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Aperiodic Dynamics and the Self-Organization of Cognitive Maps in Autonomous Agents

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Aperiodic dynamics are known to be essential in the formation of perceptual mechanisms and representations in biological brains. Complex systems concepts are helping us to understand the properties of nonlinear systems that are fundamental for the emergence of complex spatio-temporal patterns in natural and biological systems. Advances in neuroscience and computational neurodynamics are applying these concepts of selforganization to understanding the spatio-temporal patterns observed in biological brains. In this paper we introduce a neural population model that is capable of replicating the important aperiodic dynamics observed in biological brains. We use the model to selforganize cognitive maps in an autonomous agent.

Keywords: Self-Organization; Nonlinear Dynamics; Cognitive Maps.

1. Introduction

The study of nonlinear dynamics has expanded in all areas of science in the past decades for many reasons. Nonlinear dynamics provide new conceptual and theoretical tools that allow us to understand and examine complex phenomena that we have never been able to model before. Nonlinear dynamics seem to show up everywhere, in physical systems such as electrical circuits, lasers, optical and chemical systems. Such dynamics are especially prevalent in the biological world, from fractal growth patterns in biological development and city formation to the self-organizing characteristics of population models, and the importance in regulating healthy biological rhythms such as the beating of the heart.

Nonlinear systems in critical states have many interesting properties. Phenomenon such as stochastic and chaotic resonance are known which enable such systems to detect the presence of signals much better in noisy environments than linear systems are capable of doing.¹ Their greatest interest lies, however, in their fundamental relationship to self-organization and emergence of complex patterns and behaviors in complex environments. Aperiodic dynamics are both an indication of and a mechanism for the emergence of such self-organizing properties.

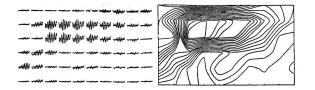
Insights in nonlinear systems theory are beginning to be applied to understanding the dynamics of the brains, and how such processes produce cognition. ^{2,3,4} Aperiodic dynamics are know to play a fundamental role in the mechanisms for the self-organization of meaning in mammalian perceptual systems. ^{5,6} Neurological evidence has shown that perceptual meanings (of recognized smells) are created through the formation and dissolution of chaotic attractors in the olfactory bulb. We will discuss this example of the self-organization of a perceptual pattern of meaning. We use this type of organization in aperiodic systems to model the formation of cognitive maps in the hippocampus of biological organisms.

2. K-Sets: A Neurodynamical Population Model of Brain Dynamics

2.1. Aperiodic Dynamics in Olfactory Systems

In their influential paper, Skarda and Freeman argued that chaos, as an emergent property of intrinsically unstable neural masses, is very important to brain dynamics. ⁵ In experiments carried out on the olfactory system of trained rabbits, Freeman was able to demonstrate the presence of chaotic dynamics in EEG recordings and mathematical models. In these experiments, Freeman and associates conditioned rabbits to recognize smells, and to respond with particular behaviors for particular smells (e.g. to lick or chew). They performed EEG recordings of the activity in the olfactory bulb, before and after training for the smells.

The EEG recordings revealed that in fact chaotic dynamics, as shown by the observed strange attractors, represented the normal state when the animal was attentive, in the absence of a stimulus. These patterns underwent a dramatic (nonlinear) transition when a familiar stimulus was presented and the animal displayed recognition of a previously stored memory, through a behavioral response. The pattern of activity changed, very rapidly, in response to the stimulus in both space and time. The new dynamical pattern was much more regular and ordered, very much like a limit cycle, though still chaotic of a low dimensional order. The spatial pattern of this activity represented a well defined structure that was unique for each type of odor that was perceptually significant to the animal (e.g. conditioned to recognize). Figure 1 shows an example of such a recorded pattern after recognition of a stimuli of the EEG signals and the associated contour map. In this figure after recognition, all of the EEG waves are firing in phase, with a common frequency which Freeman called the carrier wave. The pattern of recognition is encoded in the heights (amplitude modulations) of the individual areas. The amplitude patterns, though regular, are not exact limit cycles and exhibit low dimensional chaos. In other words, different learned stimuli were stored as a spatio-temporal pattern of neural activity, and the strange attractor characteristic of the attention state (before recognition) was replace by a new, more ordered attractor related to the recognition process. Each (strange) attractor was thus shown to be linked to the behavior the system settles into when it is under the influence of a particular familiar input odorant.



