

# **Over Some Open 2D/3D Shape Features Extraction and Matching Problems**

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## **MOTIVATION**

The amount of digital images increased enormously over the last few years in Internet, satellite, medical, and environmental imaging.

Thus the problem for efficient image database management and quick, **automatic**, image retrieval is emerging as an active area of research that attracts the attention of mathematicians, computer scientists and engineers.

On the other hand, there is a lack of efficient image retrieval tools on the software market.

It makes the problem promising and attractive for the software developing corporations and agencies working in medical, satellite and/or geo imaging.

## Image Database Querying

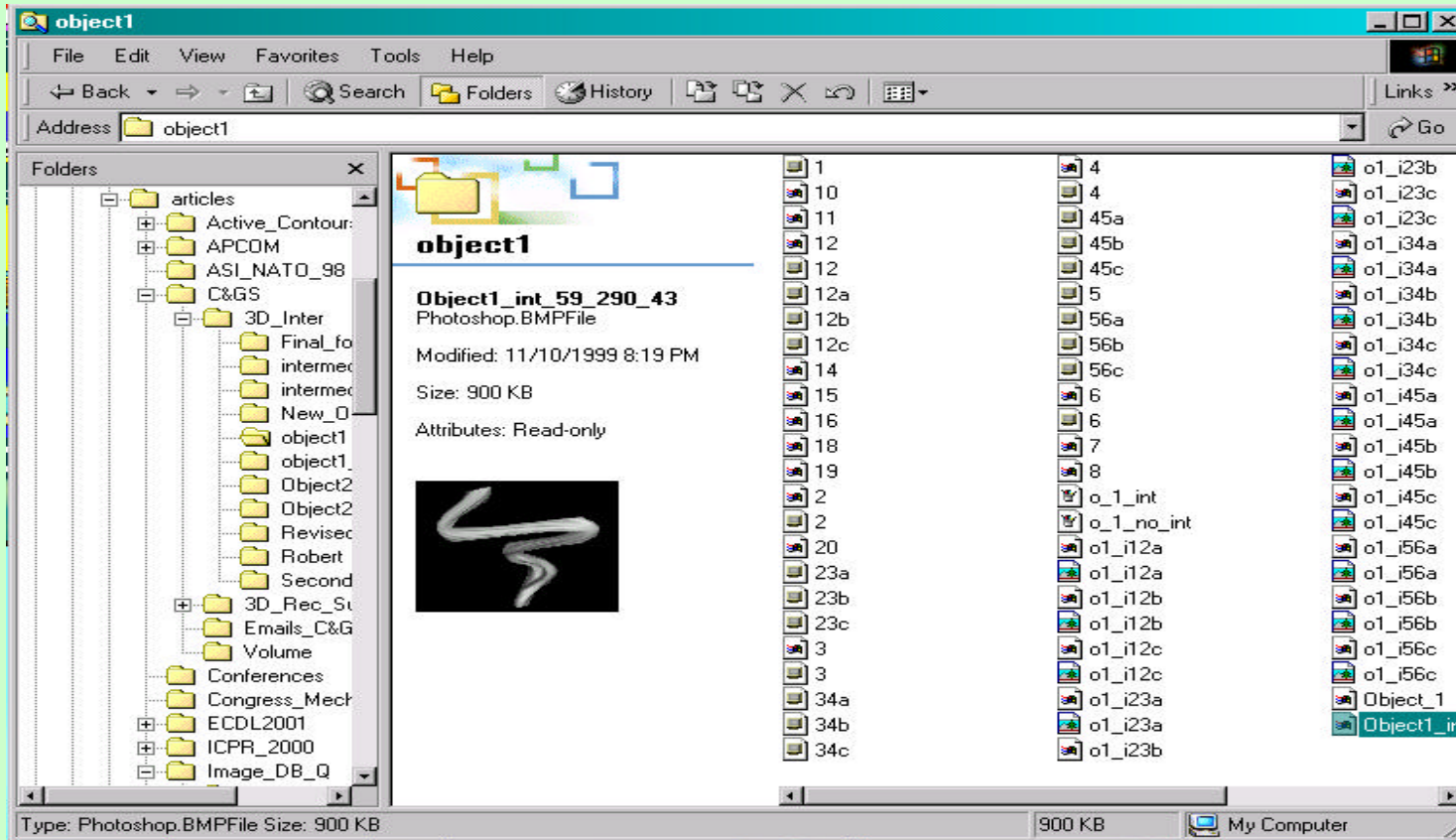
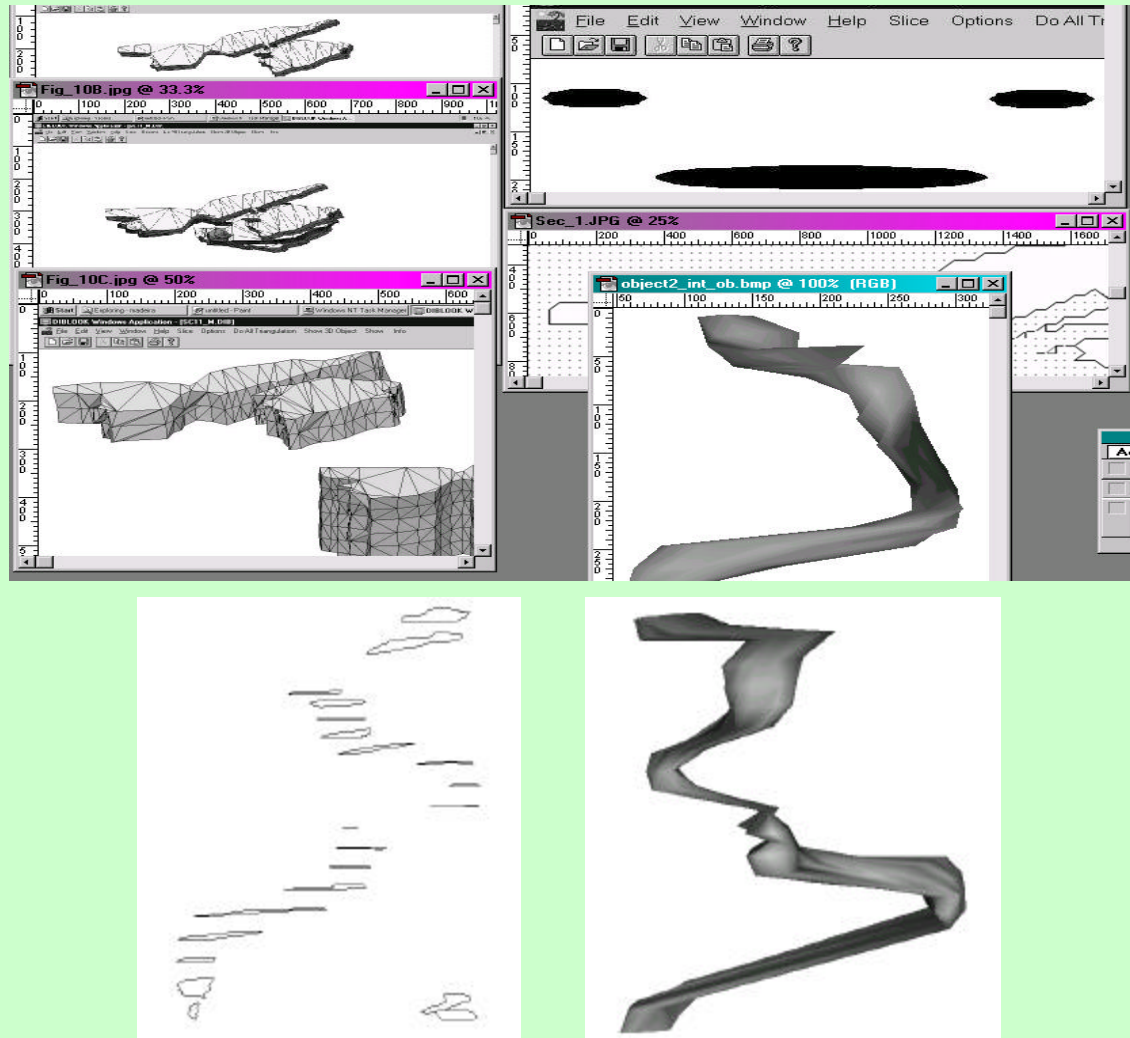


Figure 1. Windows Explorer for browsing images.

