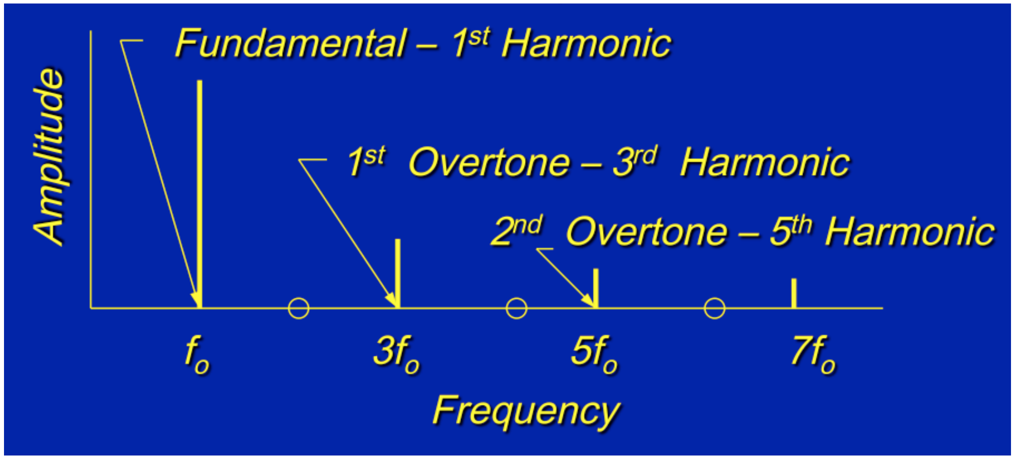
Summary: Synthesis of a square wave



<u>Fourier Analysis</u> is the decomposition of a wave into the sine wave components from which it can be built up.

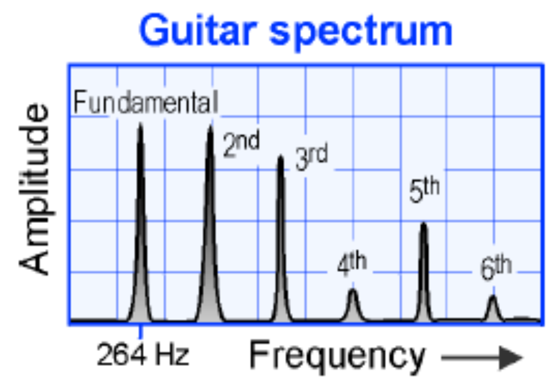


*A **<u>Fourier Analysis</u>** is a representation of all the components that comprise a waveform, **<u>amplitude</u>** versus frequency and **<u>phase</u>** versus frequency.*



Sound Spectrum of Musical Instruments

- Each musical instrument has its own characteristic sounds - quite complex!
- Any note played on an instrument has fundamental + harmonics of fundamental.
- Higher harmonics - brighter sound
- Less harmonics - mellower sound



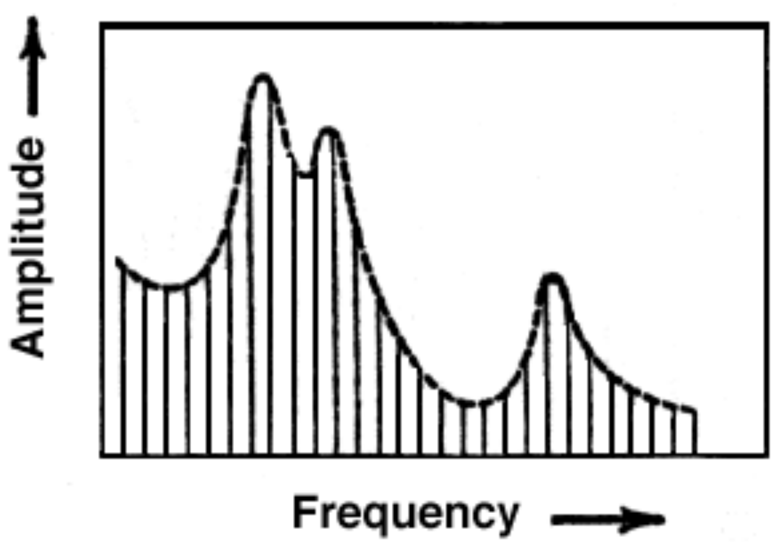
- Harmonic content of note can/does change with time:
  - Takes time for harmonics to develop - "attack" (leading edge of sound)
  - Harmonics don't decay away at same rate (trailing edge of sound)
  - Higher harmonics tend to decay more quickly
- Sound output of musical instrument is not uniform with frequency
  - Details of construction, choice of materials, finish, etc. determine *resonant structure* (**formants**) associated with instrument - mechanical vibrations!

Formants

• Musical instrument may have formant.

• The human voice has formant regions determined by the size and shape of the the vocal tract.

• Formant regions are not directly related the fundamental frequency and may remain more or less constant as the fundamental changes. If the fundamental is well below or low in the formant range, the quality of the sound is rich, but if the fundamental is above the formant regions the sound is thin and in the case of vowels may make them impossible to produce accurately - the reason singers often seem to have poor diction on the high notes.

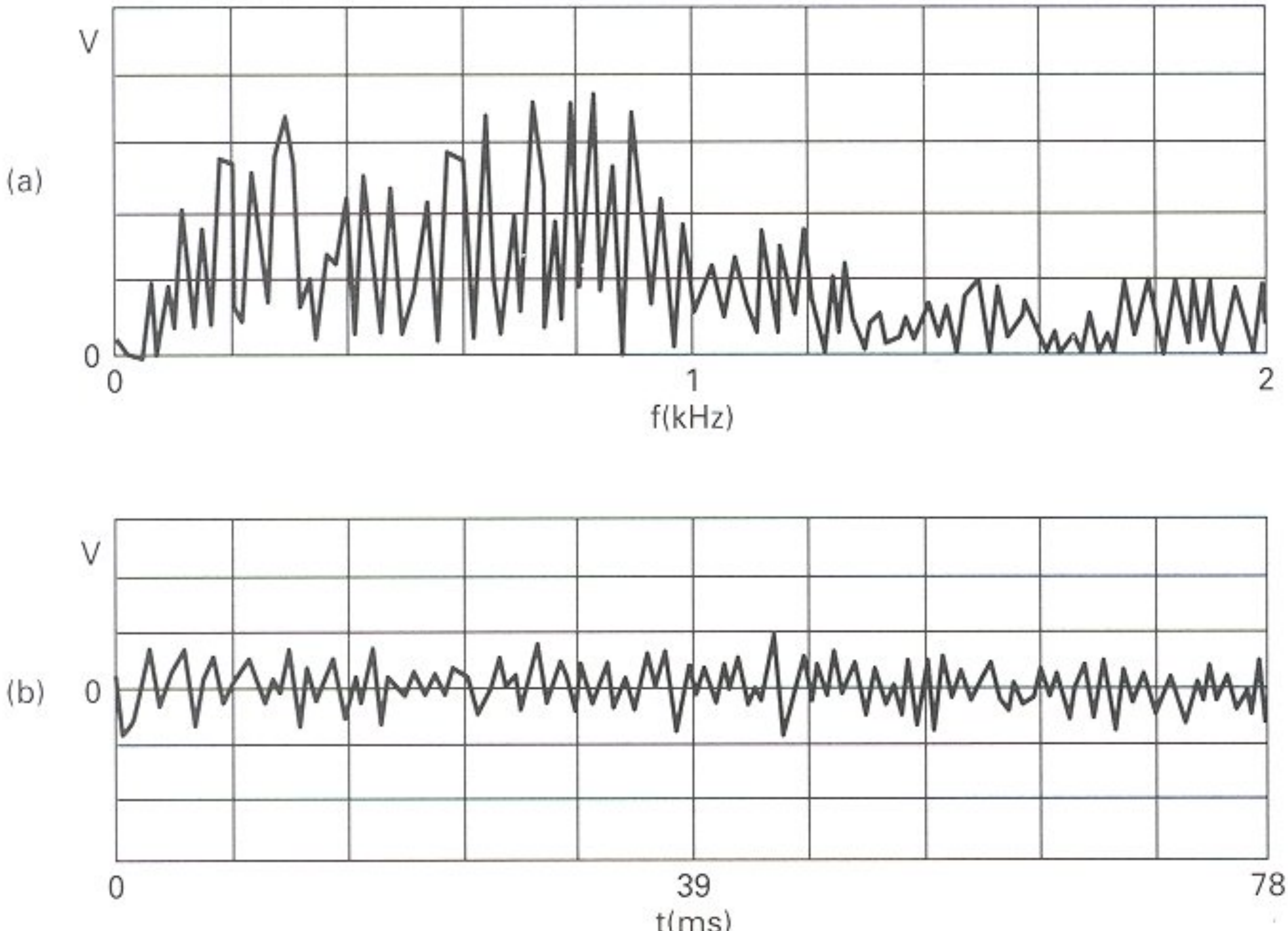


• Spectrum of the vowel "ah" showing three formant regions.