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Fig. 1. EEG carrier wave patterns (left) and contour map (right) of olfactory cortex activity in response to a recognized smell stimulus (from Freeman, 1991, p. 80)

Figure 2 shows the effects on the spatial attractor pattern due to learning. Every time a new odor was learned by the animal, all of the existing attractor patterns changed. In this figure the contour pattern of activity for sawdust is shown (before learning the banana odor), for the banana odor, and then again for sawdust. Notice that the spatial pattern for sawdust no longer resembles its previous pattern. Whenever an odor becomes meaningful in some way, changes in the synaptic connections between neurons in different parts of the olfactory cortex take place. Just as in the Hopfield model and other neural networks, these changes are able to create another attractor, and all existing attractors are modified as a result of this learning. However, in real brains, the attractors of perceptual meaning are not simple point attractors, but are specific strange attractors.

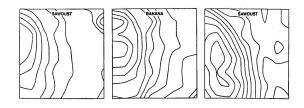


Fig. 2. Change in contour maps of olfactory bulb activity with the introduction of a new smell stimulus (from Freeman, 1991, p. 81)

Freeman suggests that "an act of perception consists of an explosive leap of the dynamic system from the basin of one (high dimensional, in the attentive state) chaotic attractor to another (low dimensional state of recognition). ⁶ These results suggest that the brain maintains many chaotic attractors, one for each odorant an animal or human being can discriminate. Freeman and Skarda speculate on many reasons why these chaotic dynamics may be advantageous for perceptual categorization. For one, chaotic activity continually produces novel activity patterns which can provide a source of flexibility in the individual. But since chaos is a ordered state, such flexibility is under control. As Kelso remarks, ⁷ such fluctuations continuously probe the system, allowing it to feel its stability and providing opportunities

to discover new patterns. Another advantage of chaos is that it allows for very rapid switching between attractors, which random activity is not able to do. Freeman also proposed that such patterns are crucial to the development of nerve cell assemblies.

2.2. K-Set Model of Aperiodic Dynamics

The K-set hierarchy, developed by Freeman and associates, ^{8,2,5,6} is both a model of neural population dynamics and a description of the architectures used by biological brains for various functional purposes. The original purpose of the K-set was to model the dynamics observed in the olfactory perceptual system. The lowest level of the hierarchy, the K0 set, provides a basic unit that models the dynamics of a local population of tens of thousands of neurons. The dynamics of the K0 set are described by a second order ordinary differential equation feeding into an asymmetric sigmoid function:

$$ab\frac{d^2x(t)}{dt^2} + (a+b)\frac{dx(t)}{dt} + x(t) = f(t)$$
(1)

This equation was determined by measuring the electrical responses of isolated neural populations to stimulation and other conditions. The *a* and *b* parameters are time constants that were determined through such physiological experiments. x(t)is the pulse density of the modeled neural population, in other words the average number of neurons that are pulsing in the population at any given point in time. f(t) is a nonlinear asymmetric sigmoid function describing the influence of incoming activation, and is given in equation 2.

$$f(t) = k[1 - exp(-\frac{e^{v-1}}{k})]$$
(2)

A K0 unit models the dynamics of an isolated neural population. From the basic K0 unit can be built up architectures that capture the observed dynamics of increasingly larger functional brain areas. The KI models excitatory-inhibitory feedback populations. KII models interacting excitatory-inhibitory populations and correspond to organized brain regions such as the olfactory bulb (OB) or the prepyriform cortex (PC). KIII combine 3 or more KII populations to model functional brain areas such as perceptual cortex or hippocampus, and are capable of aperiodic dynamics of the type observed in these regions to, for example, derive meaning from perceptual senses. In the simulations presented in this paper, we use a discretized version of the K-model (described in 9,10) developed for use in large-scale autonomous agent simulations.

