Musical Acoustics

Lecture 16

Interval, Scales, Tuning and Temperament - I

Notes and Tones

- Musical instruments cover useful range of **27 to 4200 Hz.**

- **Ear:** pitch discrimination of 0.03 semitones → **30 distinguishable pitches in one semitone.** (much more than needed!). (one semitone = 1/12 of an octave)

- Musicians select discrete frequencies in an array: **SCALE**

- One of the frequencies = **NOTE**

- Note is also a symbol in a musical staff, or refers to a key on a piano, etc.

- Note is sometimes synonymous to **TONE**

Scale and Temperament

SCALE – A sucession of notes in ascending order (e.g., Pythagorean, just, meantone, equal temperament).

TUNING – Adjustment of pitch to correspond to an accepted norm.

TEMPERAMENT – A system of tuning in which intervals deviate from acoustically pure (Pythagorean).

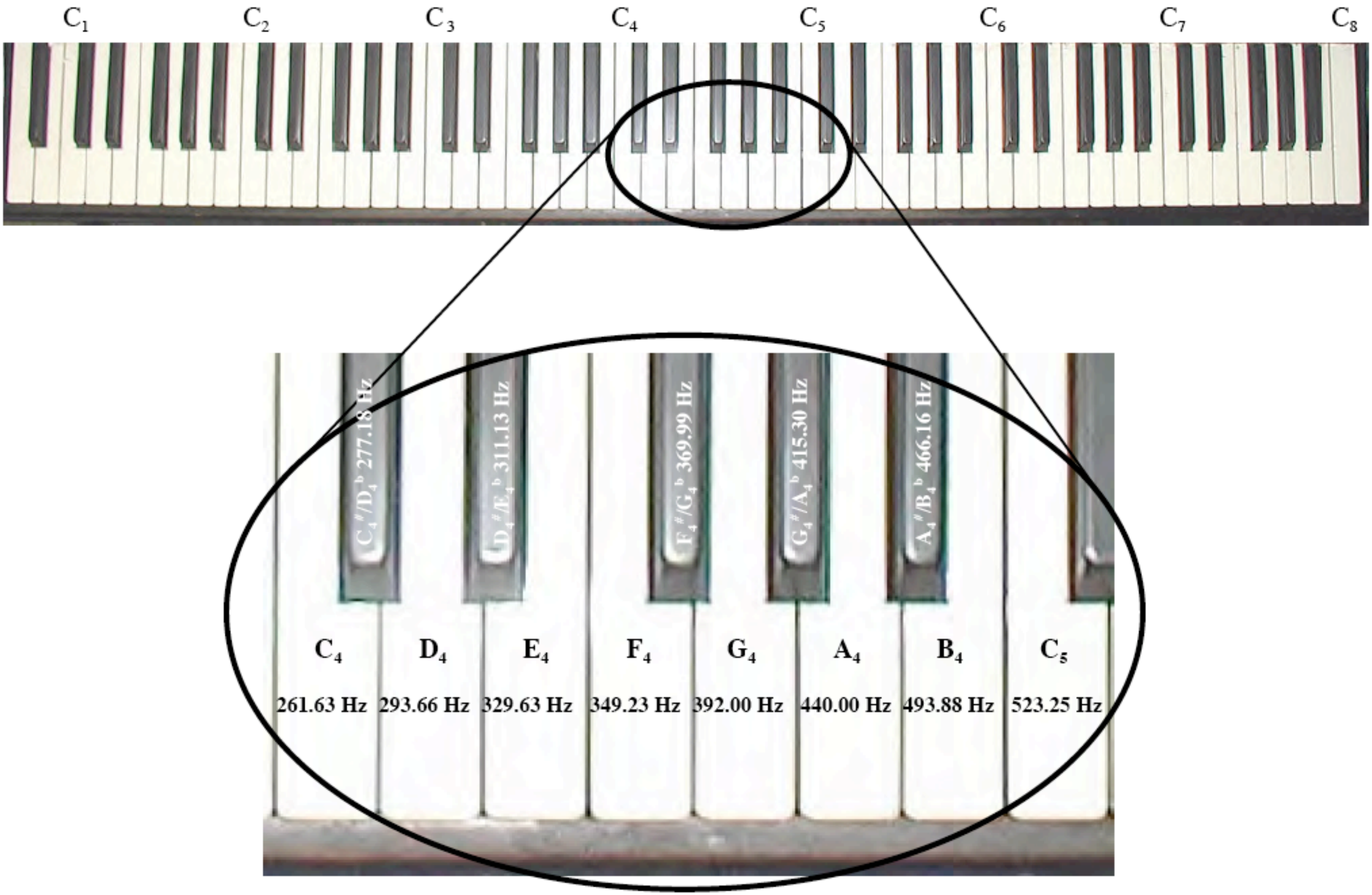
INTONATION – Degree of accuracy with which pitches are produced.

