Musical Acoustics

Lecture 2

Physical Quantities

International System of Units

Length	meter	[m]
Mass	kilogram	[kg]
Time	second	[s]
Electric current	ampere	[A]
Temperature	Kelvin	[K]
Amount of substance	mole	[mol]
Sound intensity	bel	[B]

Metric Prefixes (Big)

• 10^{24}	yotta	Y
• 10^{21}	zetta	Z
• 10^{18}	exa	E
• 10^{15}	peta	P
• 10^{12}	tera	T
• 10^{9}	giga	G
• 10^{6}	mega	M
• 10^{3}	kilo	k
• 10^{2}	hecto	h
• 10^{1}	deka	da

Metric Prefixes (Small)

• 10^{-1}	deci	d
• 10^{-2}	centi	c
• 10^{-3}	milli	m
• 10^{-6}	micro	μ
• 10^{-9}	nano	n
• 10^{-12}	pico	p
• 10^{-15}	femto	f
• 10^{-18}	atto	a
• 10^{-21}	zepto	z
• 10^{-24}	yocto	y

Meter is the Unit of Length

- **The meter is the length of the path traveled by light in vacuum during a time interval of 1/299 792 458 of a second.**

- The meter was intended to equal 10^{-7} or one ten-millionth of the length of the meridian through Paris from pole to the equator.

- The first prototype was short by 0.2 millimeters because researchers miscalculated the flattening of the earth due to its rotation.

- Platinum-iridium bar was replaced as a unit to this length.

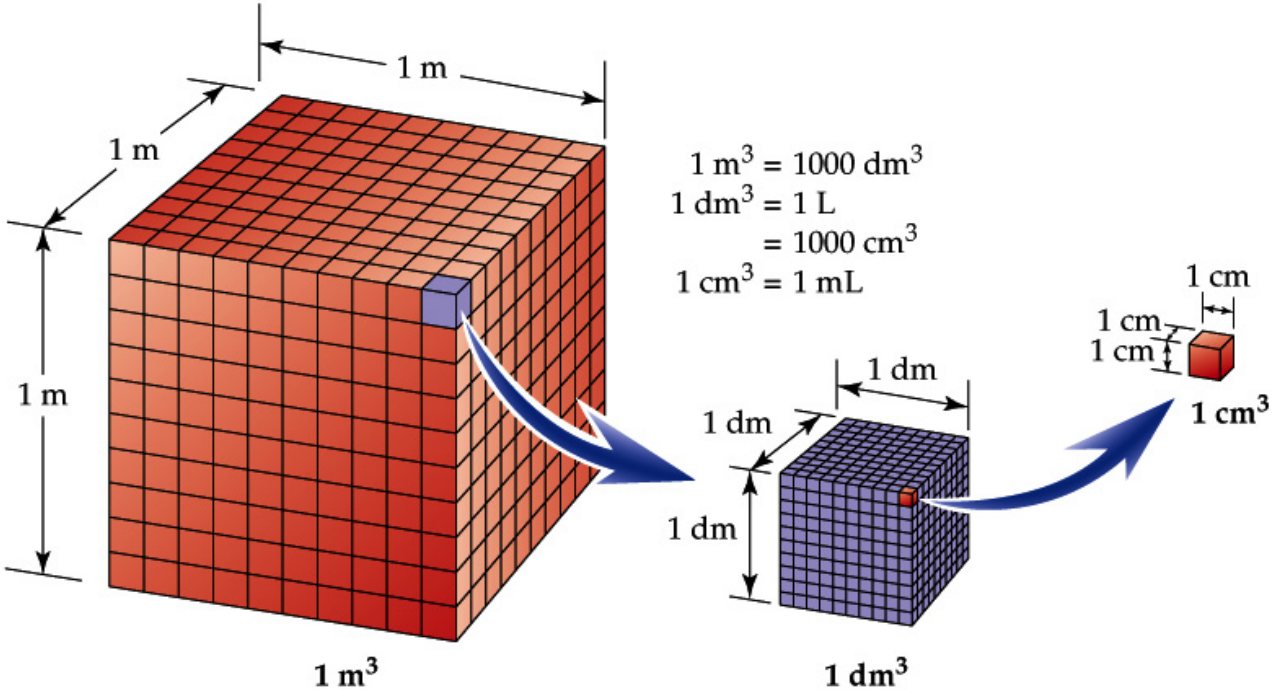
Kilogram is the Unit of Mass

- **A kilogram is equal to the mass of the international prototype of the kilogram.**

- At the end of the 18th century, a kilogram was the mass of a cubic decimeter of water. In 1889, scientists made the international prototype of the kilogram out of platinum-iridium.

