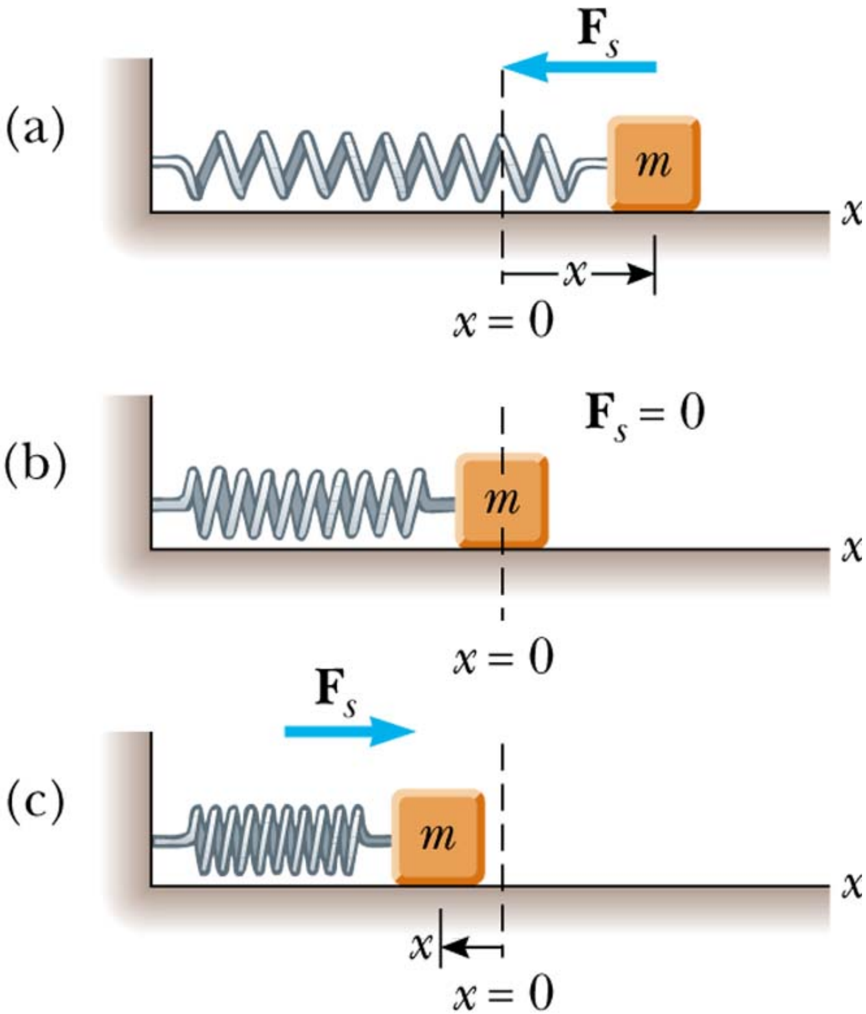
Musical Acoustics

Lecture 4

Simple vibrating systems I

Hooke's law

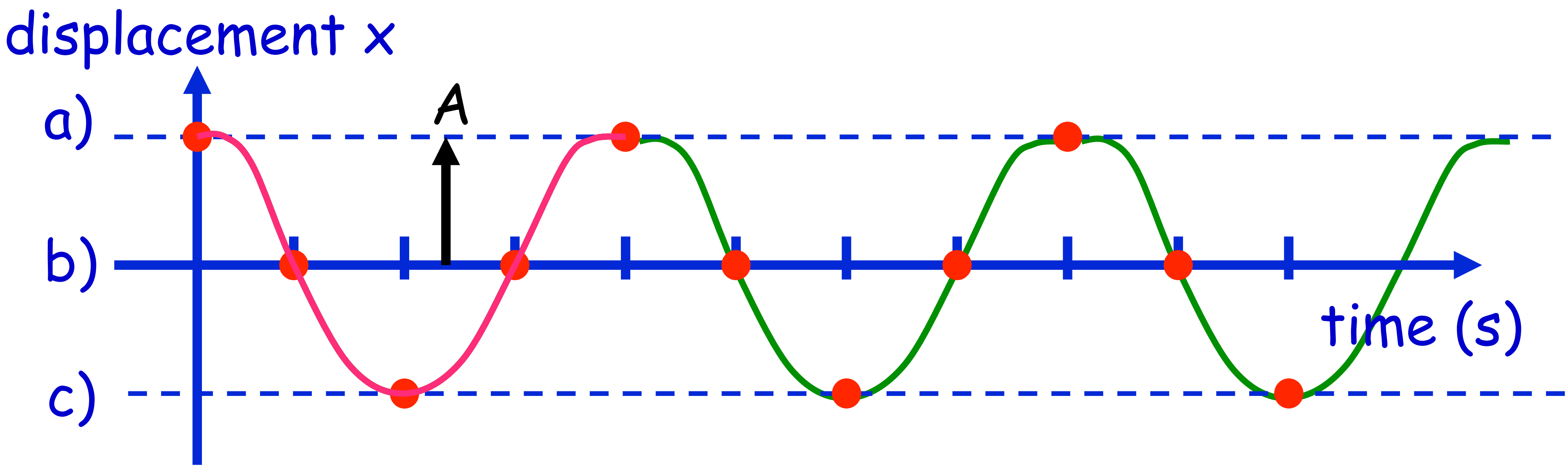


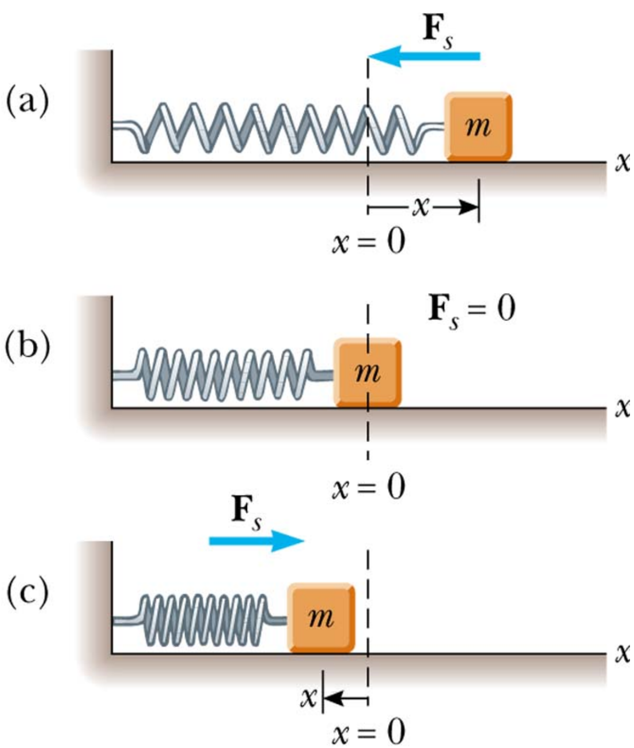
$F_s = -kx$ Hooke's law

If there is no friction, the mass continues to oscillate back and forth.

If a force is proportional to the displacement x, but opposite in direction, the resulting motion of the object is called: simple harmonic oscillation

