Exam #1: Please work as many problems as possible showing ALL work! Even if you have to write your thought process in English, please do NOT simply state the answers.

1. List the elements of the given set that are natural numbers, integers, rational numbers, and irrational numbers. (Enter your answers as comma-separated lists.)

   \( \{0, -15, 20, 25, 1, \frac{22}{7}, 0.538, \sqrt{8}, \pi, -\frac{1}{3}, \sqrt{2}\} \)

   (a) natural numbers
   \( 20, 25, 1 \)

   (b) integers
   \( 0, -15, 20, 25, 1 \)

   (c) rational numbers
   \( 0, -15, 20, 25, 1, \frac{22}{7}, 0.538, -\frac{1}{3} \)

   (d) irrational numbers
   \( 2\sqrt{2}, \pi, \sqrt{2} \)

2. Find the indicated set if given the following.

   \( A = \{x|x \geq -8\}, \ B = \{x|x < 5\}, \ C = \{x|-1 < x \leq 6\} \)

   (a) \( A \cap C \)
   \( \{x|-8 \leq x < 5\} \)
   \( \{x|-8 \leq x \leq 5\} \)
   \( \{x|-1 < x \leq 6\} \)
   \( \{x|-1 \leq x \leq 6\} \)
   none of these

   (b) \( A \cap B \)
   \( \{x|-8 \leq x < 5\} \)
   \( \{x|-8 \leq x \leq 5\} \)
   \( \{x|-1 < x \leq 6\} \)
   \( \{x|-1 \leq x \leq 6\} \)
   none of these
3. Simplify the expression and eliminate any negative exponent(s).

(a) \( \frac{6a^5b^{-3}}{2a^{-7}b^6} \)

(b) \( \left( \frac{y}{2x^{-2}} \right)^{-3} \)

4. Factor the trinomial.

\( 3x^2 - 25x + 42 \)

\( (x - 6)(3x - 7) \)

5. Factor the trinomial.

\( (2x + 3)^2 + 6(2x + 3) + 8 \)

\( (2x + 5)(2x + 7) \)

6. Use a Special Factoring Formula to factor the expression.

\( (x + 5)^2 - 16 \)

\( (x + 1)(x + 9) \)

7. Factor the expression by grouping terms.

\( 2x^3 + 7x^2 - 6x - 21 \)

\( (2x + 7)(x^2 - 3) \)

8. Find the domain of the expression.

\( \frac{1}{\sqrt{x - 1}} \)

- \( x \geq 1 \)
- \( x > 1 \)
- \( x < 1 \)
- \( x \leq 0 \)
- all real numbers
9. Question Details

Perform the addition or subtraction and simplify. (Give your answer in factored form.)
\[
\frac{1}{x + 9} + \frac{5}{x - 8} = \frac{6x + 37}{(x + 9)(x - 8)}
\]

10. Question Details

Solve the equation by factoring.
\[6x(x - 1) = 25 - 11x\]
\[x = \begin{array}{c}
\frac{-5}{2} \\
\frac{5}{3}
\end{array}\] (smaller value)
\[x = \begin{array}{c}
\frac{5}{3} \\
6
\end{array}\] (larger value)

11. Question Details

Evaluate the function at the indicated values. (If an answer is undefined, enter UNDEFINED.)
\[f(x) = x^2 - 6\]
\[f(-3) = 3\]
\[f(3) = 3\]
\[f(0) = -6\]
\[f\left(\frac{1}{2}\right) = -5.75\]
\[f(10) = 94\]

12. Question Details

Evaluate the function at the indicated values. (If an answer is undefined, enter UNDEFINED.)
\[f(x) = 6x^2 + 2x - 12\]
\[f(0) = -12\]
\[f(2) = 16\]
\[f(-2) = 8\]
\[f\left(\sqrt{2}\right) = 2\sqrt{2}\]
\[f(x + 1) = 6x^2 + 14x - 4\]
\[f(-x) = 6x^2 - 2x - 12\]
### Evaluate the piecewise defined function at the indicated values.

$$f(x) = \begin{cases} \ x^2 & \text{if } x < 0 \\
\ x + 3 & \text{if } x \geq 0 \end{cases}$$

| \( f(\ -4) \) | 16 |
| \( f(\ -3) \) | 9  |
| \( f(\ 0) \)  | 3  |
| \( f(\ 3) \)  | 6  |
| \( f(\ 4) \)  | 7  |

### Use the Vertical Line Test to determine whether the curve is the graph of a function of \( x \).

(a) ![Graph](image)(a)

- is a function
- is not a function

(b) ![Graph](image)(b)

- is a function
- is not a function

(c) ![Graph](image)(c)

- is a function
- is not a function

(d) ![Graph](image)(d)

- is a function
- is not a function
Consider the following graph.

