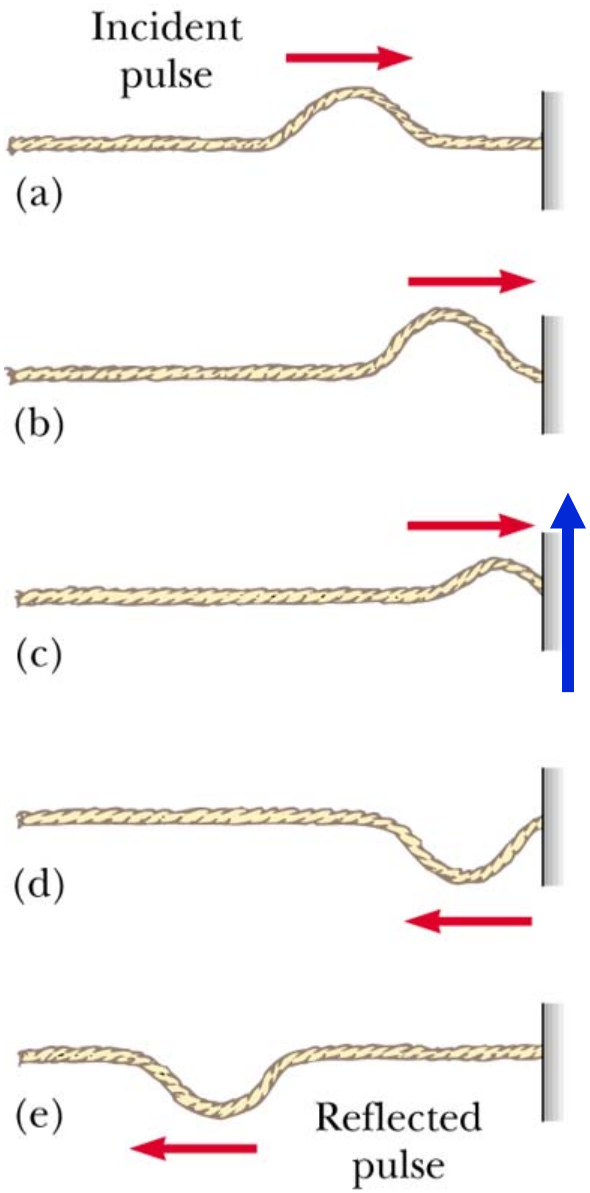
Musical Acoustics

Lecture 9

Harmonics in strings, pipes and drums - 1

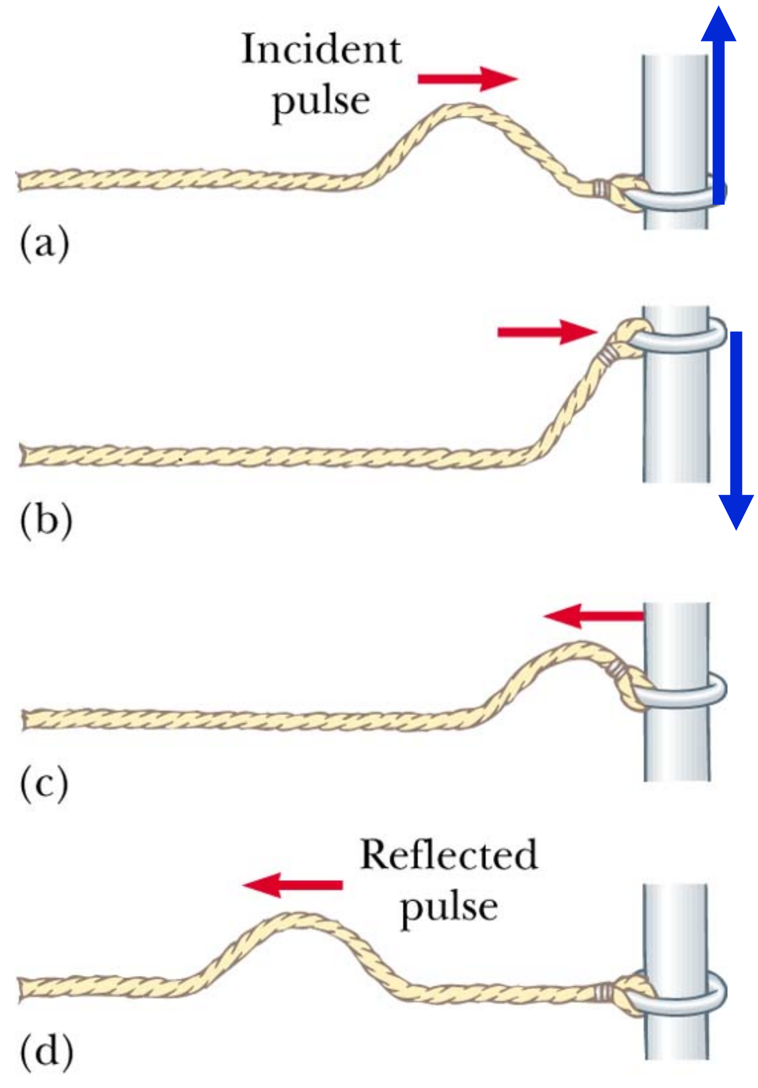