In the original K model, the purpose of the KIII set was to model the chaotic dynamics observed in rat and rabbit olfactory systems. ^{11,12,13} KII are capable of oscillatory behavior, as described above. When three or more oscillating systems (KII) of different frequencies are connected through positive and negative feedback, the incommensurate frequencies can result in aperiodic dynamics. The dynamics of the KIII are produced in just this manner, by connecting three or more KII units of

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differing frequencies together. The KIII set was not only capable of producing time series similar to those observed in the olfactory systems under varying conditions of stimulation and arousal, but also of replicating power spectrum distributions characteristics of biological and natural systems in critical states. ^{14,15}

The power spectrum is a measure of the power of a particular signal (or time series as for example that obtained from an EEG recording of a biological brain) at varying frequencies. The typical power spectrum of a rat EEG (see Figure 3, top) shows a central peak in the 20-80 Hz range, and a $1/f^{\alpha}$ form of the slope. The measured slope of the power spectrum varies around $\alpha = -2.0$. $1/f^{\alpha}$ type power spectra are abundant in nature and are characteristic of critical states, between order and randomness, at which chaotic processes operate. Power spectra of biological brains have been observed to vary from $\alpha = -1.0$ to $\alpha = -3.0$. The atypical part of the experimental EEG spectra is the central peak, indicating stronger oscillatory behavior in the γ frequencies. This central peak in the 20-80 Hz range is known as the γ frequency band, and is associated with cognitive processes in biological brains. The K-models are capable of replicating the power spectra of biological EEG signals, as shown in Figure 3, bottom. ^{9,6}

The KIII sets are capable of organizing perceptual categories in the fashion observed in biological perceptual systems. The KIII used as such a pattern classifier is very robust and compares well with more standard methods of pattern classification. 1

3. Hippocampal Simulation

Meanings, or perceptual categories, seem to be a property of the formation of aperiodic attractors in the spatio-temporal activation of neuronal groups. The normal baseline background state in perceptual systems appears as a high-dimensional chaotic attractor. When sensory information is received and impinges on the receptors the dynamics of the neuronal populations change dramatically. When the sensory information is remembered as being something experienced in the past, the neuronal dynamics fall into a new chaotic attractor that represents the recognition of the stimuli. Failure of the perceptual system to recognize the stimuli results in the attractor remaining in the high-dimensional basal background state. If this unrecognized stimuli is associated with a pain or pleasure signal, a new attractor will be formed to capture the meaning of the new stimuli.

The same basic mechanisms of attractor formation in perception are also believed to be used by brains in other areas to form longer-term memory and behavior producing structures. ^{16,?} We use the basic KA-III architecture, described previously, to simulate the formation of cognitive maps in the hippocampus of an autonomous agent. One function of the hippocampus appears to be the formation of long-term representations of the environment, which can be used to navigate to goal locations, remember where food sources and shelter are located, etc. These environmental representations are usually referred to as cognitive maps. In this experiment,

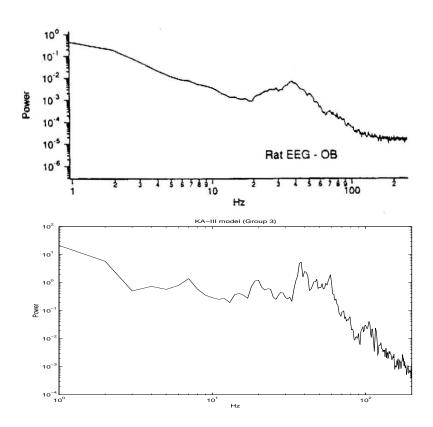


Fig. 3. The power spectrum of a rat Olfactory Bulb EEG is simulated with the KA-III model. The calculated "1/f" slope of the EEG and model is approximately -2.0. Rat OB data from (Kay 04), KA power spectrum from (Harter 2004)

we show how aperiodic attractors can be shaped and associated with locations in the agents environment, in a way that we speculate is similar to the processes used by the real hippocampus to form cognitive maps. These attractors have interesting spatio-termporal properties. For example, attractors for locations that are close to one another will be more similar to each other than locations further apart. We will next show how these types of representations can be formed in an autonomous agent, and how they might be used to perform goal-directed navigation and route planning tasks.