• Vertical lines are harmonics produced by vibration of the vocal cords and based on a low fundamental.

• The vowel has a characteristic spectral shape.

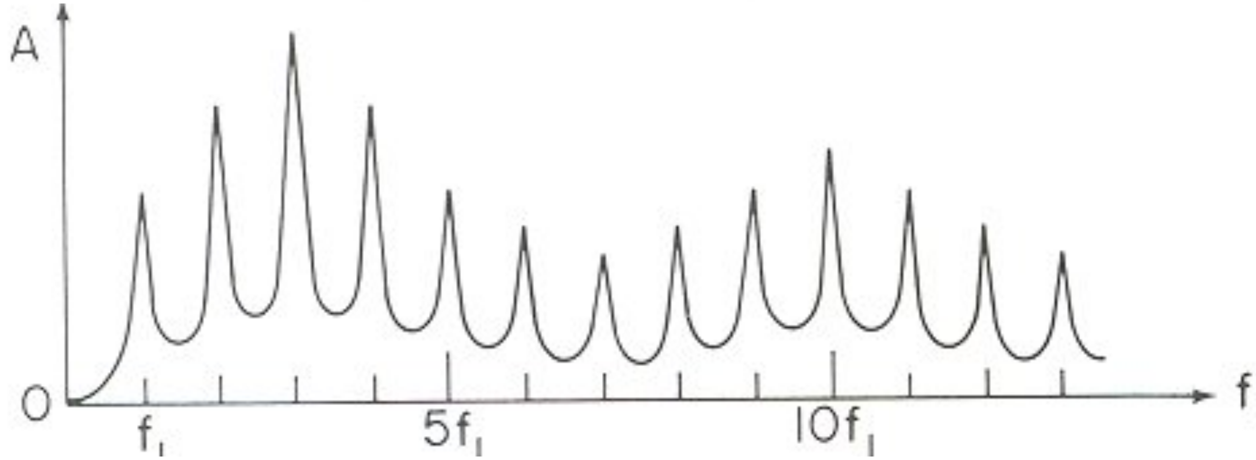
Noise



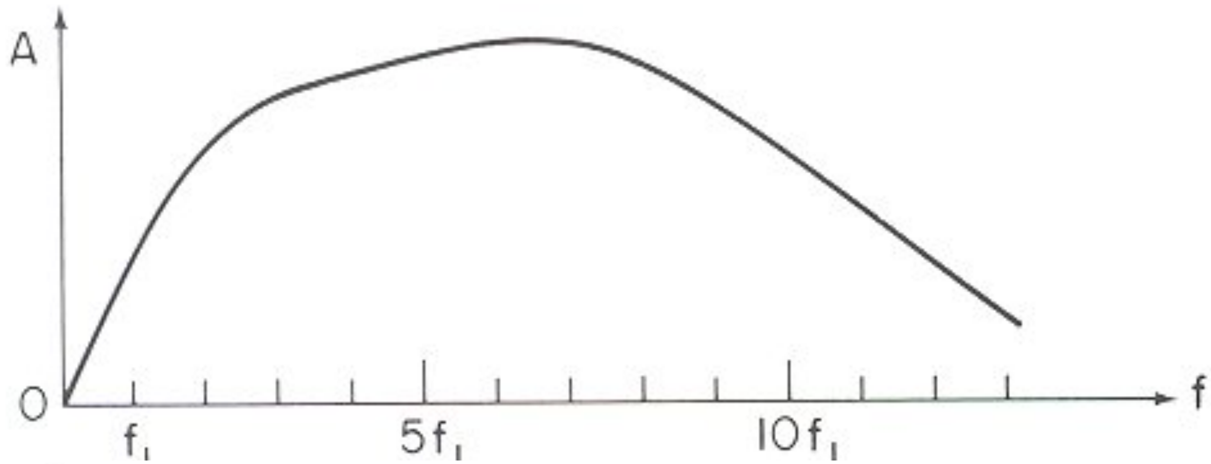
Blowing gently across a microphone

Hypothetical instrument's timbre

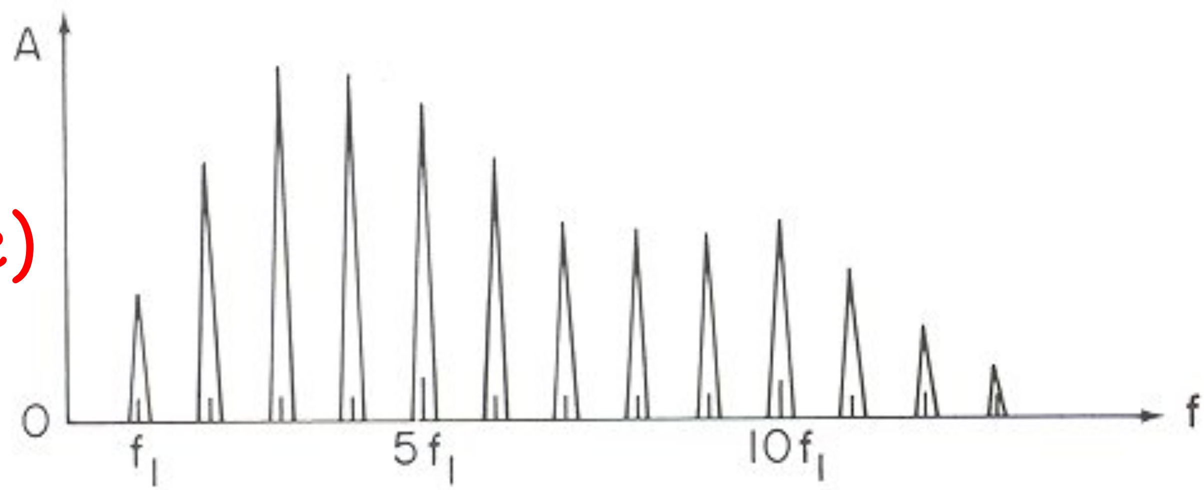
Resonance curve



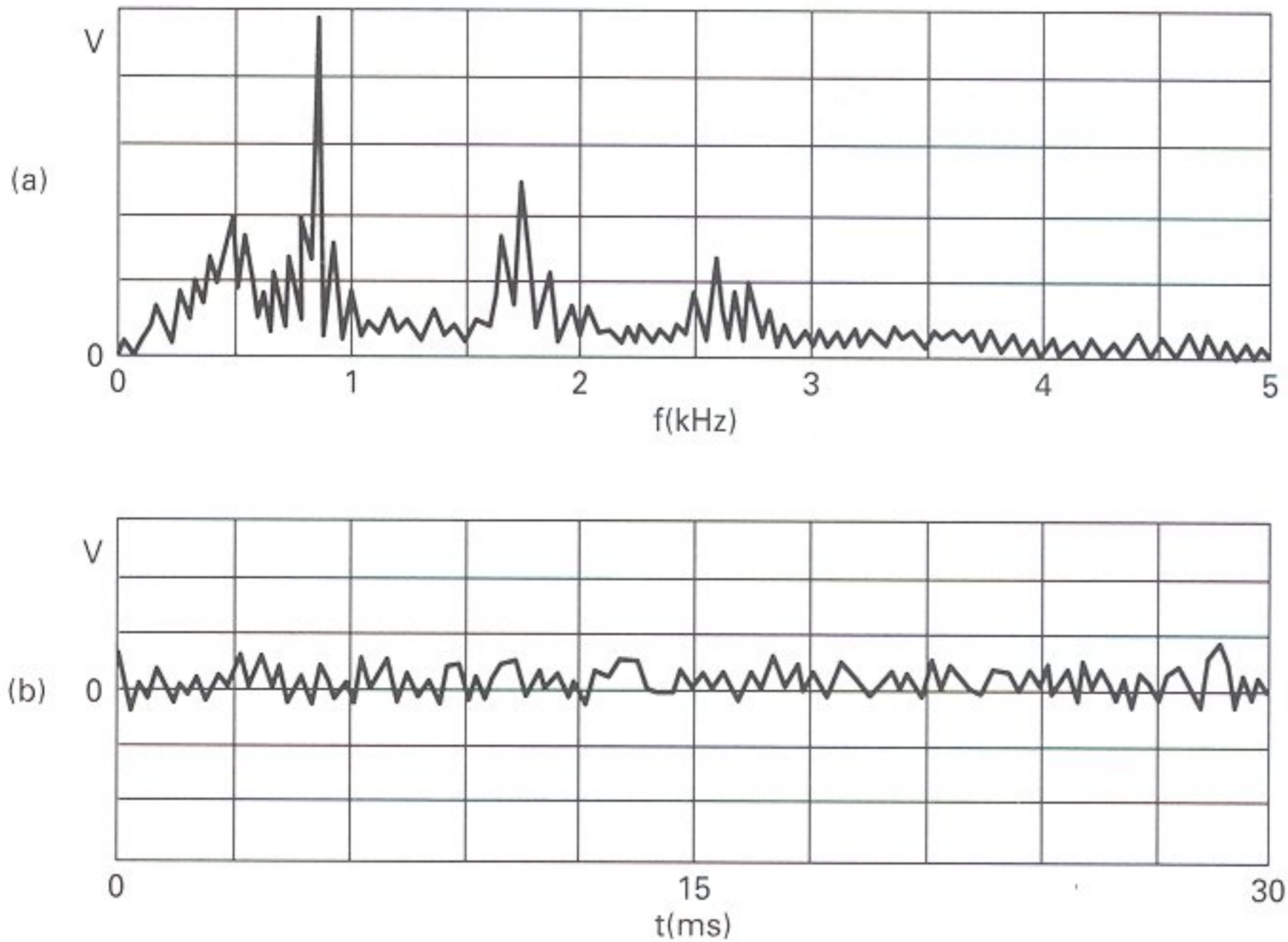
Noise spectrum



Fourier spectrum
(Resonance minus noise)



Blowing into open tube tube

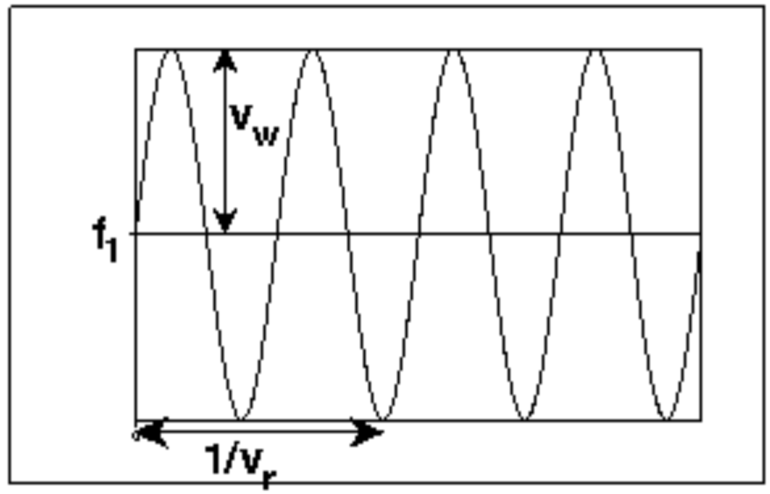


Frequency Modulation and Vibrato