Strings

Reflection of waves



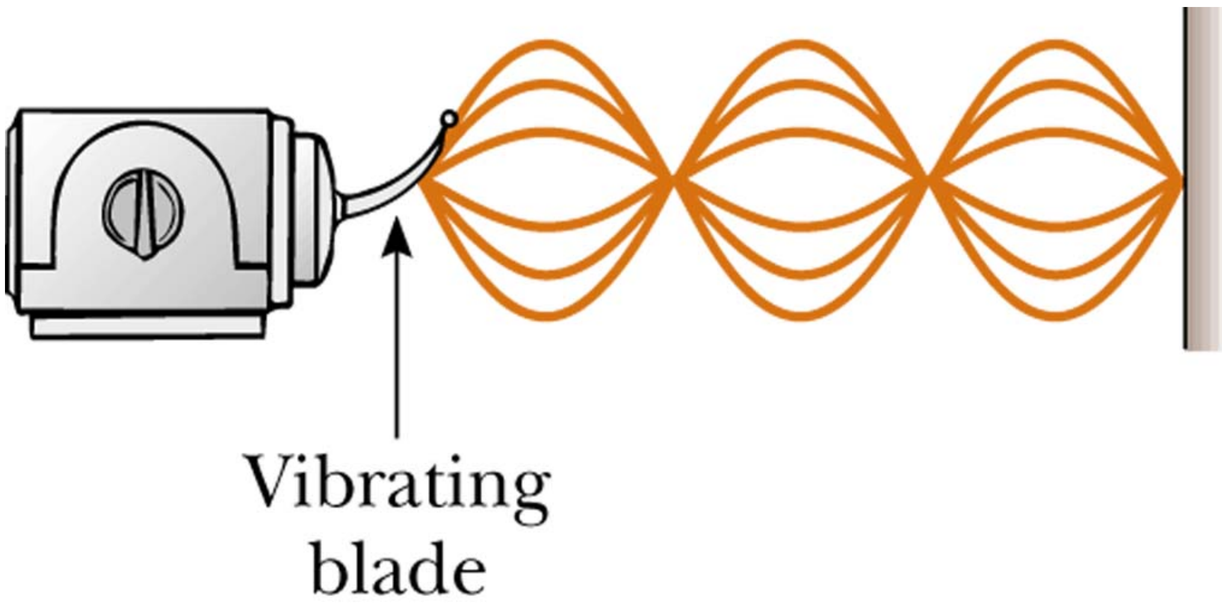
$F_{\text{rope on wall}} = -F_{\text{wall on rope}}$