Liter is a Volume Unit

- A liter (abbreviated either l or L) is equal to 1 $dm^3 = 10^{-3}\ m^3$



Volume Units

Time Units

- Minute min 1 min = 60 s
- Hour h 1 h = 60 min = 3600 s
- Day d 1 d = 24 h = 86,400 s

- Second can be abbreviated " (a double tick).
- Minute can be abbreviated ´ (a single tick).

Temperature

- The **Kelvin**, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water (i.e. when water, ice and vapor coexist).

- Temperature T, is commonly defined in terms of its difference from the reference temperature $T_0 = 273.15$ K, the ice point.

- This temperature difference is called a **Celsius** temperature, symbol t, and is defined by the quantity equation

$$t = T - T_0.$$

Mole is the Unit of Amount of Substance

- A **mole** is the amount of substance of a system which contains as many elementary entities as there are atoms in 12 gram of carbon 12. Moles of other substances are obtained by comparing to this amount of carbon.

- **"Avogadro's Number"** is an honorary name attached to the calculated value of the number of atoms, molecules, etc. in a gram molecule of any chemical substance.

- 12 grams of pure carbon, whose molecular weight is 12, will contain 6.023×10^{23} molecules.

Conversion Table

METRIC TO ENGLISH			ENGLISH TO METRIC		
From Metric	To English	Multiply by	From English	To Metric	Multiply by
meters	yards	1.09	yards	meters	0.91
meters	feet	3.28	feet	meters	0.30
centimeters	inches	0.39	inches	centimeters	2.54
kilometers	miles	0.62	miles	kilometers	1.61
grams	ounces	0.035	ounces	grams	28.35
kilograms	pounds	2.20	pounds	kilograms	0.45
liters	quarts	1.06	quarts	liters	0.95
liters	gallons	0.26	gallons	liters	3.78

Example: Use of the Prefixes for Mass

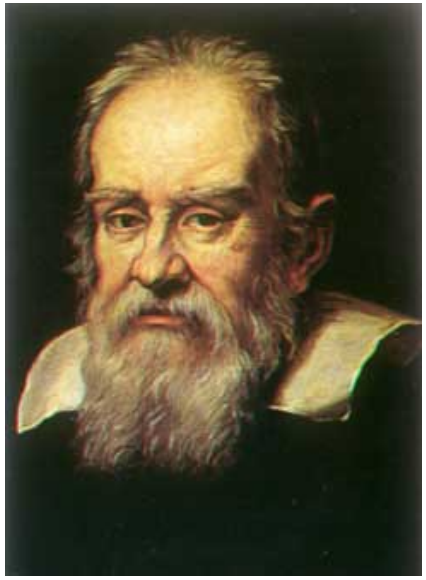
• Kilogram	kg	10^3 g
• Gram	g	1 g
• Milligram	mg	10^{-3} g
• Microgram	μg	10^{-6} g
• Nanogram	ng	10^{-9} g
• Picogram	pg	10^{-12} g
• Femtogram	fg	10^{-15} g

Density

$$\rho = \frac{M}{V} \quad (kg/m^3)$$

Substance	ρ(kg/m^3)	Substance	ρ(kg/m^3)
Ice	0.917×10^3	Water	1.00×10^3
Aluminum	2.70×10^3	Glycerin	1.26×10^3
Iron	7.86×10^3	Ethyl alcohol	0.806×10^3
Copper	8.92×10^3	Benzene	0.879×10^3
Silver	10.5×10^3	Mercury	13.6×10^3
Lead	11.3×10^3	Air	1.29
Gold	19.3×10^3	Oxygen	1.43
Platinum	21.4×10^3	Hydrogen	8.99×10^{-2}
Uranium	18.7×10^3	Helium	1.79×10^{-1}

Galileo Galilei



1581	Constancy of period of pendulum
1589	Showed that objects fall at the same rate independent of mass
1592	Suggests that physical laws of the heavens are the same as those on Earth
1592	Primitive thermometer
1600	**Study of sound and vibrating strings**
1604	distance for falling object increases as square of time
1609	builds a telescope
1610	Observes the phases of Venus
1610	Observes moons of Jupiter
1610	Observes craters on the moon
1610	Observes stars in the Milky Way
1610	Observes structures around Saturn
1612	Hydrostatics
1613	Principle of inertia
1624	Theory of tides
1632	Galilean relativity
1632	Support for Copernicus' heliocentric theory
1638	Motion and friction

Then he died in house arrest due to religious intolerance of his time (he dared to claim that the Earth was not the center of the Universe).