- Vibrato - Periodic variation of frequency
- A simple signal: **sin[2πf(t)t]**
- with vibrato

$$f(time) = freq_1 + vib_{wid} \sin(2\pi . vib_{rate} . time)$$

- vib_{wid} = vibrato width
  - amount of vibrato
- vib_{rate} = vibrato rate
  - frequency of vibrato



- Typical vibrato values:

  vib_{rate} = 5 Hertz

  normal range:

  1-6 Hertz, with slight acceleration during tone

  vib_{wid}

  minimum: 0 (none)

  usual maximum for instruments:

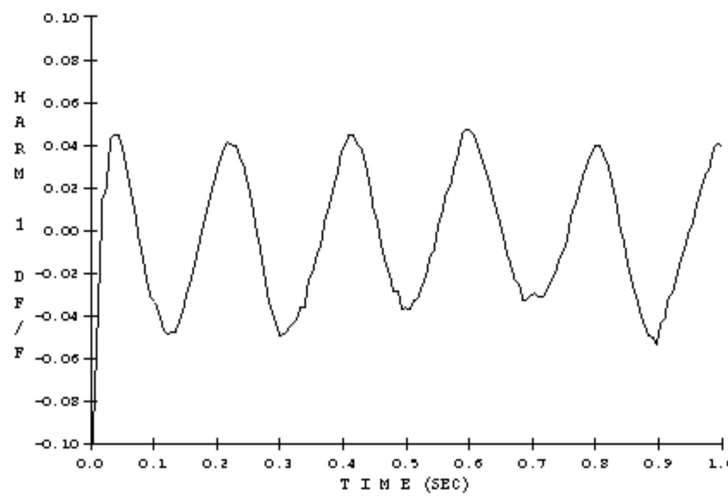
  $.01*freq_1$ (1%)

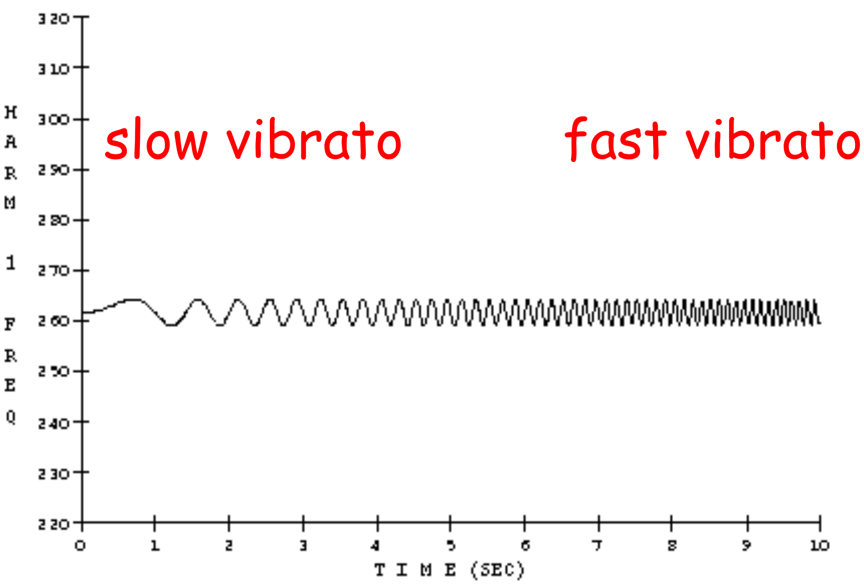
  usual maximum for voices:

  $.05*freq_1$ (5%)

Example: Tenor Voice Vibrato

Tenor voice has 5 Hertz vibrato rate and vibrato width of 4.5%





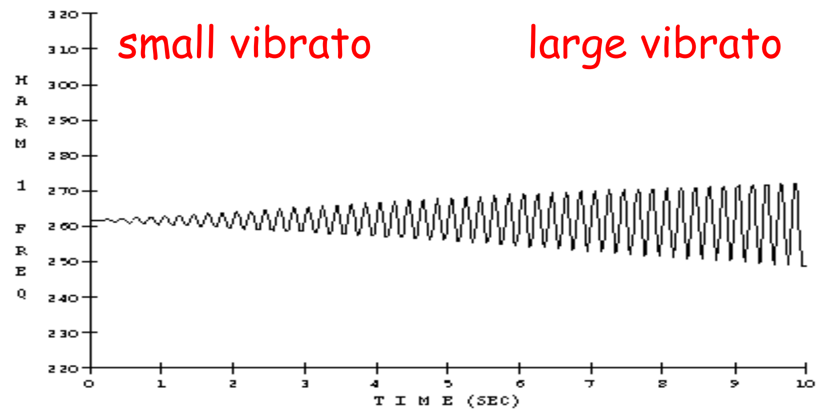
Vibrato rate

vib_{rate} = 0 -----> 10 Hertz (over 10 seconds)

vib_{wid} = .01 * $freq_1$ (1%)

Vibrato Amount

- vib_{rate} = 5 Hertz
- vib_{wid} = 0 -----> .05 * $freq_1$ (0-5% over 0:10 s)



A sine wave with vibrato becomes a full spectrum when vib_{rate} is in the audio range (above 20 Hz), especially as vib_{rate} approaches $freq_1$.

Since it is no longer vibrato, we use the term *modulation frequency* instead of vibrato rate.

When vib_{rate} is above 20 Hz:

$$freq_{mod} = vib_{rate}$$

Vibrato → FM

With FM, we may not get the frequency out that we put in. We call the base frequency of the outer sine wave the carrier frequency f_{car}:

$$f(time) = f_{car} + vib_{wid} \sin(2\pi . f_{mod} . time)$$

Vibrato → FM

- FM uses a modulation index as well as the vibrato width to describe the amount of modulation. The relationship between them is

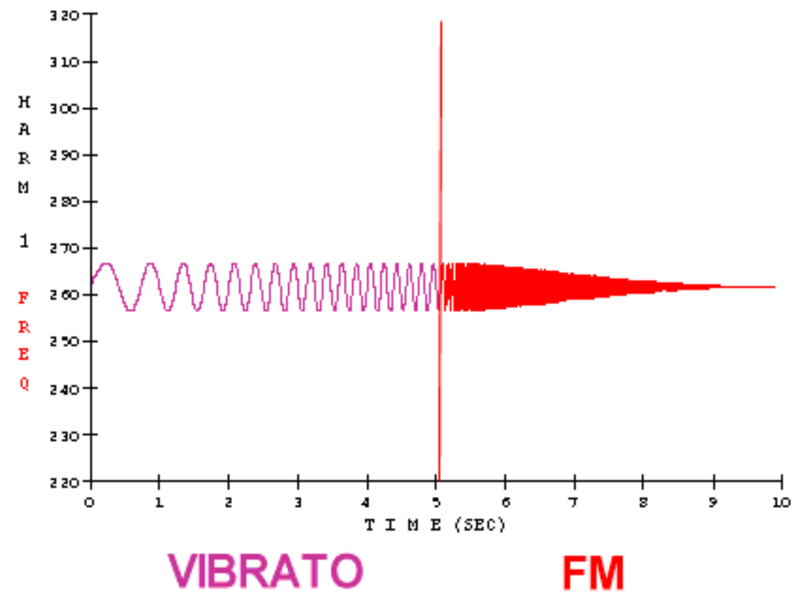
$$vib_{wid} = Index * freq_{mod}$$

or

$$Index = vib_{wid} / freq_{mod}$$

- Typical values for modulation index:

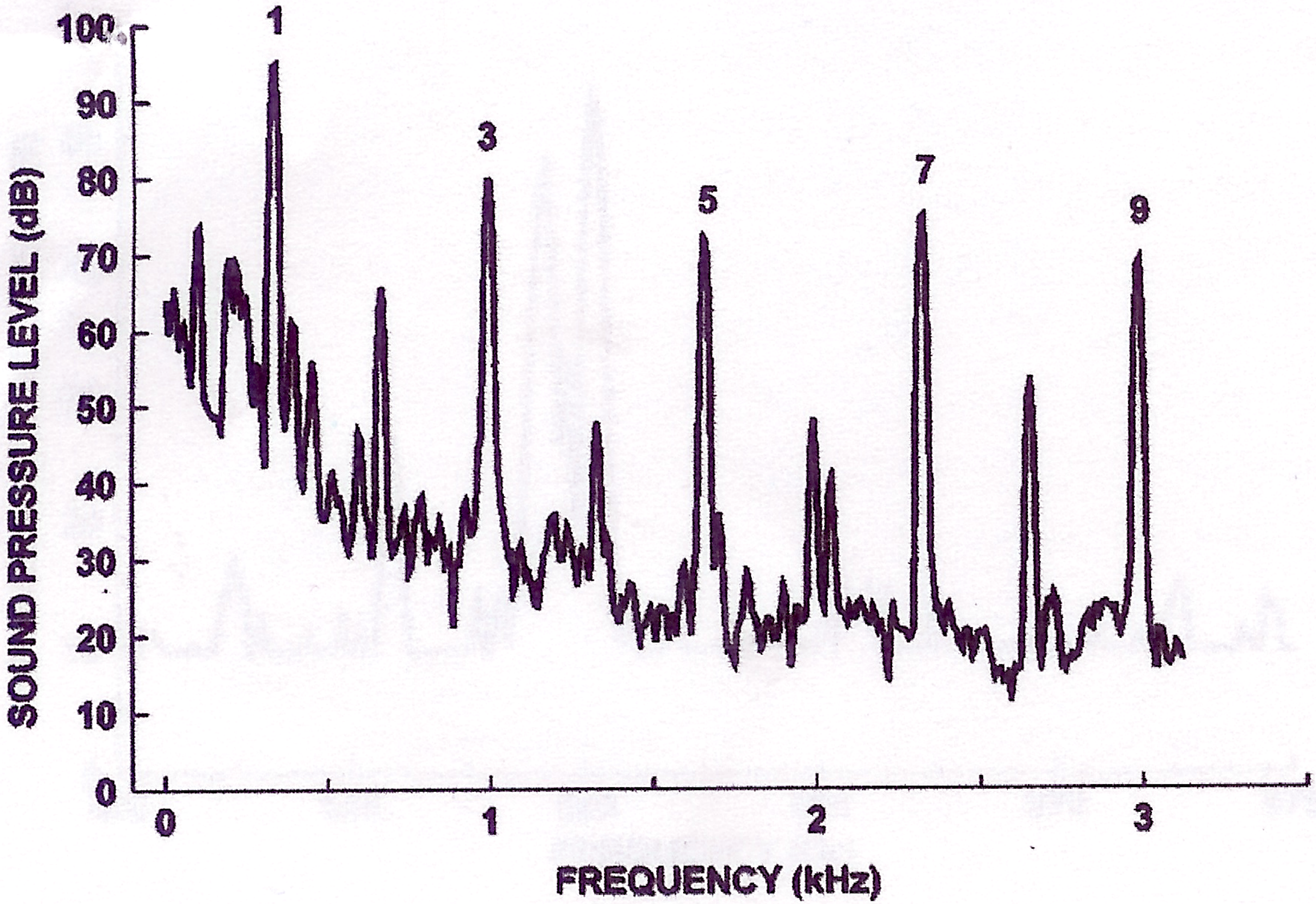
$$0 <= Index <= 10$$



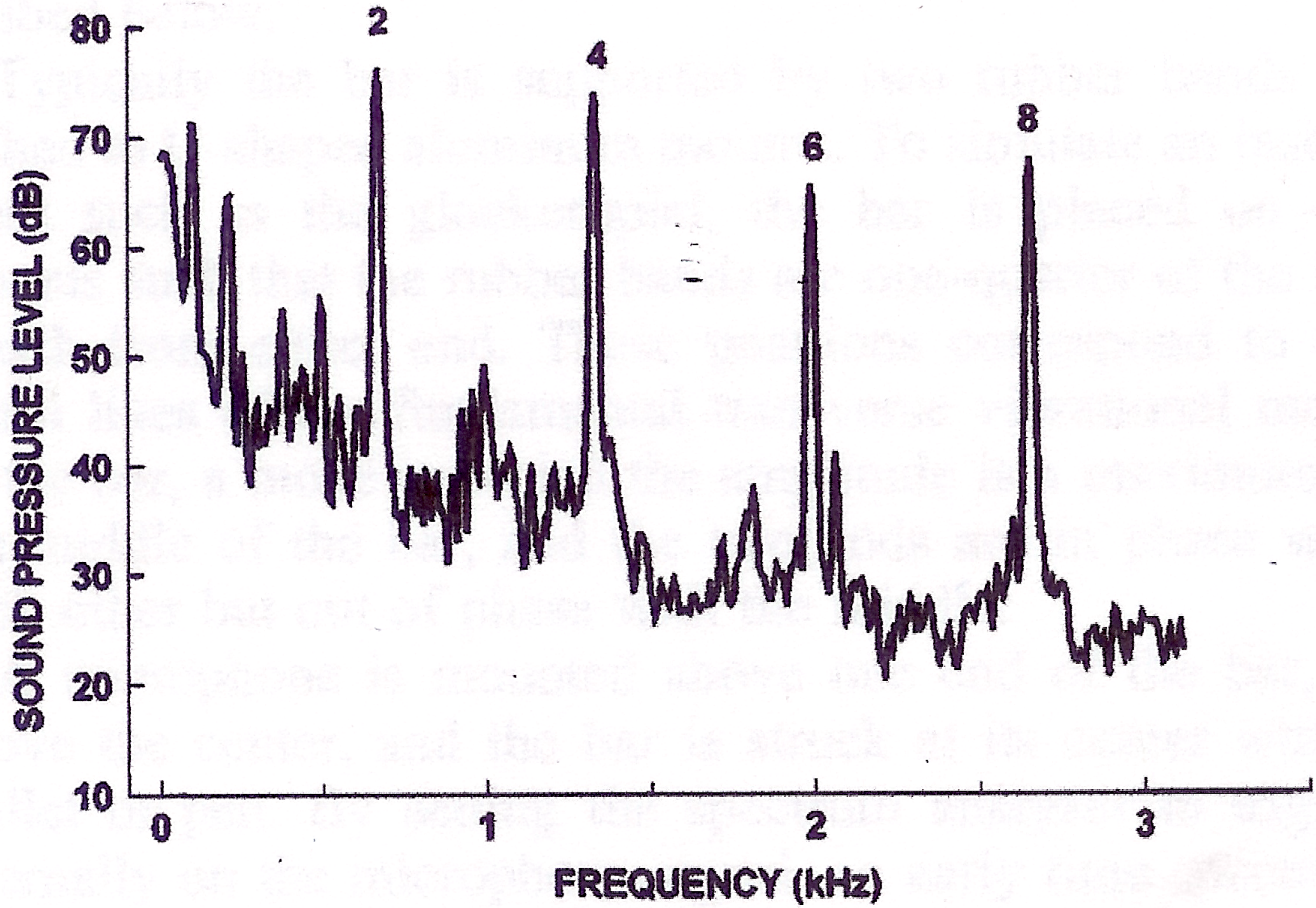
Example

- $freq_{mod}$ = 1 ---> 6 Hz ---> 261.6 Hz (= vib_{rate})
  (vib) (FM)
- Index = .02
  - (vib_{wid} = .02 * $freq_{mod}$)

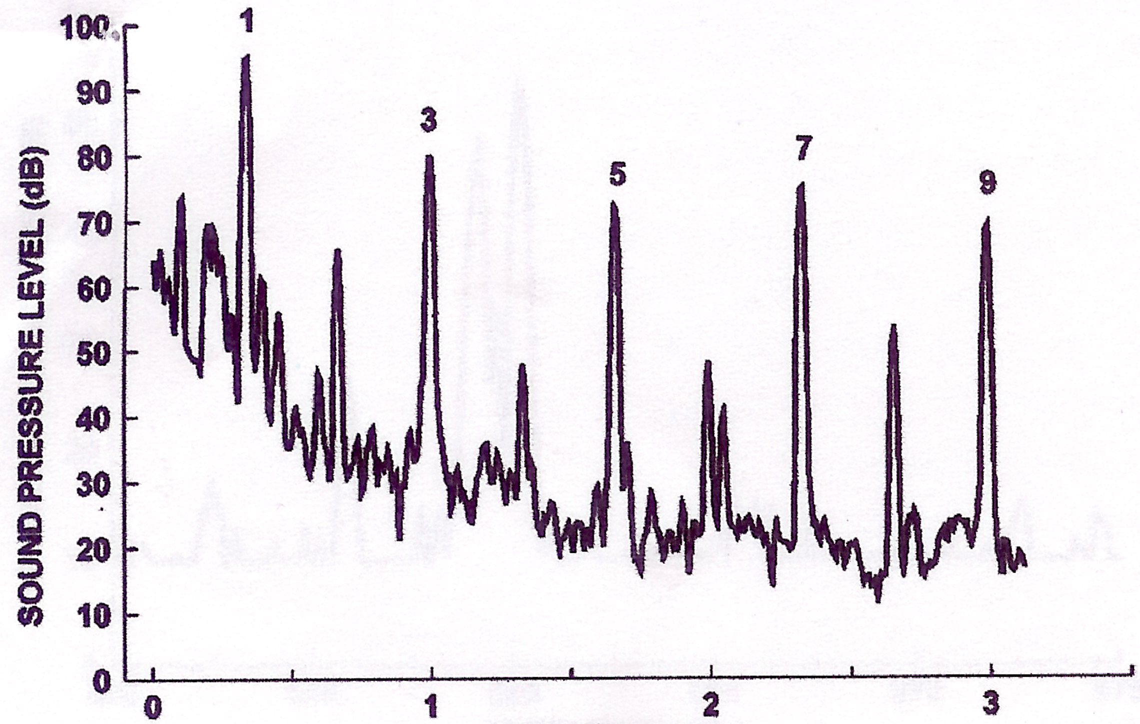
- When the frequency of the modulator reaches 6 Hertz (at 5 seconds), the effect changes from vibrato to FM.
- The frequency changes of all the harmonics get much faster during FM.