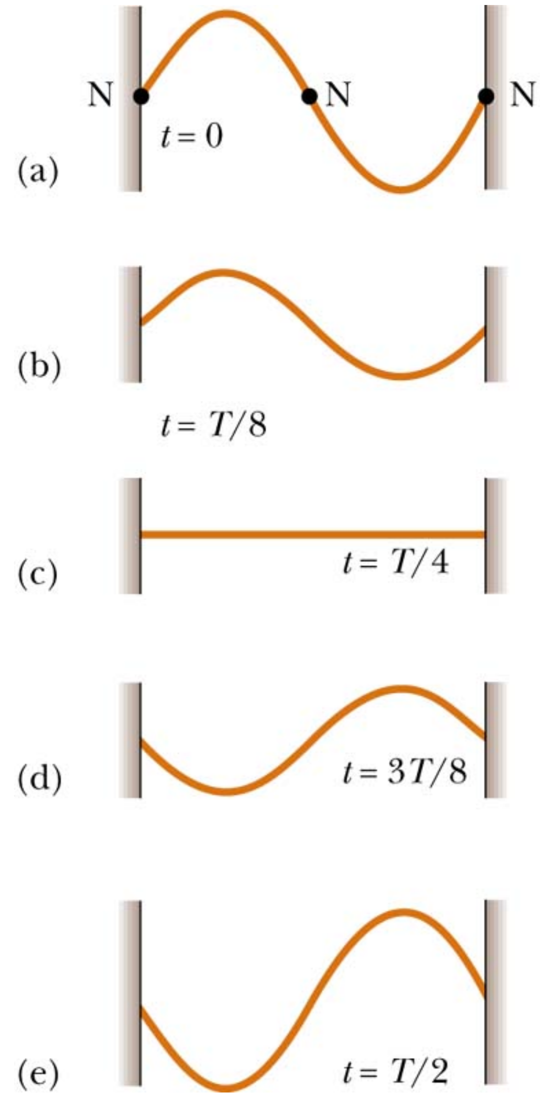
FIXED END: pulse inversion



FREE END: no inversion

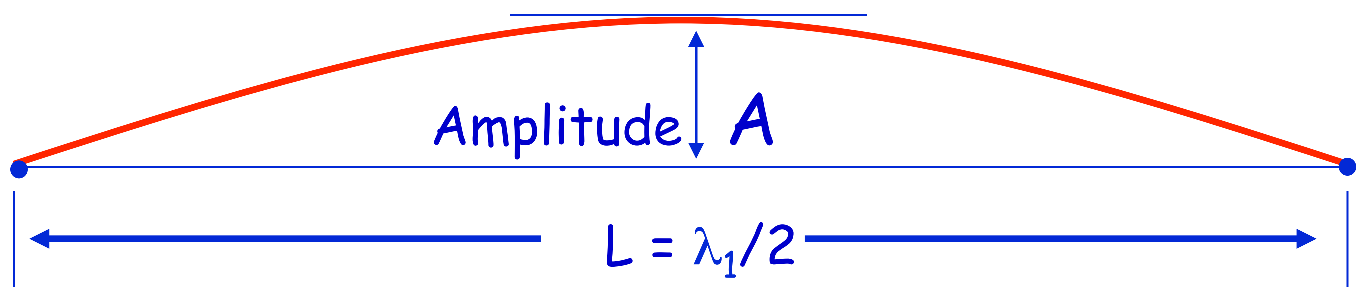
How to create standing waves: a rope





The oscillations in the rope are reflected from the fixed end (amplitude is reversed) and create a standing wave.

Standing Waves on a Stretched String



snapshot of standing wave at one instant of time, t

- For string of length L with fixed ends, the lowest mode of vibration has frequency $f_1 = v/2L$ ($v = f_1\lambda_1$) (f in cycles per second, or Hertz (Hz))