Use the Vertical Line Test to determine whether the curve is the graph of a function of $x$.

- Yes, the curve is a function of $x$.
- No, the curve is not a function of $x$.

If the curve is a function, state the domain and range. (Enter your answers using interval notation. If the curve is not a function enter NONE.)

- Domain: $[-3, 2]
- Range: $[-2, 2]$
Consider the following graph.

Use the Vertical Line Test to determine whether the curve is the graph of a function of $x$.

- Yes, the curve is a function of $x$.
- No, the curve is not a function of $x$.

If the curve is a function, state the domain and range. (Enter your answers using interval notation. If the curve is not a function enter NONE.)

- **domain**: NONE
- **range**: NONE

---

Determine whether the equation defines $y$ as a function of $x$. (See Example 9.)

$$x^2 + (y - 3)^2 = 8$$

- is a function
- is not a function

---

Determine whether the equation defines $y$ as a function of $x$. (See Example 9.)

$$x^2y + y = 2$$

- is a function
- is not a function

---

Fill in the blank with the appropriate axis ($x$-axis or $y$-axis).

(a) The graph of $y = -f(x)$ is obtained from the graph of $y = f(x)$ by reflecting in the __x-axis__.

(b) The graph of $y = f(-x)$ is obtained from the graph of $y = f(x)$ by reflecting in the __y-axis__.
20. Question Details SPreCalc 2.5.011. [2452058]

Suppose the graph of \( f \) is given. Describe how the graph of each function can be obtained from the graph of \( f \).

(a) \( y = 7f(x + 6) - 2 \)
- shift right 6 units, stretch horizontally by a factor of 7, then shift up 2 units
- shift right 6 units, stretch horizontally by a factor of 7, then shift down 2 units
- shift left 6 units, stretch vertically by a factor of 7, then shift up 2 units
- shift left 6 units, stretch horizontally by a factor of 7, then shift down 2 units
- shift left 6 units, stretch vertically by a factor of 7, then shift down 2 units

(b) \( y = 7f(x - 6) + 2 \)
- shift right 6 units, stretch vertically by a factor of 7, then shift up 2 units
- shift left 6 units, stretch horizontally by a factor of 7, then shift up 2 units
- shift right 6 units, stretch horizontally by a factor of 7, then shift up 2 units
- shift right 6 units, stretch vertically by a factor of 7, then shift down 2 units
- shift left 6 units, stretch horizontally by a factor of 7, then shift down 2 units

21. Question Details SPreCalc 2.7.004. [1613541]

True or false?

(a) If \( f \) has an inverse, then \( f^{-1} \) is the same as \( \frac{1}{f(x)} \).
- true
- false

(b) If \( f \) has an inverse, then \( f^{-1}(f(x)) = x \).
- true
- false

22. Question Details SPreCalc 2.5.051. [1615694]

A function \( f \) is given, and the indicated transformations are applied to its graph (in the given order). Write the equation for the final transformed graph.

\( f(x) = \sqrt[4]{x}; \)  reflect in the \( y \)-axis and shift upward 2 units

\( y = \frac{(-x)^{\frac{1}{4}}}{4} + 2 \)
23. The graph of a function $f$ is given. Determine whether $f$ is one-to-one.

- [ ] is one-to-one
- [x] is not one-to-one

24. The graph of a function $f$ is given. Determine whether $f$ is one-to-one.

- [ ] is one-to-one
- [x] is not one-to-one

25. Determine whether the function is one-to-one.

$f(x) = 3x - 7$

- [x] is one-to-one
- [ ] is not one-to-one
This is Exam #2. Please work all the problems, showing ALL necessary work.

1. Find the missing coordinate of \( P \), using the fact that \( P \) lies on the unit circle in the given quadrant.

\[
\begin{array}{c|c}
\text{Coordinates} & \text{Quadrant} \\
\hline
\left(-\frac{4}{5}, \ -\frac{3}{5}\right) & \text{III} \\
\end{array}
\]

2. Find the missing coordinate of \( P \), using the fact that \( P \) lies on the unit circle in the given quadrant.

\[
\begin{array}{c|c}
\text{Coordinates} & \text{Quadrant} \\
\hline
\left(\frac{2\sqrt{5}}{5}, \ \frac{1}{5}\right) & \text{II} \\
\end{array}
\]

Consider the following.