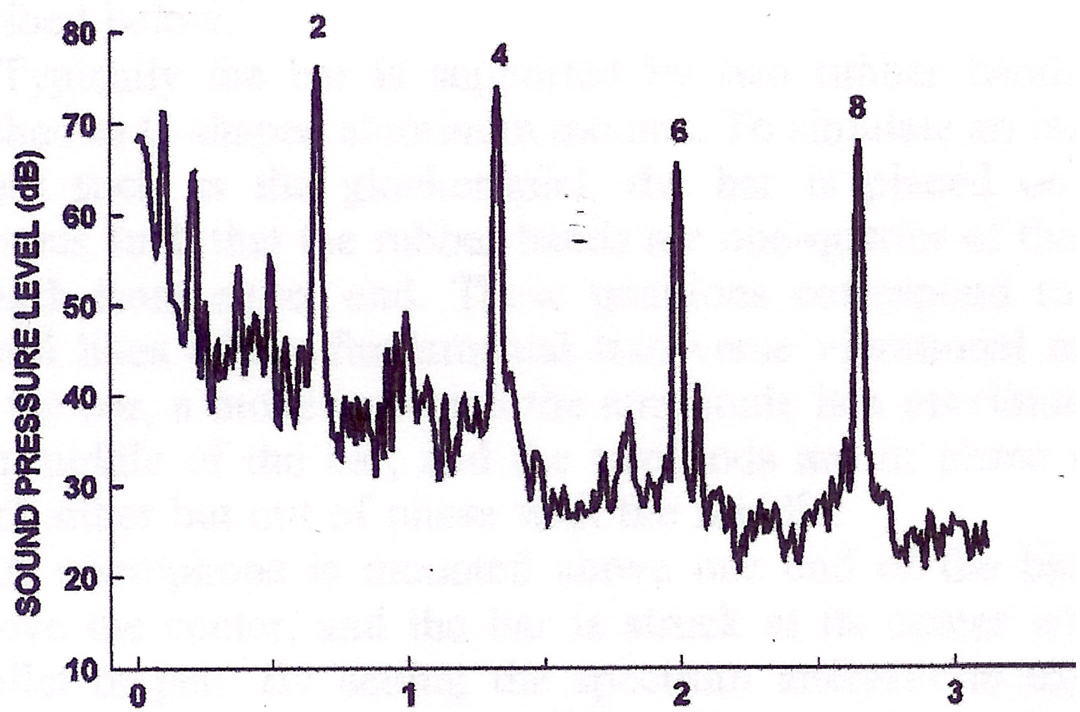
Guitar E_4 string plucked near its midpoint. The nearly symmetric disturbance created results in a spectrum of mostly odd harmonics.



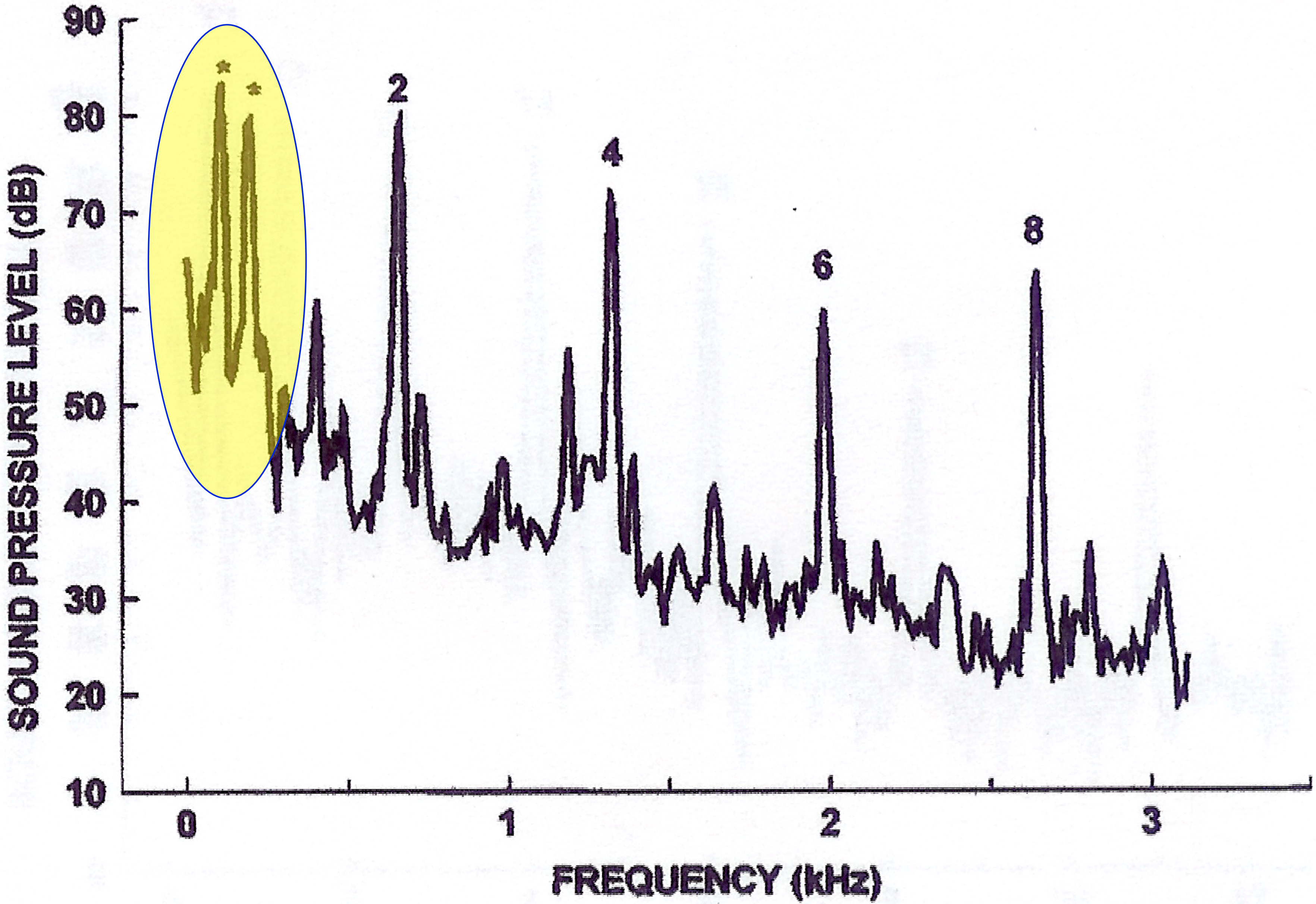
Guitar E_4 string struck while lightly touched at its midpoint, producing a spectrum of even harmonics only.