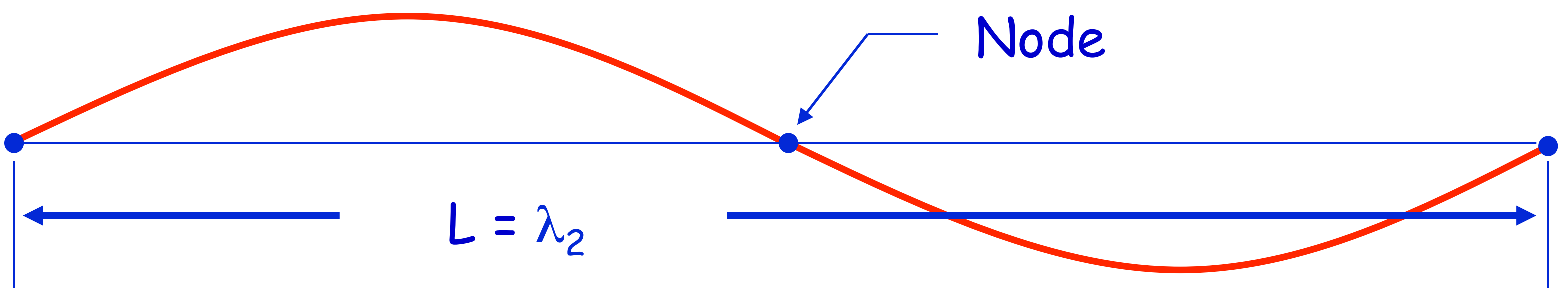
- Frequency of vibration, $f = 1/T$, where T = period = time to complete 1 cycle

- Wavelength, λ_1 of lowest mode of vibration has $\lambda_1 = 2L$ (in meters)

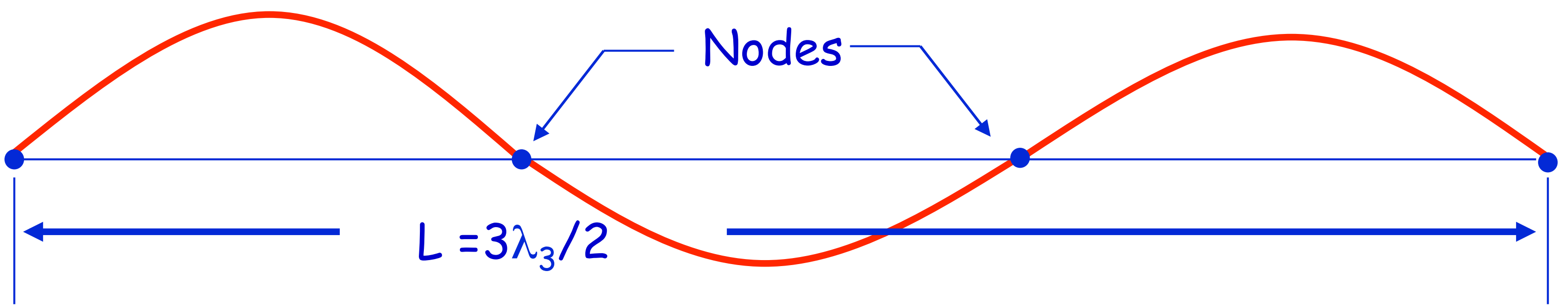
- Amplitude of wave (maximum displacement from equilibrium) is *A*

String can also vibrate with higher modes:

• Second mode of vibration of standing wave has $f_2 = 2v/2L = v/L$ with $\lambda_2 = 2L/2 = L$



• Third mode of vibration of standing wave has $f_3 = 3v/2L$ with $\lambda_3 = 2L/3$