Pythagoras and the monochord

Ancient Greeks - Aristotle and his followers - discovered using a ***Monochord*** that certain combinations of sounds with ***rational number (n/m)*** frequency ratios were pleasing to the human ear.

$$f \propto \frac{1}{L} \quad \Rightarrow \quad \frac{f_1}{f_2} \propto \frac{L_2}{L_1}$$

Jump few centuries: Piano keyboard

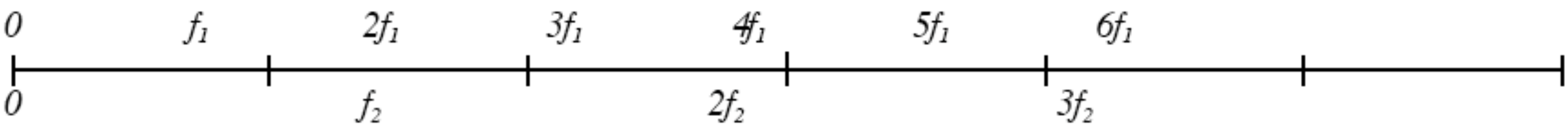


Do Re Me Fa So La Ti Do

Consonance

- Frequencies in consonance are neither similar enough to cause beats nor within the same critical band.

- Many of the overtones of these two frequencies coincide and most of the ones that don't will neither cause beats nor be within the same critical band.



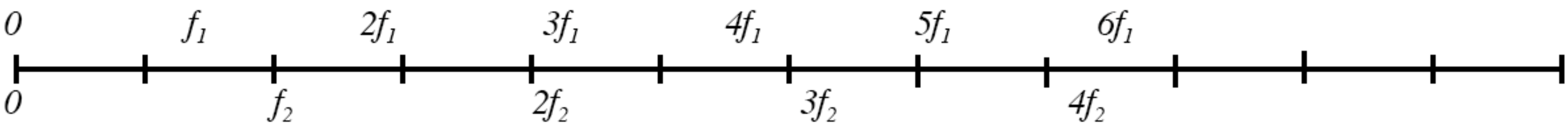
Ex 1: $f_2/f_1 = 2$

Frequency 1 (Hz)	Frequency 2 (Hz)
$f_1 = 100$	$f_2 = 200$
$2f_1 = 200$	$2f_2 = 400$
$3f_1 = 300$	$3f_2 = 600$
$4f_1 = 400$	
$5f_1 = 500$	
$6f_1 = 600$	

Frequencies in consonance sound nearly the same.

Consonance

Ex 2: $f_2/f_1 = 3/2$



Match of harmonics not quite as good, but the harmonics of f_2 that don't match those of f_1 are still different enough from the harmonics of f_1 that no beats are heard and they don't fall within the same critical band.

Frequency 1 (Hz)
$f_1 = 100$
$2f_1 = 200$
$3f_1 = 300$
$4f_1 = 400$
$5f_1 = 500$
$6f_1 = 600$

Frequency 2 (Hz)
$f_2 = 150$
$2f_2 = 300$
$3f_2 = 450$
$4f_2 = 600$

$$\frac{f_1}{f_2} \propto \frac{L_2}{L_1}$$

Pythagorean scale

Ancient Greeks - Monochord most pleasant sounds with $f_2/f_1 = 2$ and $f_2/f_1 = 3/2$ → $L_1/L_2 = 2$ and $L_1/L_2 = 3/2$

Building a scale (Pythagoras) – To get more "pleasant" tones multiply, or divide, strings by 3/2.