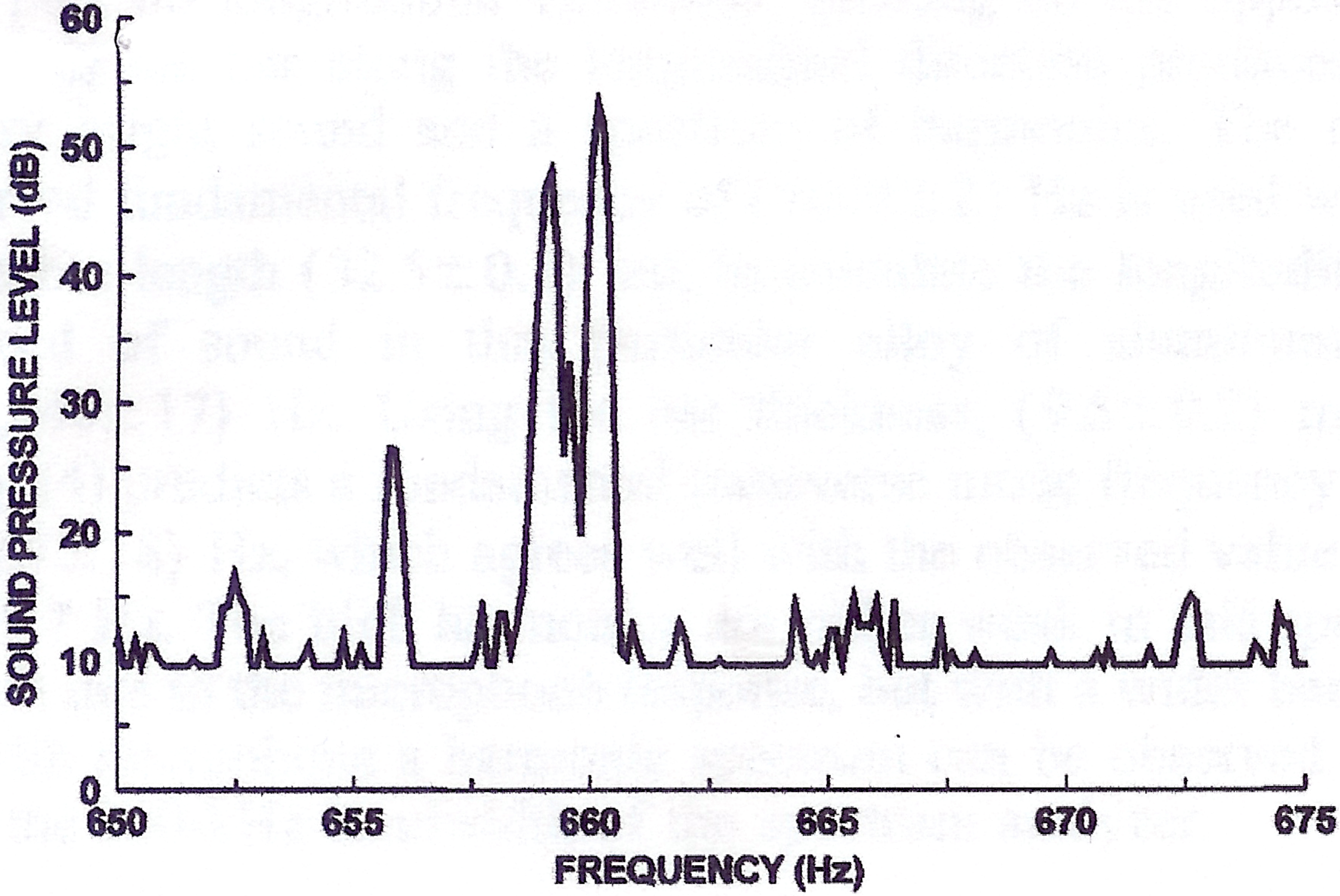
Plucked at Midpoint



Touched at Midpoint



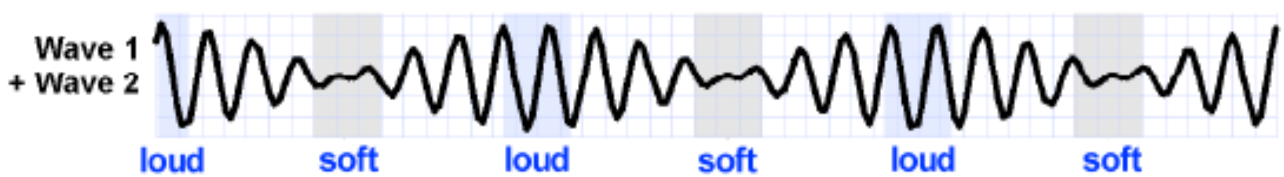
Guitar E_4 string struck and touched at the midpoint, but with the microphone placed over the soundhole, rather than behind the bridge.



High resolution spectrum of the second partial of E_4, with additional peaks from the harmonics of the E_2 and A_2 strings, caused by energy transfer through the bridge.