Galileo's father

- Vincenzo Galilei was born in Florence. He made his living as a lutenist, composer, theorist, singer, and teacher.
- Published a number of books of musical scores for the lute and several books on musical theory.

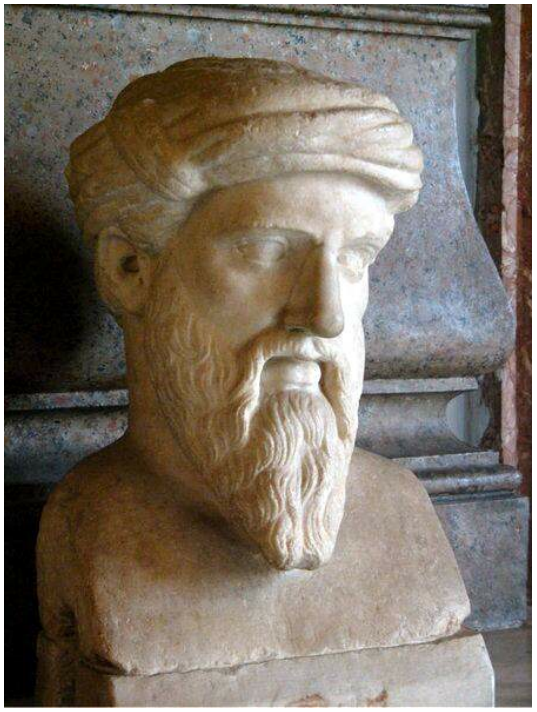
- He combined the practice and theory of music. Since antiquity, the theory of music had consisted of a mathematical discussion of harmony, in other words what are the mathematical ratios of the lengths of strings producing consonances, and how does one divide the octave?

- It had always been thought that not only was the ratio of lengths of two strings sounding an octave 2:1, but that so also was the ratio of the tensions of strings of equal lengths tuned an octave apart.
- Galilei showed that this is not the case: the ratio of tensions is 4:1. He found that ratio by hanging weights from strings.

- Galileo probably helped with these experiments.

APPENDIX – Trigonometry

Pythagoras (570 BC - 495 BC)



Bust of Pythagoras of Samos in the Capitoline Museums, Rome

Pythagoras, depicted on a 3rd-century coin

Mathematician, Philosopher, Inventor, etc.

According to legend, the way Pythagoras discovered that **musical notes** could be translated into mathematical equations was when one day he passed blacksmiths at work, and thought that the sounds emanating from their anvils being hit were beautiful and harmonious and decided that whatever scientific law caused this to happen must be mathematical and could be applied to music. He went to the blacksmiths to learn how this had happened by looking at their tools, he discovered that it was because the hammers were "simple ratios of each other, one was **half** the size of the first, another was **2/3** the size, **3/4** of the size, and so on."

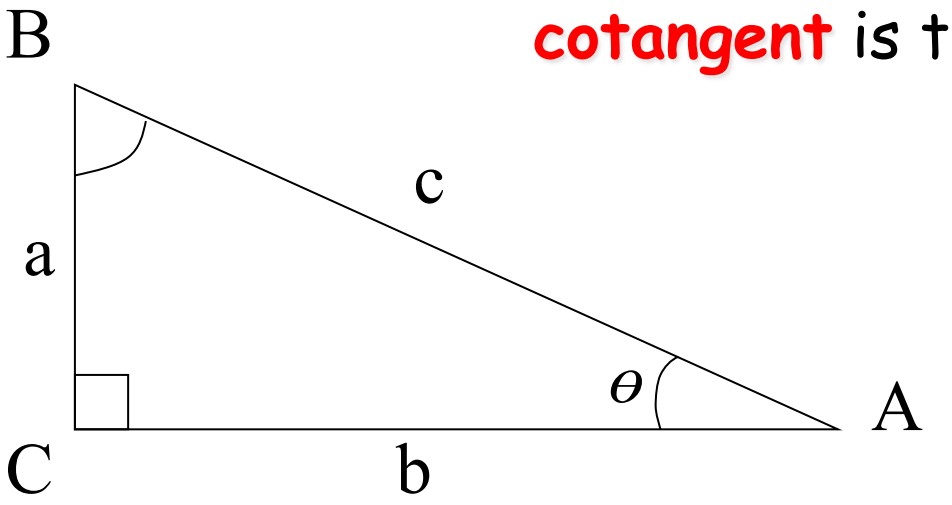
FROM WIKIPEDIA

APPENDIX - Trigonometry

Sines, Cosines, Tangents

A **trigonometric function** is a ratio of certain parts of a right triangle. The names of these ratios are: The **sine, cosine, tangent, cosecant, secant, cotangent**.