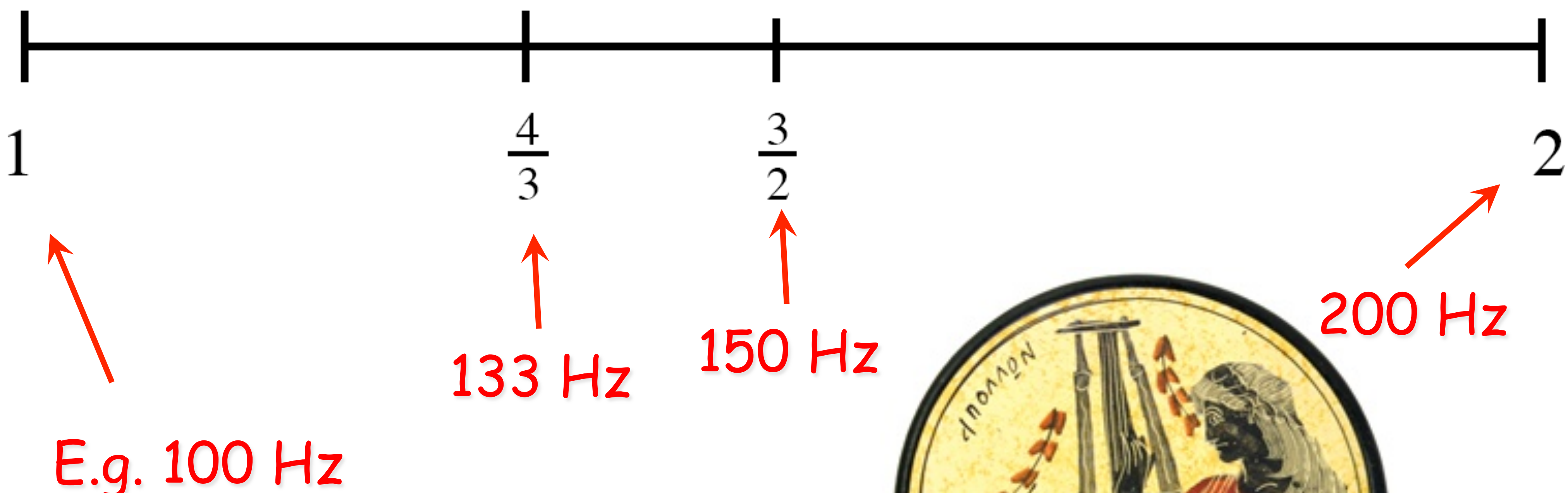
Problem: new string length might be shorter than the shortest string or longer than the longest string.

Solution: cut in half or double in length (even repeatedly) because strings that differ by a ratio of 2:1 sound virtually the same.

Building a Pythagorean scale

- Assume shortest string length = 1 **(whatever units)**.
  Longest one length = 2.

Let us start: $1 \times \frac{3}{2} = \frac{3}{2}$ and $\frac{2}{3/2} = 2 \times \frac{2}{3} = \frac{4}{3}$



This four-note scale is thought to have been used to tune **ancient lyre**

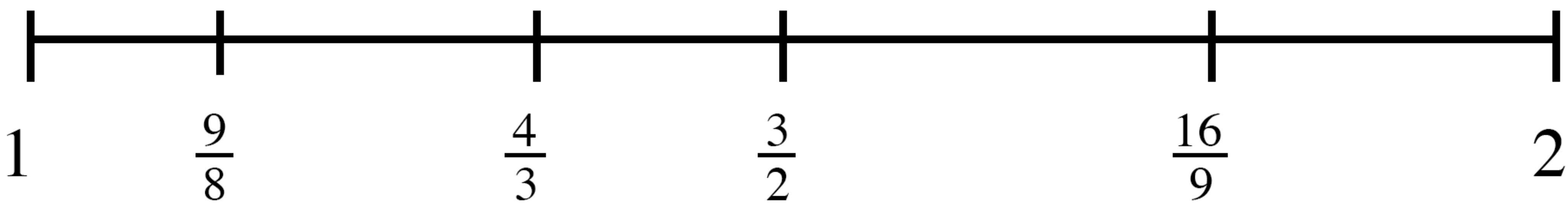
Building a Pythagorean scale - continued

Let try more (using intermediate frequencies 4/3 and 3/2):

$$\frac{4/3}{3/2} = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9} \quad \text{and} \quad \frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$$

But 8/9 < 1 and 9/4 > 2. Solution: divide or multiply by 2, as they will sound nearly the same.