3.1. Experimental Architecture

In this experiment, we used the Webots virtual environment to simulate a Khepera robot moving in and learning an environment. ¹⁷ Figure 4 (bottom left) shows the morphology of the Khepera agent. The Khepera robot is a simple agent that contains 8 infra-red and 8 light sensors. The sensors are positioned around the periphery of the body, with most sensors concentrated in the front of the robot. The agent has two

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independently controlled wheels that allow it to move forward, backward, and turn left or right in place. The environment for this experiment is shown in figure 4. In the environment we place 8 light sources, which will be used as salient environmental locations. The light sources can be thought of as locations of food, or some other type of positive environmental feature. The light sources are detectable to the agent at a distance, and the detection range is indicated in Figure 4. In addition to the 8 salient environmental locations, there are 4 landmarks. The landmarks are always detectable to the agent, and it knows the distance and direction to each of the 4 landmarks as part of its sensory information. These landmarks provide a means of localization for the agent, so that it can judge its relative position in the environment.

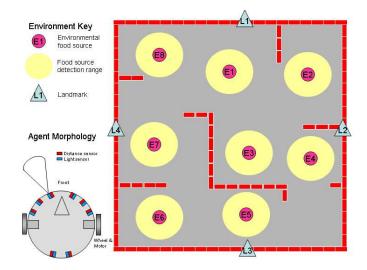


Fig. 4. Agent morphology (bottom left) and environmental setup for hippocampal simulations. The environment contains landmarks, used as allocentric reference points by the agent, and salient environmental locations, such as food sources. The agent is only able to detect the presence of a food source when it is within a particular range of it.

The architecture of the simulated hippocampus is shown in Figure 5. The portions of the architecture that form the cognitive map of the environment are simulated by a KA-III. These are the CA1, CA2 and CA3 layers, and are based on biological evidence of the structure of the hippocampus. The CA3 and CA1 layers each contain an 8x8 array of KA-II units (for a total of 64 units in each CA region). The CA2 layer has a single KA-II unit. Within the CA3 and CA1 layers, the KA-II are connected to one another via lateral connections. Each of the four nearest neighbors are connected to one another (the edges wrap around to technically form a torous out of the layers). The e_1 units are connected to the four closest neighboring e_1 's, and similarly the i_1 units are also connected to the four closest

neighbors.

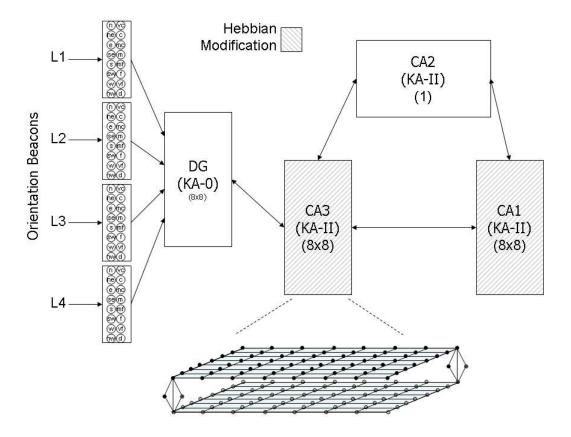


Fig. 5. Architecture of KA-III hippocampal simulations. The CA1, CA2 and CA3 areas form a KA-III. CA1 and CA3 are 8x8 matrices of KA-II units. CA2 contains a single KA-II. An example of the CA3 layer is shown below. Each e_1 unit (top sheet) is connected to its four nearest neighbors as is each i_1 unit (bottom sheet). The units at the edges have connections that wrap around, so technically the sheets form tori. The top sheet of excitatory units form KA-II with the bottom sheet of inhibitory units by connecting with the unit immediately above/below along with two other units. We show an example of the KA-II formations in the lower-left and upper-right (only two examples of the actual 64 KA-II formed are shown in the figure). Hebbian modification occurs only among lateral connections in CA1 and CA3. Input from the landmarks feeds into an 8x8 matrix of KA-0 units (DG). Projecting connections between the CA1, CA2 and CA3 are formed by fan-out connectivity, where each unit fans-out and connects with a number of units in the projecting layer. See text for full description.

There are various projecting connections between the CA layers. For simplicity, the layers in this experiment have been fully connected to one another, and we indicate this using bi-directional arrows in Figure 5. When a layer projects to another layer, each of the e_1 (or sometimes i_1) units projects to multiple units in the

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other layer. We use a fan-out parameter to determine the level of connectivity. For example, if we have a fan-out of 10 between layers CA1 to CA3, each unit in CA1 is connected to 10 other units in CA3. The projecting connections that fan-out to other layers are chosen at random. So, in the previous example, the 10 projecting fan-out connections would have been chosen at random in the target CA3 layer. The fan-out parameter for projecting connections is chosen so that the layers are fairly sparsely connected, and reflect to some degree the observed projecting connectivity in the biological hippocampus.