The **Cosecant** is the inversion of the sine, the **secant** is the inversion of the cosine, the **cotangent** is the inversion of the tangent.



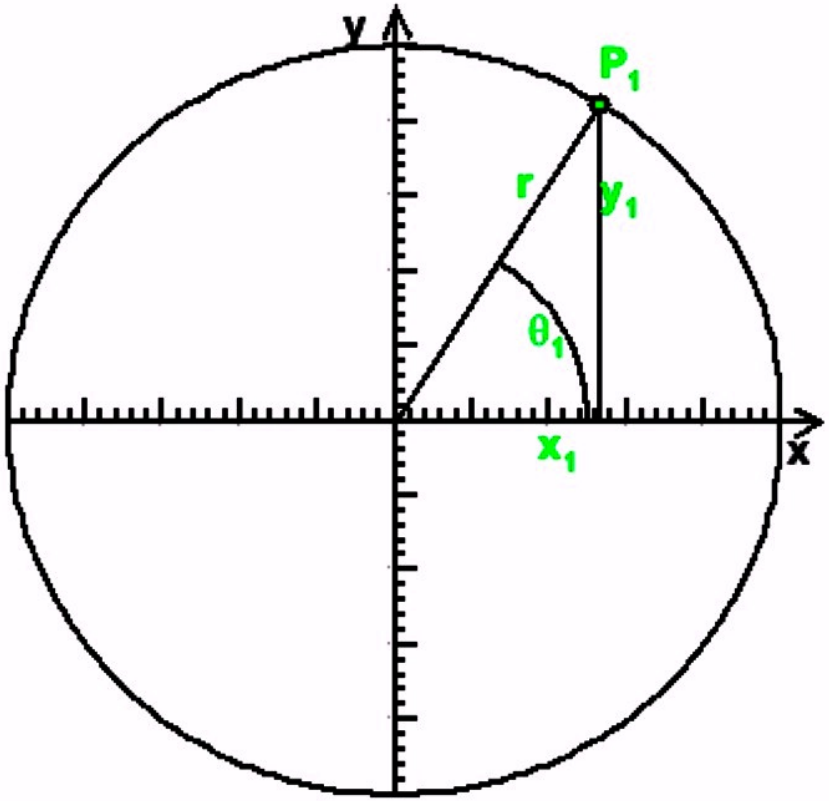
$$Sin\theta = \frac{\text{Side Opposite}}{\text{Hypothenuse}} = \frac{a}{c}$$

$$Cos\ \theta = \frac{\text{Side Adjacent}}{\text{Hypothenuse}} = \frac{b}{c}$$

$$Tan\ \theta = \frac{\text{Side Opposite}}{\text{Side Adjacent}} = \frac{a}{b}$$

Trigonometry and Circles

- The point $P_1=(x_1,y_1)$ lies on a circle of radius r.
- The line from the origin to P_1 makes an angle θ_1 w.r.t. the x-axis.
- The trigonometric functions sine and cosine are defined by the x- and y-components of P_1:
  - $x_1 = r\cos(\theta_1)$: $\cos(\theta_1) = x_1 / r$
  - $y_1 = r\sin(\theta_1)$: $\sin(\theta_1) = y_1 / r$
  - Tangent of $(\theta_1) = y_1 / x_1$
  - $\tan(\theta_1) = [\sin(\theta_1)] / [\cos(\theta_1)]$