Simple harmonic motion





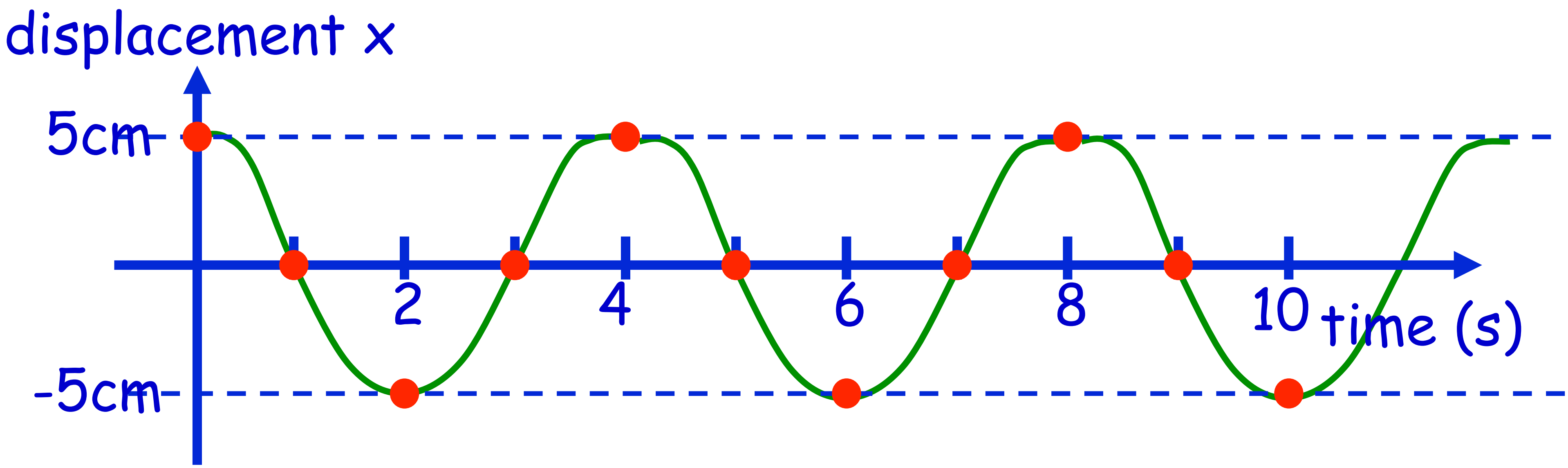
Amplitude (A): maximum distance from equilibrium (unit: m)

Period (T): Time to complete one full oscillation (unit: s)

Frequency (f): Number of completed oscillations per second (unit: 1/s = 1 Herz [Hz])

$f=1/T$

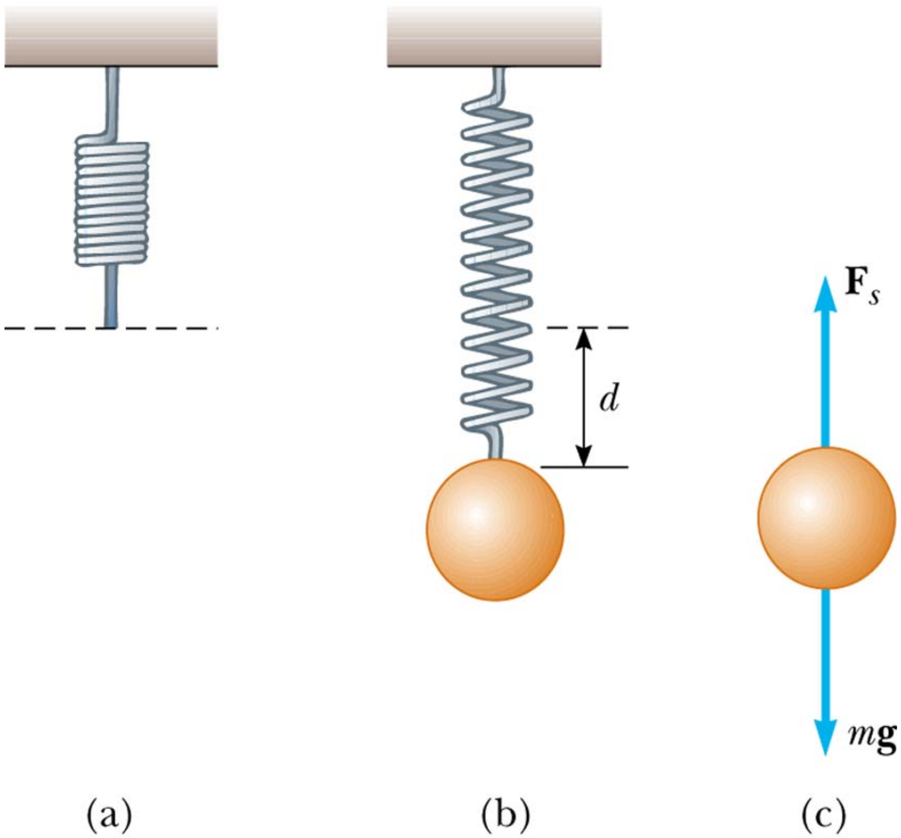
Example: Simple harmonic motion



a) what is the amplitude of the harmonic oscillation?
b) what is the period of the harmonic oscillation?
c) what is the frequency of the harmonic oscillation?