Orientation beacons are fed into the hippocampal simulation through the DG region (Figure 5, left). The DG layer contains an 8x8 matrix of KA-0 units. Orientation signals from the 4 landmarks are fed into the DG units. Each of the 4 landmarks has 8 units associated with the direction to the landmark, and 8 units associated with the distance. Directions are broken into 8 cardinal units, North, NorthEast, East, SouthEast, South, SouthWest, West and NorthWest. Units are sensitive to the direction of a particular landmark, though we use a graded response with a normal distribution, instead of a simple winner-takes-all configuration. [?] Similarly there are 8 cardinal distance values VeryClose, Close, MediumClose, Medium, MediumFar, Far, VeryFar, Distant. Again a graded response with normal distribution is applied to the units. The DG area feeds into the CA3 area.

3.2. Method

Learning is performed in this experiments using Hebbian modification. The lateral connections between units in the CA1 and CA3 layers are the only ones subjected to Hebbian modification. All other weights, including KA-II internal weights and weights projecting between layers, are held constant. All weights are initialized to small random values within some range. The modification of the lateral connections in the CA1 and CA3 areas causes the attractor dynamics to be shaped in response to and by the stimulation from the environmental landmarks. Hebbian modification, however, is not turned on constantly. Hebbian modification only occurs when the agent is within the detection range of a environmental location. Proximity to a salient location causes a type of positive reinforcement signal. Therefore, attractors are only shaped and affected when the robot is in environmentally salient regions.

We use a simple sensory-motor mechanism to cause wandering and exploration behavior in the agent. This exploration behavior is not affected by the cognitive map learning, but we will talk later about how the cognitive maps built by the agent using aperiodic attractors may be used to perform goal-directed navigation. The wander behavior is implemented using KA-0 units and is described in.⁹

The agent is allowed to roam in the environment at random for 60 minutes. While moving in the environment, the agent learns and builds a cognitive map. At the end of this learning period, we examine the attractors that have been formed to discover their properties.

We use two types of learning in the simulation, Hebbian modification as we

have indicated and habituation. Hebbian modification only occurs when the robot is within a certain range of a light source. When the robot is not within proximity to a light source, no reinforcement signal is produced. During these times habituation of the stimulus occurs. This has the effect of lessening the response of the simulated hippocampus to unimportant regions in the environment. ¹

The expected effects of this simulation with the two types of learning, Hebbian modification and habituation, is to form 8 distinct attractors (one for each location). The aperiodic attractors will display complex spatio-termporal characteristics. However, they should be identifiable such that when the agent is within proximity to a particular location, the observed attractor that is generated in the CA1 layer is unique. We discuss the results of this simulation next.

3.3. Results

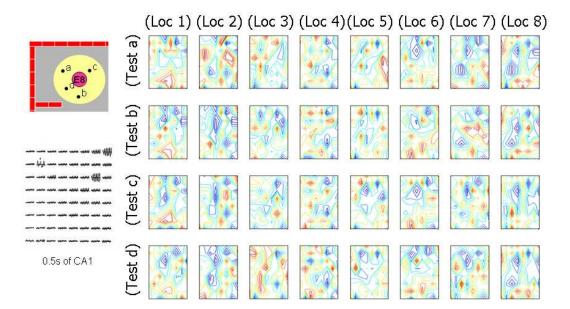
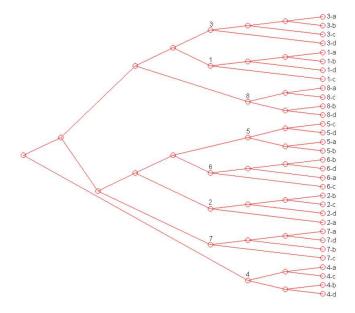


Fig. 6. Amplitude Modulation (AM) pattern formation (right) in the CA1 area for the simulation in response to four test points in proximity to each of the eight environmental locations. (Left top) shows an example of 4 test points chosen at random in proximity to location E8. (Left bottom) is an example of a half second of activity of the e_1 units in the CA1 layer in response to test point a at location E8. Amplitudes for each of these 64 time series were calculated simply by using the standard deviation. These 64 measures of amplitude were the values used to produce the contour maps that are shown. See text for full description.