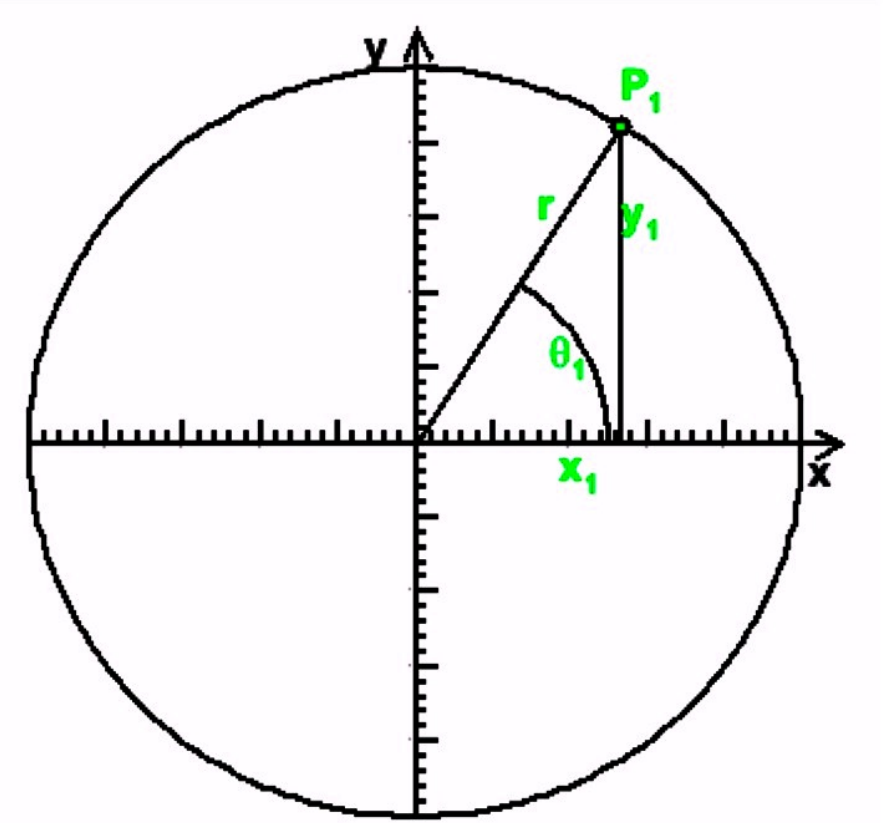


In this discussion, we always define the direction of a vector in terms of an angle counter-clockwise from the + x-axis.
Negative angles are measured clockwise.

Examples

- $\cos(0°) = 1$, $\sin(0°) = 0$
- $\cos(90°) = 0$, $\sin(90°) = 1$
- $\cos(180°) = -1$, $\sin(180°) = 0$
- $\cos(270°) = 0$, $\sin(270°) = -1$

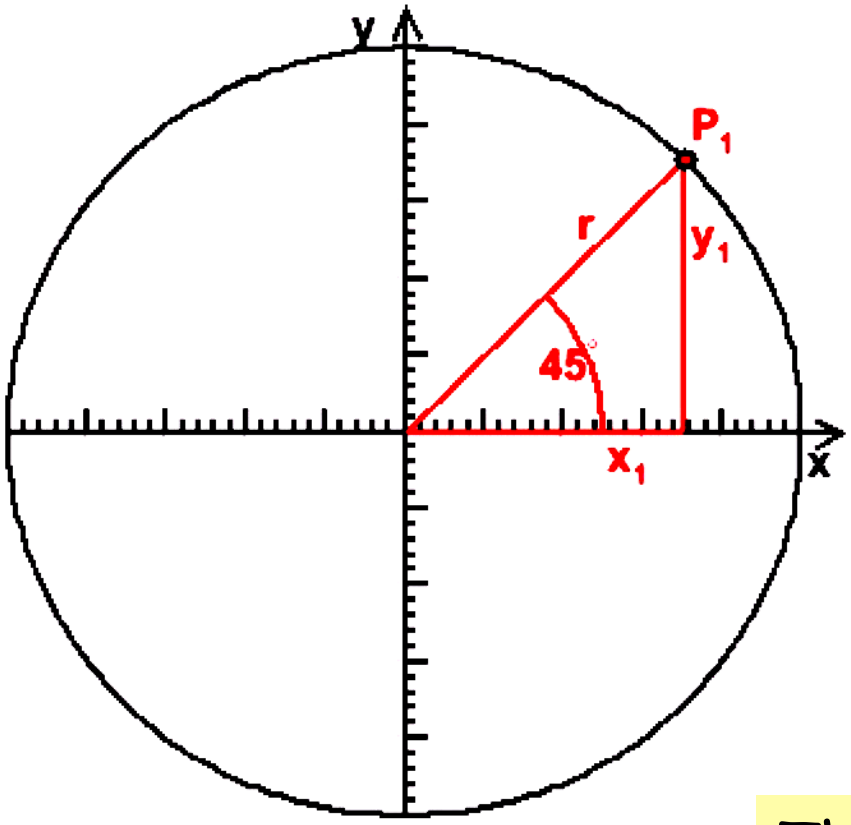
- Sine and Cosine are periodic functions:
  - $\cos(\theta+360^o) = \cos(\theta)$
  - $\sin(\theta+360°) = \sin(\theta)$