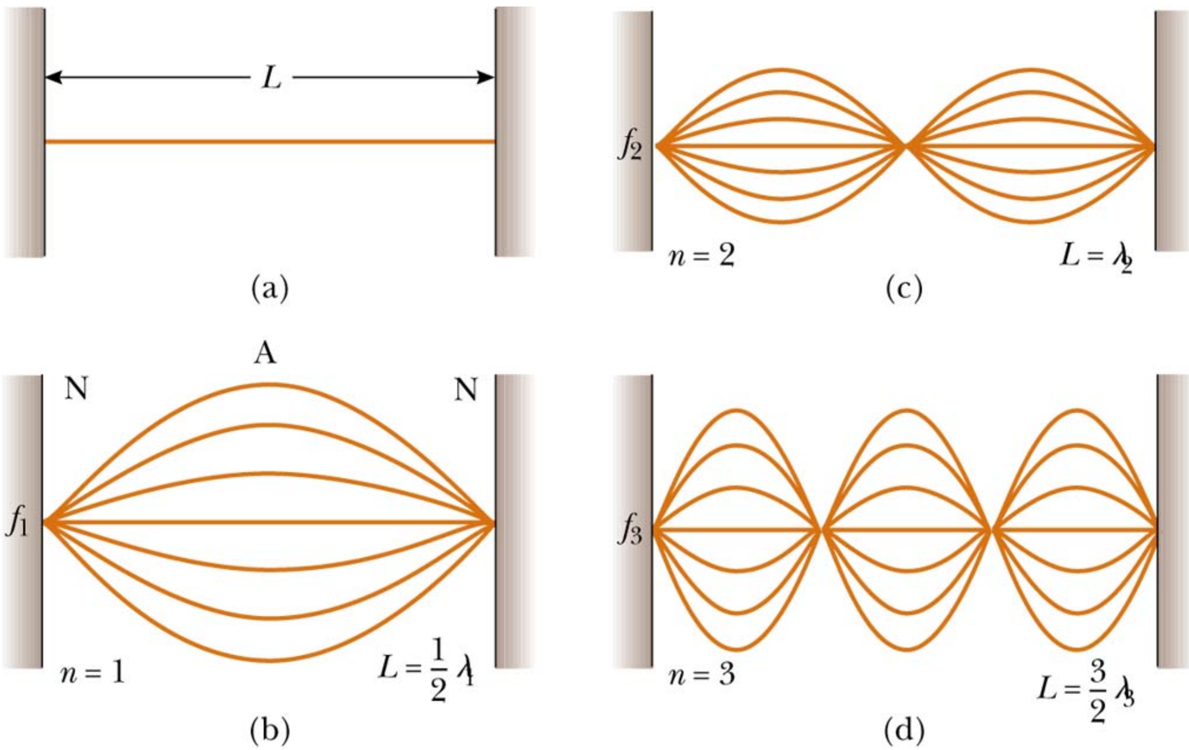
• The n^{th} mode of vibration of standing wave on a string, where n = integer = 1,2,3,4,5,.... has frequency $f_n = n(v/2L) = n\ f_1$, since $v = f_n\lambda_n$ and thus the n^{th} mode of vibration has a wavelength of $\lambda_n = (2L)/n = \lambda_1/n$

Standing waves

$$f_n = \frac{v}{\lambda_n} = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

T: tension in rope

μ: mass per unit length



$$f_1 = \frac{v}{2L}$$

$$f_2 = \frac{2v}{2L}$$

$\vdots$ n^{th} harmonics

$$f_n = \frac{nv}{2L} = nf_1$$

f_1: fundamental frequency

Example: the guitar

n^{th} harmonics: depends where and how the string is struck
note that several harmonics can be present and that non-harmonics are washed out