We first give examples of the time series produced in the CA regions. Two broad classes of activity patterns organize themselves as a result of the Hebbian and habituation weight modifications. The spatial-temporal patterns stay in a relatively



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Fig. 7. Result of cluster analysis on AM vectors. This figure demonstrates that the spatiotemporal dynamics of the CA1 layer cluster nicely into 8 attractors. The cluster analysis was performed using the 64 vector of the amplitudes for each of the 32 test cases (4 test points at each of the 8 locations). The attractors also form higher-level basins that capture, to some extent, the proximity of the locations experineced in the environment. See text for full description.

high-dimensional background state when the agent is in an uninteresting location. This pattern changes to a more regular (e.g. cyclic) pattern when the agent is close to a food containing area. The differences in these patterns come about as a direct result of Hebbian modifications being contingent on being within a meaningful area.

Evidence of this shift, between high dimensional background state and low dimensional recognition state, can be seen in Figures ??. In this figure, we show a return plot of one of the units from the CA3 area (unit 27) when it is outside of a food area (left) and when it is within (right). Notice that the dynamics for the unit are much more cyclic and regular when the agent is in a recognized area. The patterns of most of the units in the modeled hippocampus show similar transitions in their patterns from unrecognized to important areas.

Next we look at the amplitude modulation (AM) patterns produced by the hippocampal simulation. Figure ?? shows examples of the AM patterns formed in the CA3 hippocampal matrix for 2 different locations within environmental regions 2, 4, 6 and 8 respectively. The AM patterns shown are from the CA3 hippocampal region. This region has 8x8 units, for a total of 64 time series. We measure the standard deviation of each of the 64 units for a 50ms time window, and plot the results as an 8x8 contour map of the deviations of each of the units in the area. The AM pattern contour plots, therefore, give you an idea of which units are more highly

stimulated (higher amplitudes in their activity) and which are less so. As Figure ?? shows, the AM patterns are more similar to those produced from locations within the same environmental region.

As a more complete test of the formation of unique AM patterns, we feed robot with input from randomly selected locations, within the environmental food areas. AM patterns were collected for the randomly selected regions and compared to one another by calculating the euclidian distance between each pattern. This testing showed that, in fact, the patterns produced within a region are consistently more similar to one another, than those produced in another environmental region.

4. Discussion

The KA-III hippocampal simulation described here forms distinct AM patterns for the 8 salient environmental regions. These patterns are aperiodic spatio-temporal activity in the CA regions. The characteristic activity peaks in the AM patterns are examples of so called 'place cell' formation. Here we see high activity among certain regions correlated with being in a particular environmental location. For example, looking at the AM pattern for location 8 (Figure ??, right) you notice X peaks of activity among the units in the region. It is possible to interpret these peaks as being correlated with environmental locations, and therefore typical examples of the place cell.

The next step in this research is to begin to understand how such AM patterns might be used in the service of goal-directed navigation. It is known that if you measure the onset time of place cells in a biological brain, this time gradually shifts back in phase as the animal moves through the environment. This phase shift of the onset of the place cells may be evidence of the formation of navigation planning in the biological brain. One possible interpretation is that when the animal forms an intention to travel to a goal location, a sequence of AM patterns cycle through the hippocampus. This sequence can be interpreted as sequences of locations the animal intends to visit, from the current one to the next one, etc. in order to reach the goal. As the animal moves through the environment, its idea of the current location changes, and thus this whole sequence shifts back in phase in real-time to represent the next few intended steps the animal is planning to take. For this type of mechanism to be organized, the AM patterns must not simple form in an isolated way, but connections between adjacent locations must be incorporated into the mechanism. If the agent learns which AM patterns are co-located to which others, it may be possible to set up such a mechanism to produce a goal-directed planning for navigating in the environment.

5. Conclusion

The self-organization of spatio-temporal patterns in nonlinear systems are essential to cognitive mechanisms in biological brains. We need to better understand how such mechanisms operate in order to build better models of cognition and smarter

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autonomous agents. This paper has demonstrated one such self-organizational mechanism for the creation of AM patterns in a cognitive map of an agents environment.

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