3. Find \( t \) and the terminal point determined by \( t \) for each point in the figure, where \( t \) is increasing in increments of \( \pi/4 \).

\[
\begin{array}{c|c}
\text{t} & \text{Terminal Point} \\
0 & (1, 0) \\
\frac{\pi}{4} & \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \\
\frac{\pi}{2} & (0, 1) \\
\frac{3\pi}{4} & \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) \\
\pi & (0, -1) \\
\frac{5\pi}{4} & \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) \\
\frac{3\pi}{2} & (0, -1) \\
\frac{7\pi}{4} & \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \\
2\pi & (1, 0) \\
\end{array}
\]
Consider the following.

Find \( t \) and the terminal point determined by \( t \) for each point in the figure, where \( t \) is increasing in increments of \( \pi/6 \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>Terminal Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(1, 0)</td>
</tr>
<tr>
<td>( \pi/6 )</td>
<td>\left( \frac{\sqrt{3}}{2}, \frac{1}{2} \right) )</td>
</tr>
<tr>
<td>( \pi/3 )</td>
<td>\left( \frac{1}{2}, -\frac{\sqrt{3}}{2} \right) )</td>
</tr>
<tr>
<td>( \pi/2 )</td>
<td>(0, 1)</td>
</tr>
<tr>
<td>( 2\pi/3 )</td>
<td>\left( -\frac{1}{2}, \frac{\sqrt{3}}{2} \right) )</td>
</tr>
<tr>
<td>( 5\pi/6 )</td>
<td>\left( -\frac{\sqrt{3}}{2}, \frac{1}{2} \right) )</td>
</tr>
<tr>
<td>( \pi )</td>
<td>(-1, 0)</td>
</tr>
<tr>
<td>( 7\pi/6 )</td>
<td>\left( -\frac{\sqrt{3}}{2}, -\frac{1}{2} \right) )</td>
</tr>
<tr>
<td>( 4\pi/3 )</td>
<td>\left( -\frac{1}{2}, -\frac{\sqrt{3}}{2} \right) )</td>
</tr>
<tr>
<td>( 3\pi/2 )</td>
<td>(0, -1)</td>
</tr>
<tr>
<td>( 5\pi/3 )</td>
<td>\left( \frac{1}{2}, -\frac{\sqrt{3}}{2} \right) )</td>
</tr>
<tr>
<td>( 11\pi/6 )</td>
<td>\left( \frac{\sqrt{3}}{2}, -\frac{1}{2} \right) )</td>
</tr>
<tr>
<td>2\pi</td>
<td>(1, 0)</td>
</tr>
</tbody>
</table>
Find \( \sin t \) and \( \cos t \) for the values of \( t \) whose terminal points are shown on the unit circle in the figure. \( t \) increases in increments of \( \pi/4 \).

\[
\begin{array}{c|c|c}
 t & \sin t & \cos t \\
0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{2} \\
\pi/4 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\
\pi/2 & 1 & \frac{\sqrt{2}}{2} \\
3\pi/4 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\
\pi & 0 & -1 \\
5\pi/4 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\
3\pi/2 & -1 & 0 \\
7\pi/4 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\
\pi & \frac{\sqrt{3}}{2} & 1 \\
\end{array}
\]

Find the exact value of the trigonometric function at the given real number.

(a) \( \sin \frac{8\pi}{3} \)
\[ \frac{\sqrt{3}}{2} \]

(b) \( \csc \frac{8\pi}{3} \)
\[ \frac{2\sqrt{3}}{3} \]

(c) \( \cos \frac{8\pi}{3} \)
\[ -\frac{\sqrt{3}}{2} \]

Find the exact value of the trigonometric function at the given real number.

(a) \( \sec \frac{10\pi}{3} \)
\[ -2 \]

(b) \( \csc \frac{10\pi}{3} \)
\[ -\frac{2\sqrt{3}}{3} \]

(c) \( \sec \left(-\frac{\pi}{6}\right) \)
\[ \frac{2\sqrt{3}}{3} \]
Find the amplitude and period of the function.

\( y = \cos 4x \)

amplitude

period

Sketch the graph of the function.
Find the amplitude and period of the function.

$$y = 4 \sin \frac{1}{2}x$$

amplitude: 4

period: $4\pi$

Sketch the graph of the function.
The trigonometric function $y = \tan x$ has period $\pi$ and the following asymptotes.

- $x = 2n\pi$ (n is an integer)
- $x = \frac{\pi}{2} + 2n\pi$ (n is an integer)
- $x = n\pi$ (n is an integer)
- $x = \frac{\pi}{2} + n\pi$ (n is an integer)
- $x = \frac{3\pi}{2} + 2n\pi$ (n is an integer)

Sketch a graph of this function on the interval $(-\pi/2, \pi/2)$.