→ $$\frac{8}{9} \times 2 = \frac{16}{9} \quad \text{and} \quad \frac{9}{4} / 2 = \frac{9}{8}$$



Pentatonic scale: popular in many eastern cultures.

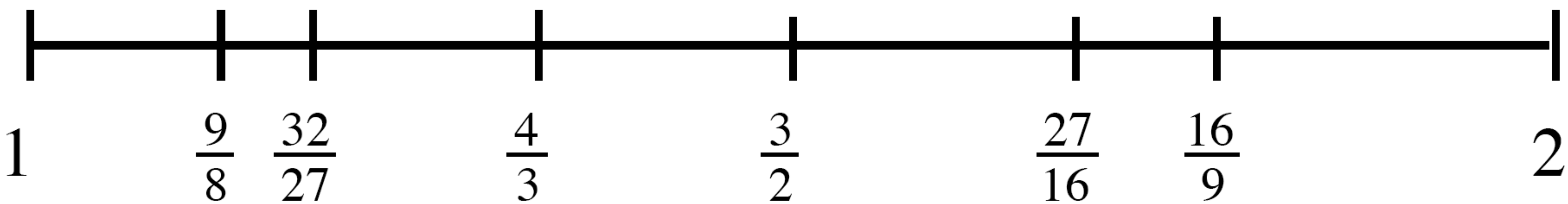
Building a Pythagorean scale - continued

Western Music has 7 notes → **Let us continue:**

$$\frac{16/9}{3/2} = \frac{16}{9} \times \frac{2}{3} = \frac{32}{27}$$

and

$$\frac{9}{8} \times \frac{3}{2} = \frac{27}{16}$$

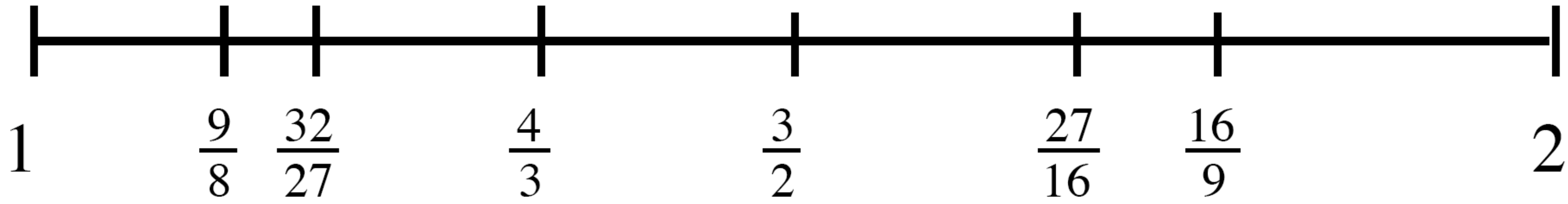


One version of the **Pythagorean scale.**

- Many frequency ratios of small integers → high levels of consonance.

- Mostly large intervals, but also two small intervals (between the second and third note and between the sixth and seventh note).

Pythagorean scale



- Across a large interval, the frequency must be multiplied by **9/8** (e.g., 32 /27 x 9 /8 = 4 /3 and 4 /3 x 9 /8 = 3/2).

- Across the smaller intervals, the frequency must be multiplied by **256/243**. In fact, 9/8 x 256/243 = 32/27, 27/16 x 256/243 = 16/9.

- 9/8 = 1.125 = change of ~12% (**whole tone**) – **W** (“full step”)
- 256/243 = 1.053 = change of ~5% (**semitone**) – **s** (“half step”)

- Going up in frequency: ***W s W W W s W***

Pythagorean scale

- Instead of ***W s W W W s W*** let us start with previous W:

	W	***W***	***s***	***W***	***W***	***W***	***s***
C_1	D	E	F	G	A	B	C_2
Do	Re	Me	Fa	So	La	Ti	Do ← ***(solfège)***

- Nonmusicians do not notice the smaller increase in pitch when going from "**Me**" to "**Fa**" and from "**Ti**" to "**Do.**"

- 7 different notes in the Pythagorean scale (8 including last note, which is one **diapson** higher than the first note, and thus essentially the same sound as the first).