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

tension can be varied by stretching the wire

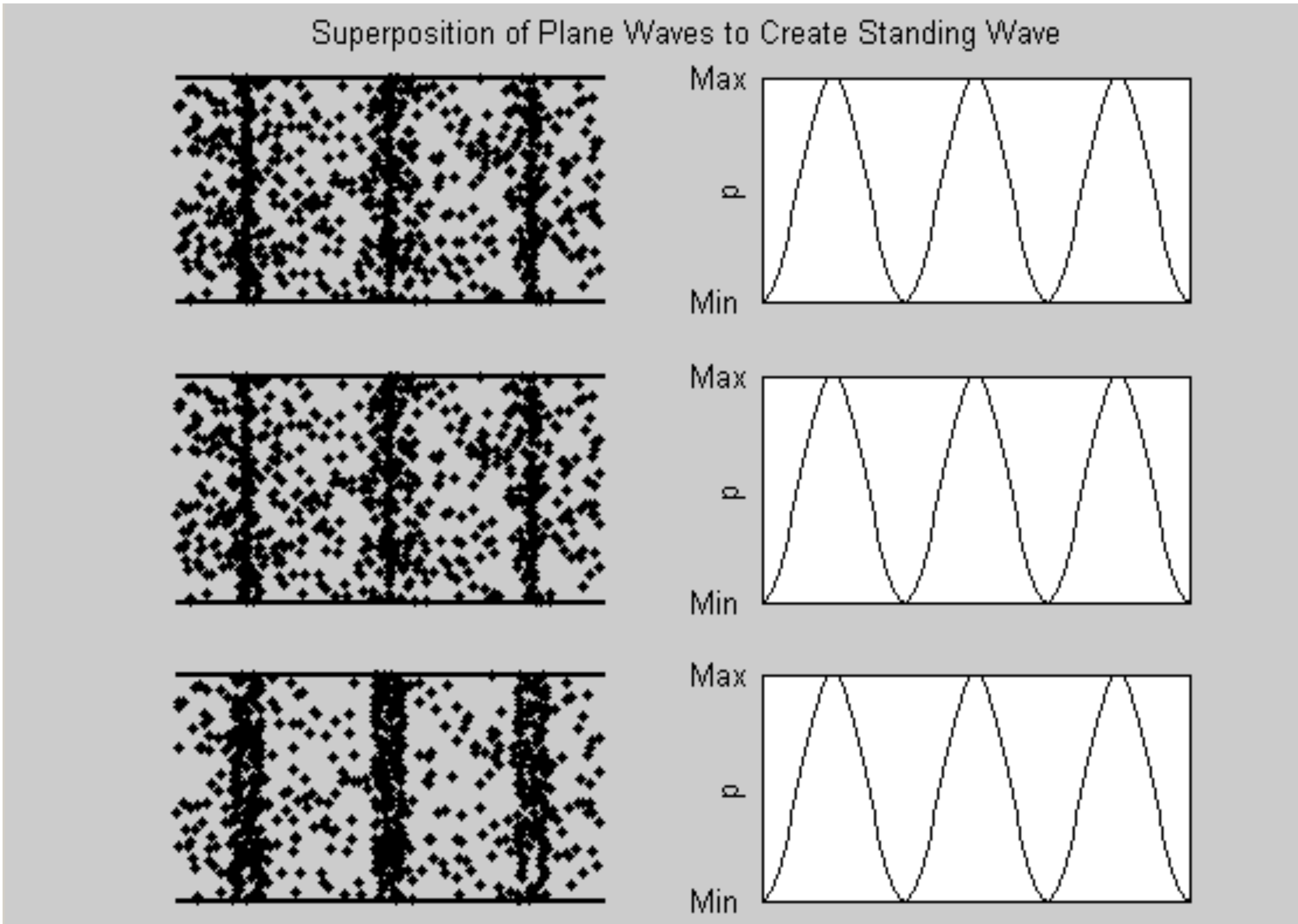
changes from string to string: bass string is very heavy