a) Amplitude: 5 cm (0.05 m)
b) period: time to complete one full oscillation: 4s
c) frequency: number of oscillations per second
   = 1/T = 0.25 Hz

The spring constant k



When the object hanging from the spring is not moving:

$F_{spring} = -F_{gravity}$

$-kd = -mg$

$k = mg/d$

Example: Find k and a

A mass of 1 kg is hung from a spring. The spring stretches by 0.5 m. Next, the spring is placed horizontally and fixed on one side to the wall. The same mass is attached and the spring stretched by 0.2 m and then released. What is the acceleration upon release?

1st step: find the spring constant k

$F_{spring} = -F_{gravity}$ or $-kd = -mg$

$k = mg/d = 1*9.8/0.5 = 19.6$ N/m

2nd step: find the acceleration upon release

Newton's second law: $F=ma \Rightarrow -kx=ma \Rightarrow a=-kx/m$

$a=-19.6*0.2/1=-3.92\ m/s^2$

Mechanical energy (ME)

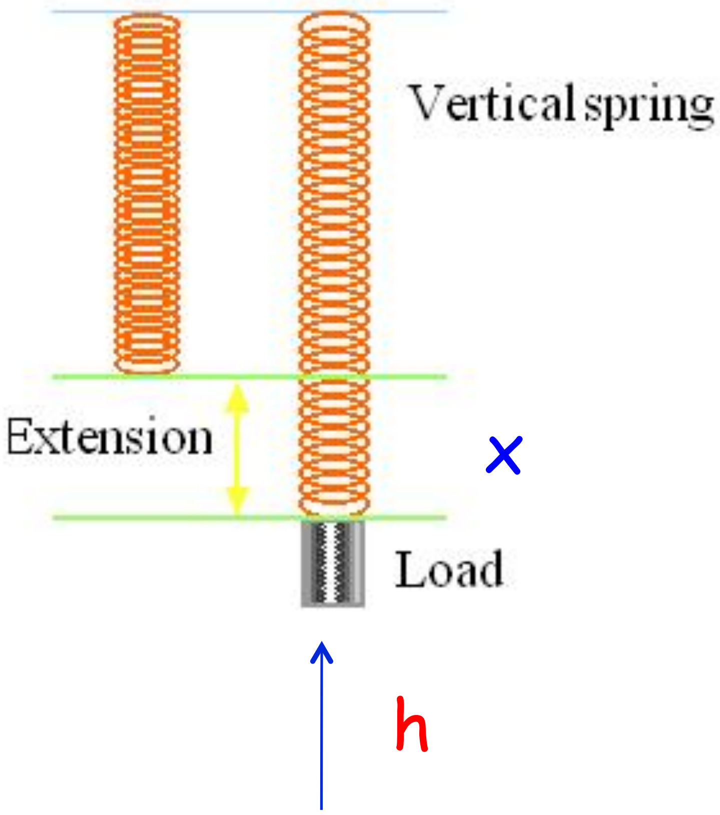
$$ME = KE + PE_{spring} + PE_{gravity}$$