- The eighth note has a ratio of 2:1 with the first note, the fifth note has a ratio of 3:2 with the first note, and the fourth note has ratio of 4:3 with the first note.

- Origin of the musical terms the **octave**, perfect **fifth** , and perfect **fourth**.

Pythagorean scale

- **"G"** sounds good when played with either the upper or the lower **"C."** It is a fifth above the lower C and a fourth below the upper C.

C_i	D	E	F	G	A	B	C_f
1	$\frac{9}{8}$	$\frac{81}{64}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{27}{16}$	$\frac{243}{128}$	2

- Multiplying the frequency of a particular **"C"** by one of the fractions in the table above gives the frequency of the note above that fraction.

- Table on right shows the full list of frequency intervals between adjacent tones.

Note change	Frequency ratio
C → D	9/8
D → E	9/8
E → F	256/243
F → G	9/8
G → A	9/8
A → B	9/8
B → C	256/243

- **Exercise:** *Assuming C_5 is defined as 523 Hz, determine the other frequencies of the Pythagorean scale.*

Just Scale (origin: Ptolemy-Greece)

- Besides **2:1**, **3:2** and **4:3**, Ptolemy also observed consonance in frequency ratio **5:4**. Ratios 4:5:6 sound particularly good → **C major scale**.

4	5	6
C_i	E	G
G	B	D
F	A	C_f

- Note in C scale are grouped in triads with frequency ratios 4:5:6

- Start with **C_i = 1** → **C_f = 2**. To get the C_i:E:G frequency ratios 4:5:6 represent C_1 as 4/4 → **E=5/4** and **G** = 6/4, or **3/2**.

C_i	D	E	F	G	A	B	C_f
1		$\frac{5}{4}$		$\frac{3}{2}$			2

- Next triad (G,B,D). Start with G = 3/2, multiply by 4/4, **5/4, and 6/4** → **G =** (3/2)x(4/4) = 12/8 = **3/2**, **B =** (3/2)x(5/4) = **15/8**, **D =** (3/2)x(6/4) = 18/8 = **9/4**.

Just Scale Intervals

- $D = 9/4 > 2 \times C_i$ → divide it by 2 to get it back within the octave bound by C_i and C_f.

C_i	D	E	F	G	A	B	C_f
1	$\frac{9}{8}$	$\frac{5}{4}$		$\frac{3}{2}$		$\frac{15}{8}$	2

Then **D =** (9/4)/2 = **9/8**

- Last triad (F, A, C_f): easier to start with C_f backwards. To the get the next set of 4:5:6 frequency ratios → multiply C_f by 4/6, 5/6, and 6/6 → **F =** 2x(4/6) = 8/6 = **4/3**, ***A* =** 2x(5/6) = 10/6 = **5/3**, **C_f =** 2x(6/6) = 12/6 = **2**.

C_i	D	E	F	G	A	B	C_f
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

Just Scale Intervals for a C major scale. Multiplying the frequency of a particular "C" by one of the fractions in the table gives the frequency of the note above that fraction.

Just Scale Intervals

• **Just Scale interval ratios.**

There are three possible intervals between notes:

9/8 (a **major whole tone** = 12.5% increase – same as Pythagorean whole tone)

10/9 (a **minor whole tone** = 11.1% increase)

16/15 (a **semitone** = 6.7% increase – slightly different than smallest Pythagorean)

Note change	Frequency ratio
C → D	9/8
D → E	10/9
E → F	16/15
F → G	9/8
G → A	10/9
A → B	9/8
B → C	16/15

Just Scale Intervals and common names →

Frequency Ratio	Interval	Interval name
2/1	C → C	Octave
3/2	C → G	Perfect fifth
4/3	C → F	Perfect fourth
5/3	C → A	Major sixth
5/4	C → E	Major third
8/5	E → C	Minor sixth
6/5	A → C	Minor third

• **Exercise:** C_4 is the frequency or note one octave below C_5 (523 Hz). Calculate the frequencies of the notes in the Just scale within this octave.