length can be chosen by placing fingers

Pipes

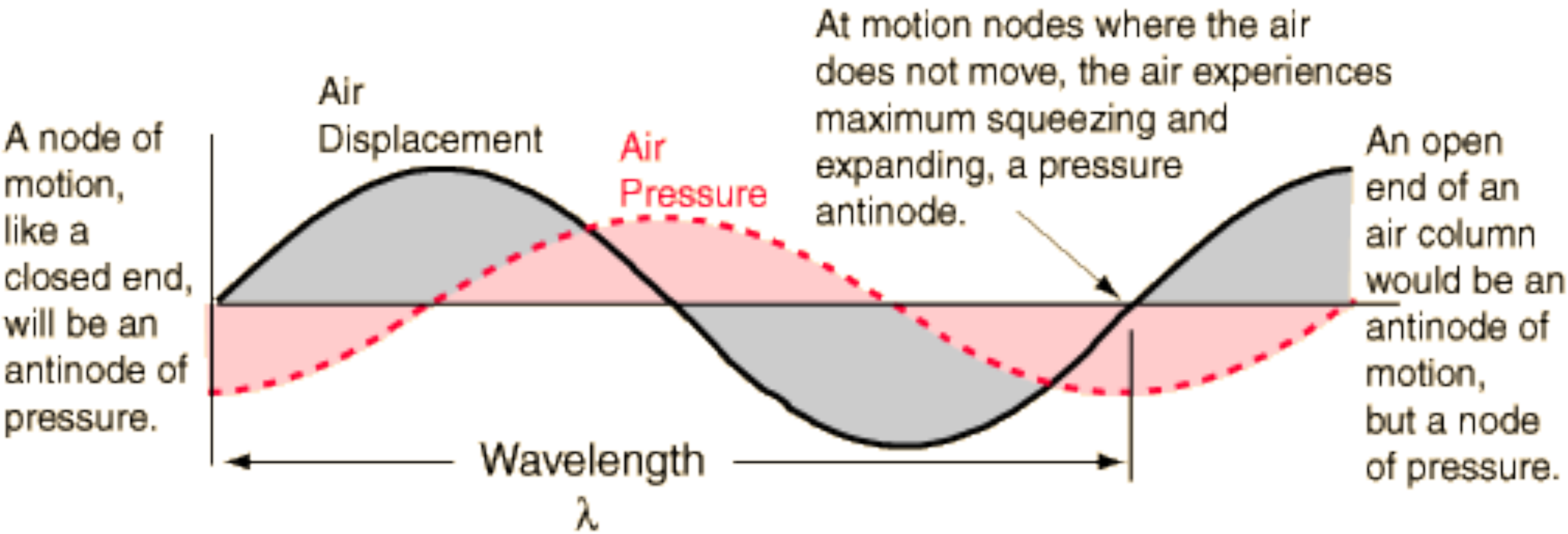
Standing waves in air columns

Just like standing waves in transverse oscillations, one can make standing waves in longitudinal oscillations as well.



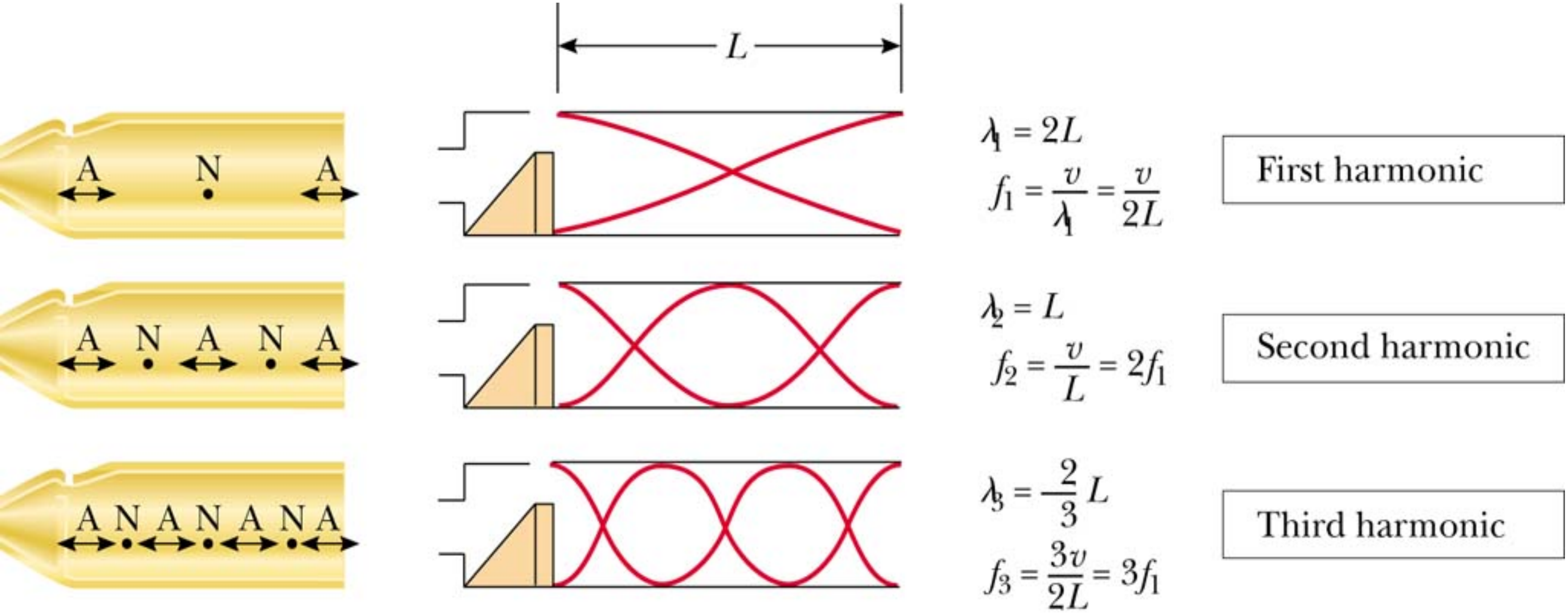
An air column (pipe)

A pipe can be open or closed on either or both sides.