More examples

- By symmetry,
  $x_1 = y_1$

- Pythagoras:
  $x_1^2 + y_1^2 = r^2$
  $2 \cdot x_1^2 = r^2$
  $x_1 = r/\sqrt{2}$



- $\cos(45°) = x_1 / r = 1/\sqrt{2}$
- $\cos(45°) = 0.7071\ldots$
- $\sin(45°) = 1/\sqrt{2}$

The first trigonometric table was apparently compiled by **Hipparchus**, (190 BC-120 BC) who is now consequently known as **"the father of trigonometry"**.

Degrees and radians

Degrees and pi radians are two methods of showing trigonometric info. To convert between them, use the following equation.

2π radians = 360 degrees

1π radians= 180 degrees

Convert 500 degrees into radians.

2π radians = 360 degrees, 1 degree = 1π radians/180,

500 degrees = π radians/180 * 500

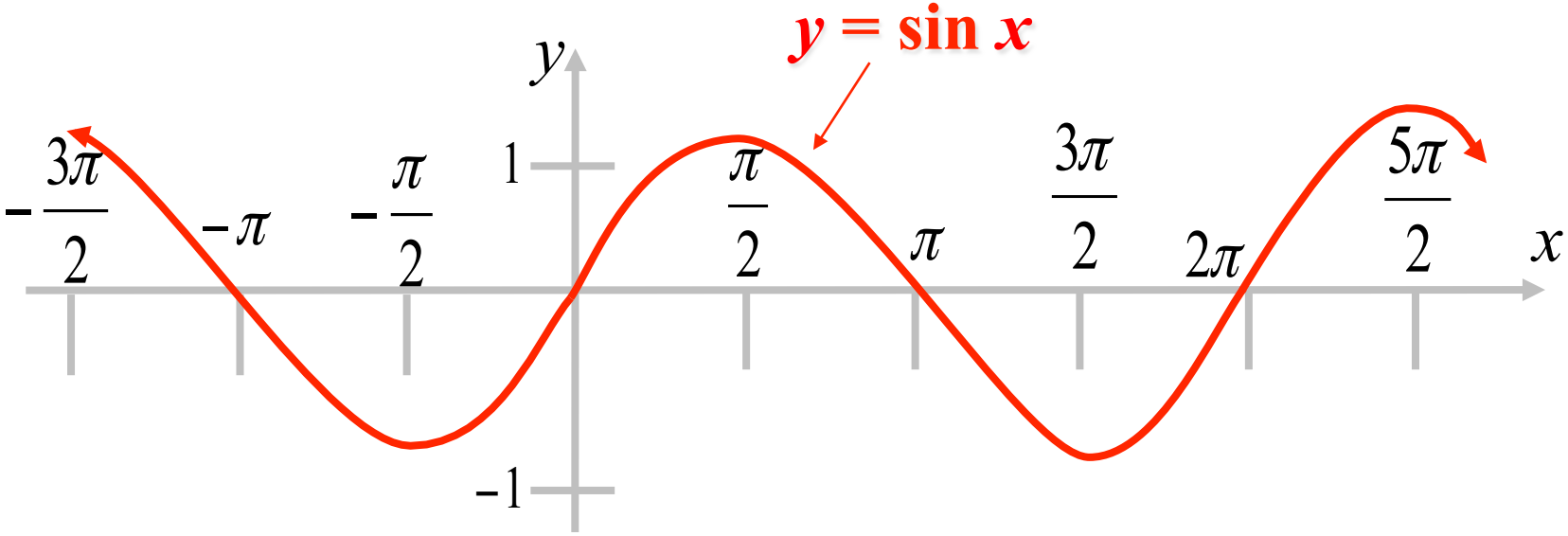
500 degrees = 25π radians/9

Graph of the Sine Function

To sketch the graph of **y = sin x** first locate the key points. These are the maximum points, the minimum points, and the intercepts.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	1	0	-1	0

Then, connect the points on the graph with a smooth curve that extends in both directions beyond the five points. A single cycle is called a **period**.



Graph of the Cosine Function

To sketch the graph of **y = cos x** first locate the key points. These are the maximum points, the minimum points, and the intercepts.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos x$	1	0	-1	0	1

Then, connect the points on the graph with a smooth curve that extends in both directions beyond the five points. A single cycle is called a **period**.