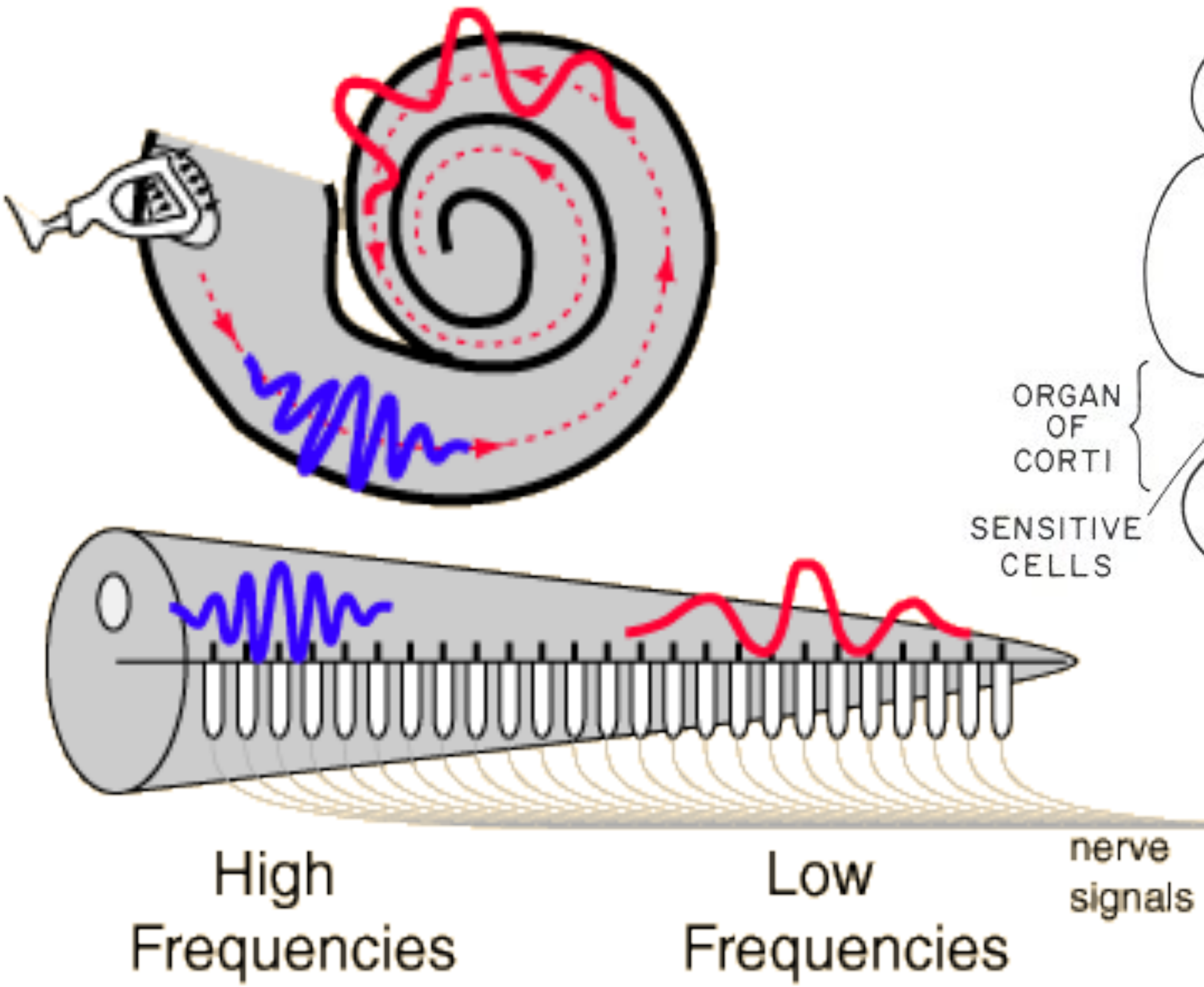
Consonance, dissonance, and beats

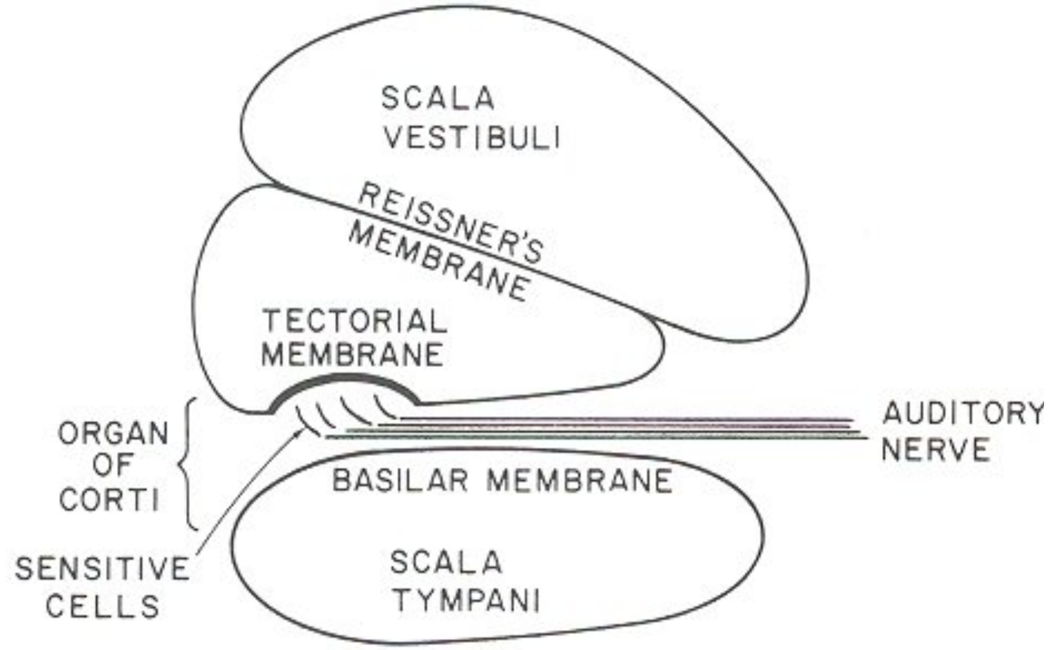
- Harmony is the study of how sounds work together to create effects desired by the composer.
- When we hear more than one frequency of sound and the combination sounds good, we call it consonance.
- When the combination sounds bad or unsettling, we call it dissonance.
- Consonance and dissonance are related to beats.
- When frequencies are far enough apart that there are no beats, we get consonance.
- When frequencies are too close together, we hear beats that are the cause of dissonance.
- Beats occur when two frequencies are close, but not exactly the same.



How we hear pitch

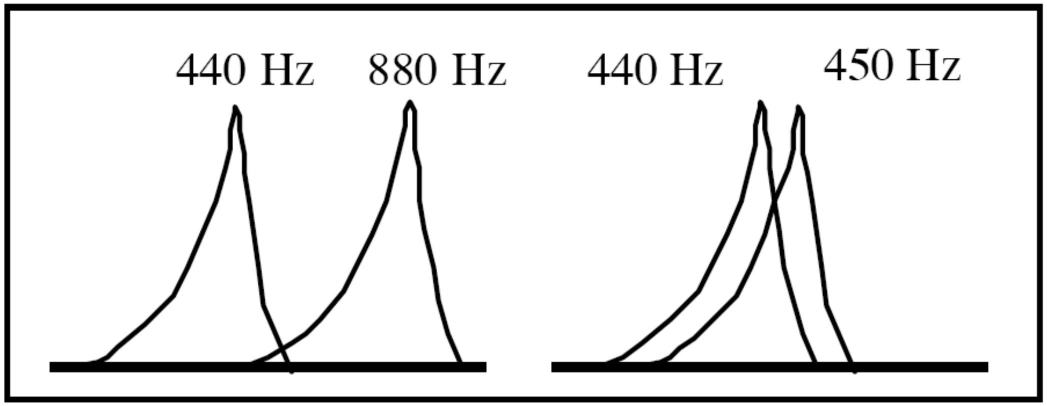
- High frequency sounds selectively vibrate the **basilar membrane** of the inner ear near the entrance port (the oval window).
- Lower frequencies travel further along the membrane before causing appreciable excitation of the membrane.





cross section of the cochlea

Consonance, dissonance, and roughness



1. Ear can't distinguish too close frequencies. An average frequency is heard, as well as the beats.

2. Lower frequency fixed and the higher raised slowly → frequencies were still indistinguishable, but the beat frequency too high to make out → a roughness to the total sound.

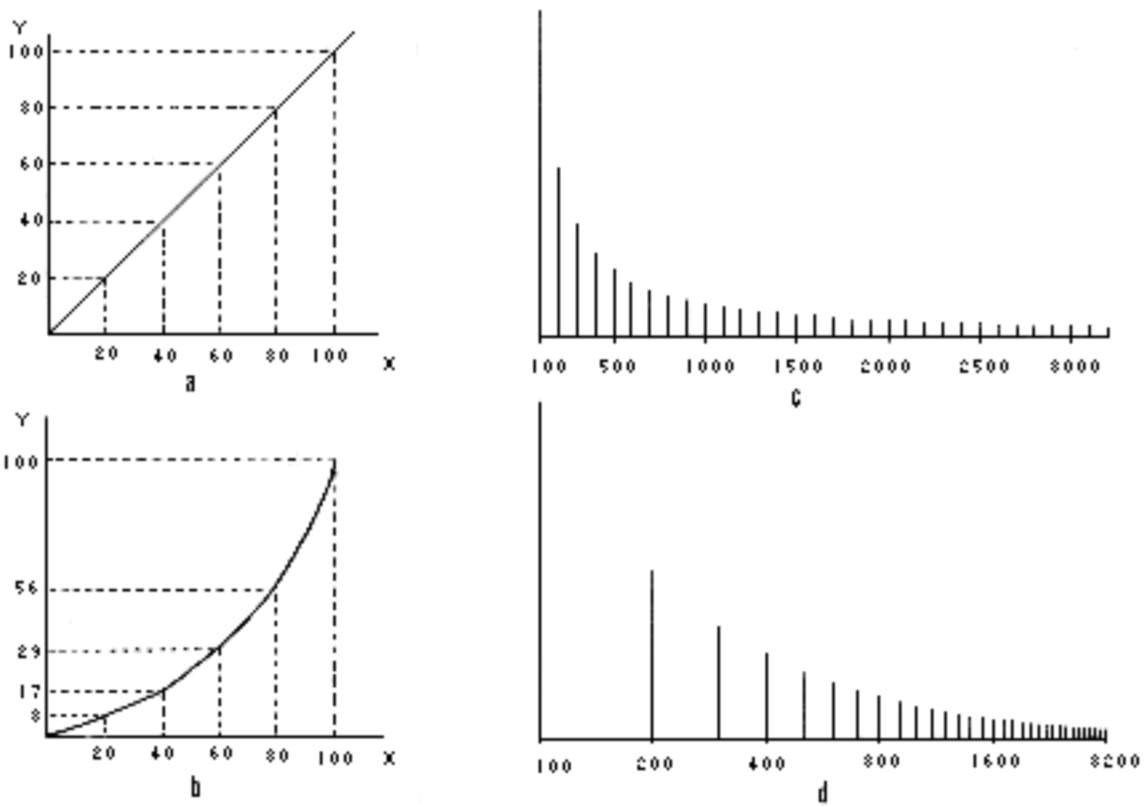
3. Dissonance continues until the higher frequency becomes distinguishable from the lower.

4. 1. and 2. are within the ***critical band***. Critical band around some "central frequency" will be stimulated by frequencies within about 15% of that central frequency.

Subjective tones

• The response of a system is **linear** when the output is directly proportional to the input, that is, any change in the input produces a proportional change in the output. When plotted on a graph, a straight line results.

•A **non-linear** system is one where such a proportional relationship between input and output does not hold, as shown in the corresponding graph.



The result of the non-linear characteristics of the ear is the addition of harmonics when the incoming sound is of sufficient intensity.

Subjective and Combination tones

- When two tones are perceived simultaneously, other tones often appear, because of **distortion effects** in the ear.

- Relatively high intensity levels are required for combination tones to be heard, and strong differences exist between individuals as to how many are heard.

- If two sine tones are played at a sufficient intensity level, one high (x) and the other low (y), the combination tones usually heard are the **difference tones**, at frequencies equal to (x - y), (2y - x) and (3y - 2x).

- The **summation tones** of frequencies (x + y) and (2x + y) are seldom if ever heard, even when in the audible range, possibly because of **masking** effects.

- The threshold of such tones varies greatly between individuals, since it depends on **non-linear** characteristics of the **inner ear**, but generally it lies between 50 and 60 dB.

- **Beats**, on the other hand, can be heard at low intensities.

- When a single tone is played loudly enough, additional harmonics will be heard that are not present in the original tone. These added tones, being frequency multiples of the original tone, are called **aural harmonics**.