For now, let's consider the air-displacements (anti-)nodes

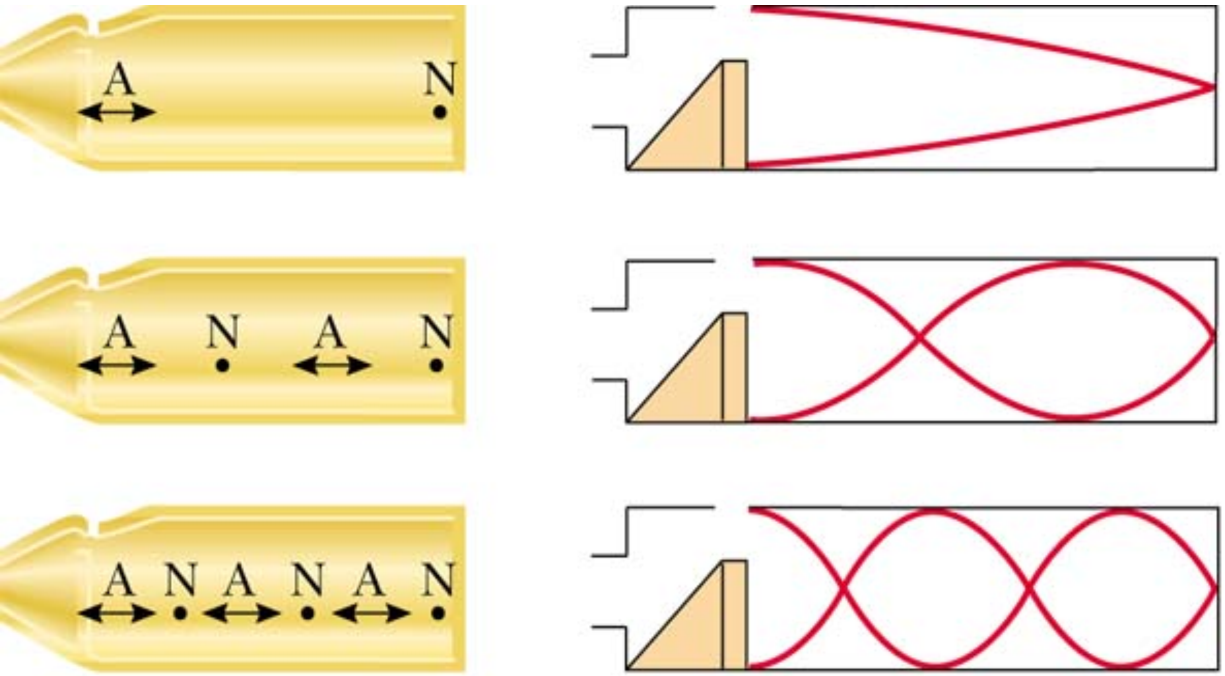
Both ends open



(a) Open at both ends

$$f_n = \frac{n v_{sound}}{2L} = n f_1 \qquad n = 1,2,3\ldots$$

One end open, one end closed



$\lambda_1 = 4L$

$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$

First harmonic

$\lambda_3 = \frac{4}{3}L$

$f_3 = \frac{3v}{4L} = 3f_1$

Third harmonic

$\lambda_5 = \frac{4}{5}L$

$f_5 = \frac{5v}{4L} = 5f_1$

Fifth harmonic

$$f_n = \frac{nv_{sound}}{4L} = nf_1 \qquad n = 1,3,5\ldots$$

even harmonics are missing!!!