$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 + mgh$$

kinetic energy

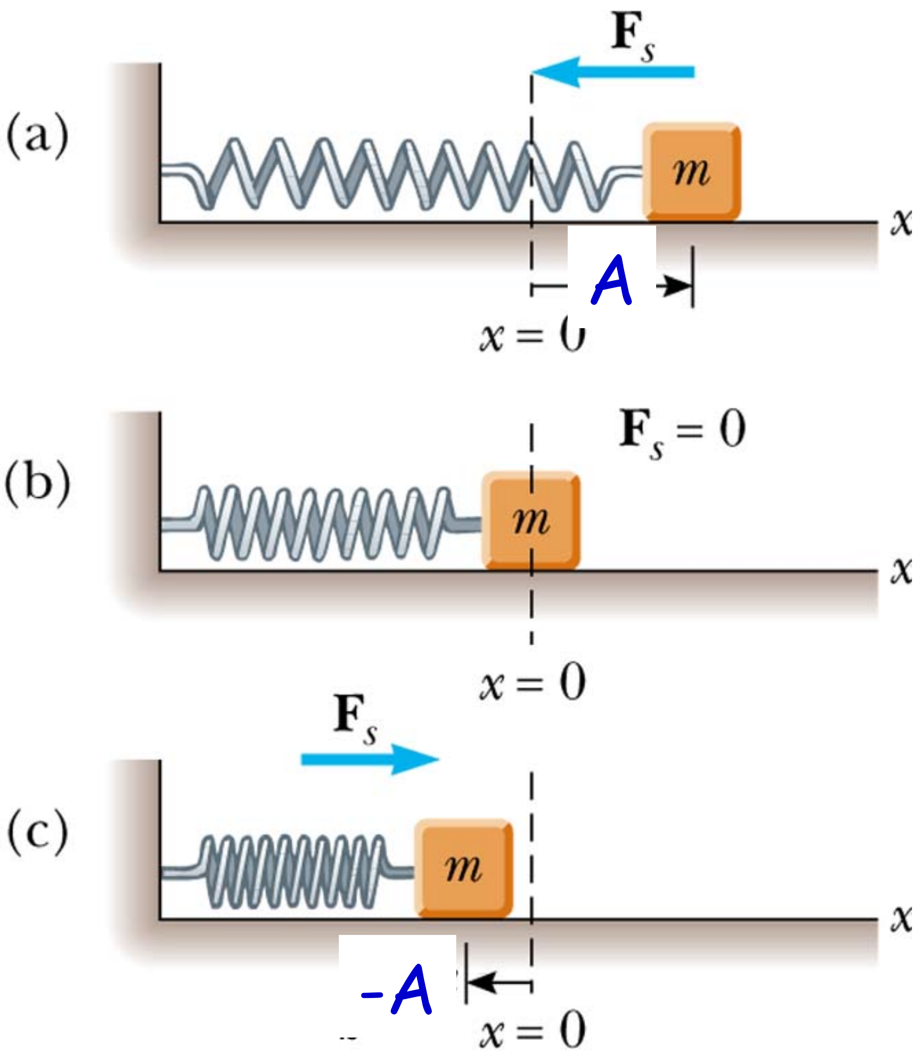
energy stored in spring

energy stored in gravity



The total **mechanical energy** is **conserved.**

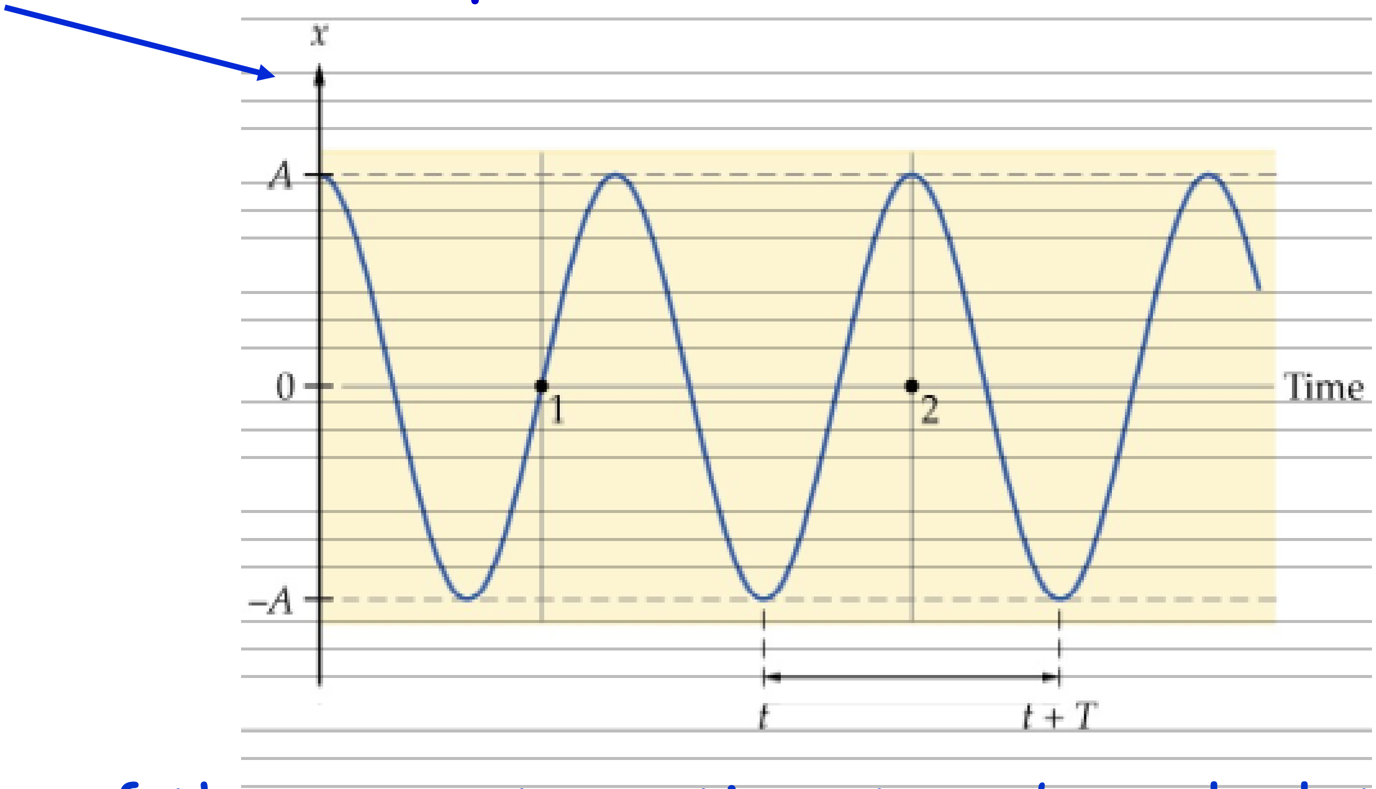
Energy and velocity (no gravity)



$E_{kin}(\frac{1}{2}mv^2)$	$E_{pot,spring}(\frac{1}{2}kx^2)$	Sum
0	$\frac{1}{2}kA^2$	$\frac{1}{2}kA^2$
$\frac{1}{2}mv^2$	0	$\frac{1}{2}mv^2$
0	$\frac{1}{2}k(-A)^2$	$\frac{1}{2}kA^2$

Simple Harmonic Motion

The amplitude (A) of the motion is the maximum **displacement** from equilibrium



The position of the mass at any time t can be calculated from

$$x = A\cos\left(\frac{2\pi}{T}t\right)$$

assuming that x is at its maximum positive value at $t = 0$.

Frequency AND Angular Frequency

Angular velocity is defined by a relation to the period of harmonic motion:

$$\omega = \frac{2\pi}{T}$$

ω is also related to the **frequency** by

$$\omega = 2\pi f = \frac{2\pi}{T}$$

We can rewrite the formula for the position of the mass on the spring as

$x(t) = A\cos(\omega t)$ if $x(0) = A$

The Period of a Mass on a Spring

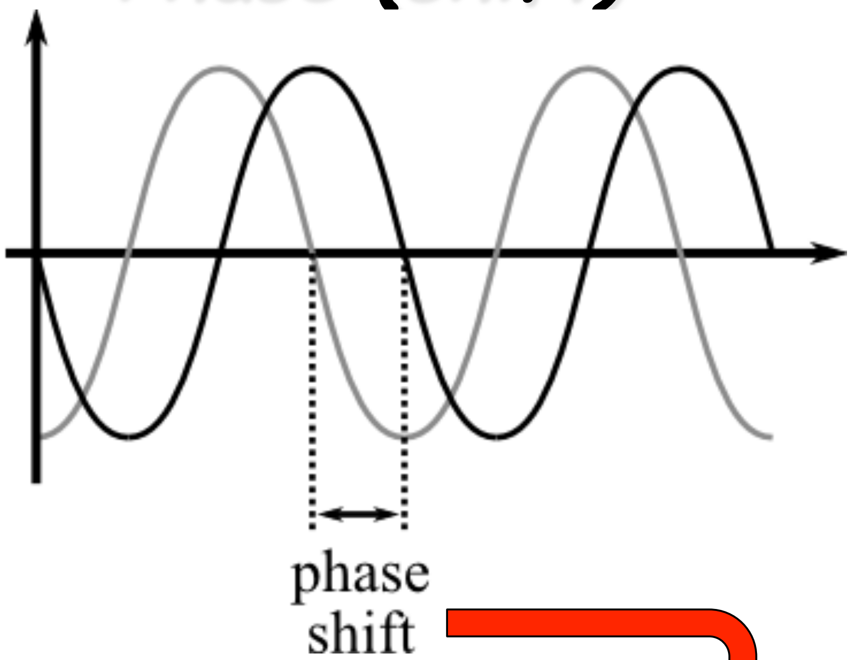
The angular frequency for the simple harmonic motion of a mass on a spring is

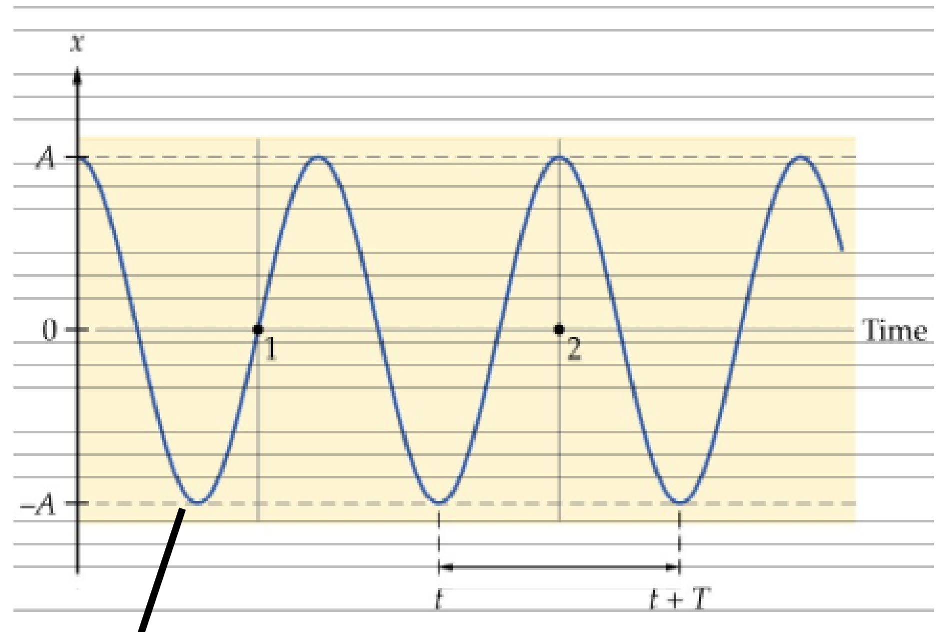
$$\omega = \sqrt{\frac{k}{m}}$$

Therefore, the period for the oscillations of a mass on a spring is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

Phase (shift)





- $x(t) = -A\sin(\omega t+\theta_0)$ $x(t) = -A\sin(\omega t+\theta_0)$
- $\omega t+\theta_0$ = phase of oscillation (not an angle of rotation).
- $\theta_0 = 0$ if position $x(t=0)$ is the maximum of the oscillation.