11. Find the degree measure of the angle with the given radian measure.

\[ \frac{\pi}{6} \]

30°

12. The measure of an angle in standard position is given. Find two positive angles and two negative angles that are coterminal with the given angle. (Enter your answers as a comma-separated list.)

135°

-585, -225, 495, 855°
13. The measure of an angle in standard position is given. Find two positive angles and two negative angles that are coterminal with the given angle. (Enter your answers as a comma-separated list.)

\[
\frac{5\pi}{6}
\]

\[
-\frac{19\pi}{6}, -\frac{7\pi}{6}, -\frac{17\pi}{6}, -\frac{20\pi}{6}
\]

14. Solve the right triangle. (Assume \( \theta = 53^\circ \).)

Find the length of the side opposite to the given angle. (Round your answer to two decimal places.)

\[46.45\]

Find the length of the hypotenuse. (Round your answer to two decimal places.)

\[58.16\]

Find the other acute angle.

\[37^\circ\]

15. Find the quadrant in which \( \theta \) lies from the information given.

\[\tan \theta < 0 \text{ and } \sin \theta > 0\]

- I
- II
- III
- IV

16. Find the values of the trigonometric functions of \( \theta \) from the information given.

\[\cos \theta = -\frac{4}{5}, \text{ } \theta \text{ in Quadrant III}\]

\[
\begin{align*}
\sin \theta &= -\frac{4}{5} \\
\tan \theta &= \frac{4}{3} \\
\csc \theta &= -\frac{5}{4} \\
\sec \theta &= -\frac{5}{3} \\
\cot \theta &= \frac{3}{4}
\end{align*}
\]

17. In triangle \( ABC \) with sides \( a, b, \) and \( c \) the Law of Sines states that

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.
\]
18. Solve the triangle using the Law of Sines. (Assume $c = 75$, $\angle A = 43^\circ$, and $\angle B = 25^\circ$. Round lengths to two decimal places.)

- $a = 55.17$
- $b = 34.19$
- $\angle C = 112^\circ$

19. For triangle $ABC$ with sides $a$, $b$, and $c$ the Law of Cosines states the following.

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$

20. Solve triangle $ABC$. (Round the length to three decimal places and the angles to one decimal place.)

- $c = 30.512$
- $\angle A = 36.6^\circ$
- $\angle B = 23.4^\circ$
Description
This is Exam #3. Please work all the problems, showing ALL necessary work.

1. Write the trigonometric expression in terms of sine and cosine, and then simplify.
   \[ \cos \frac{t}{\csc t} \]

2. Simplify the trigonometric expression.
   \[ \frac{\csc^2 x - 1}{\csc^2 x} \]

3. Verify the identity.
   \[ \frac{1 + \sec^2 x}{1 + \tan^2 x} = \cos^2 x + 1 \]

4. Use an Addition or Subtraction Formula to find the exact value of the expression, as demonstrated in Example 1.
   \[ \cos 165^\circ = -\frac{\sqrt{6} + \sqrt{2}}{4} \]

5. Use the trigonometric expression in terms of sine and cosine, and then simplify.
Find \(\sin 2x\), \(\cos 2x\), and \(\tan 2x\) from the given information.

\[
csc x = 8, \quad \tan x < 0
\]

\[
\sin 2x = \frac{3\sqrt{7}}{32}
\]

\[
\cos 2x = \frac{31}{32}
\]

\[
\tan 2x = \frac{-3\sqrt{7}}{31}
\]

6. Question Details

Find the rectangular coordinates for the point whose polar coordinates are given.

\((7, 7\pi)\)

\((x, y) = \left( \frac{7}{2}, 0 \right)\)

7. Question Details

A point in rectangular coordinates is given. Convert the point to polar coordinates \((r > 0, 0 \leq \theta < 2\pi)\).

\((-7, -7)\)

\((r, \theta) = \left( \frac{7\sqrt{2}}{4}, \frac{5\pi}{4} \right)\)

8. Question Details

Sketch a graph of the polar equation.

\(r = 4\)
Express the equation in rectangular coordinates. (Use variables $x$ and $y$.)

9. Write the complex number in polar form with argument $\theta$ between 0 and $2\pi$.
   \[ 3 + 3\sqrt{3}i \]
   \[ 6 \left( \cos \left( \frac{\pi}{3} \right) + i \sin \left( \frac{\pi}{3} \right) \right) \]

10. Find the indicated power using De Moivre's Theorem. (Express your fully simplified answer in the form $a + bi$.)
    \[ (1 + i)^{12} \]
    \[ -64 \]