## VERY BRIEF INTRODUCTION

- **Text-based search**
  - Conventional text-matching techniques
  - Tedious, labor-intensive to prepare text descriptions of images
- **Content-based search**
  - Requires features derived from image content
  - Many features can be automatically extracted
  - Similarity based on feature vectors

## Images



**Figure 2. Set of images from Environmental IDB. The images contain 2D sections and 3D modeled subsurface objects.**

# Applications

- Air and satellite imagery, GIS;
- Biomedical; Biological Imaging;
- Chemical; Agricultural Imagery

-The CBIR **is a challenging topic of current research.**

The use of extracted image features to **rapidly locate** a desired image from a **large and dynamic collection** is widely recognized as an area of active research (Lisani 2001, Yang 2002)

# CONTENT BASED IMAGE RETRIEVAL

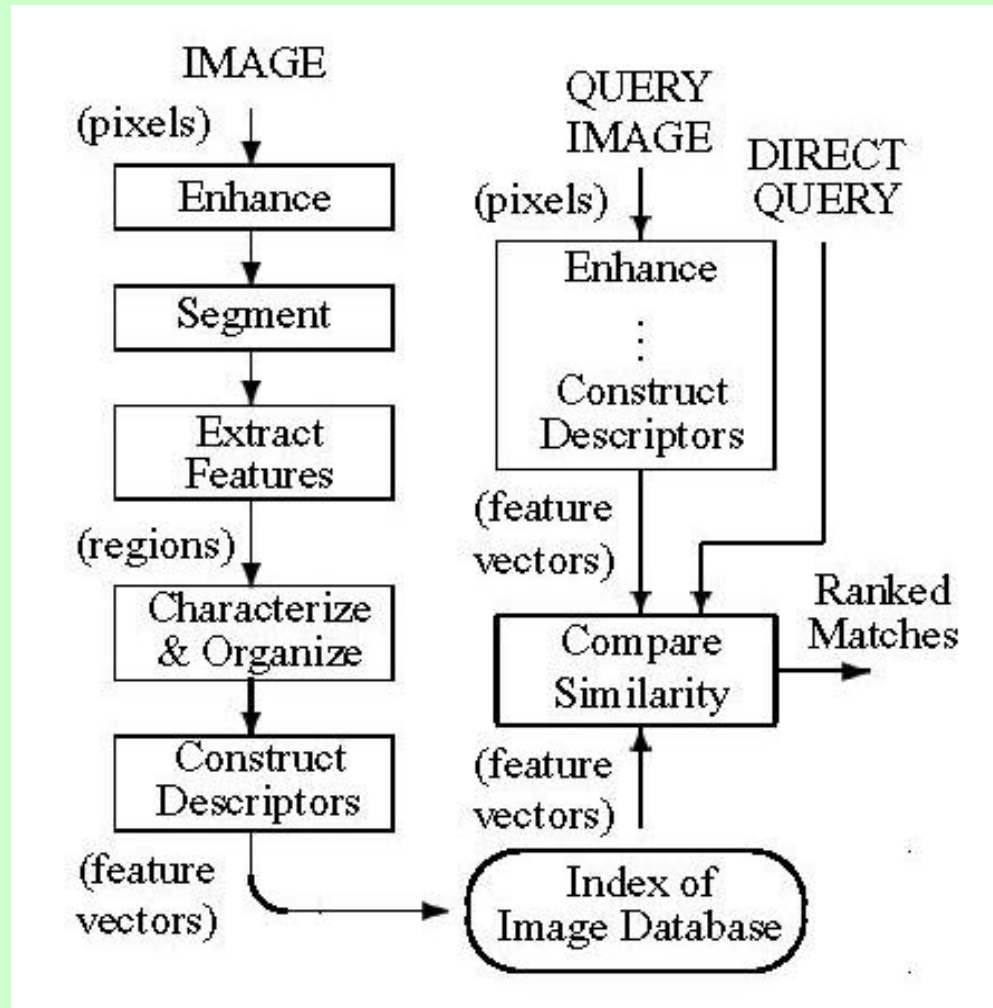
A CBIR system must be capable of:

- **Image segmentation into distinct regions (Long 1999, Zamora 2001, Yang 2002 );**
- **Feature Extraction, Classification (Long 2001);**
- **Indexing by features, (Antani 2003, Chan 2003);**
- **Image retrieval by matching (Latecki 2001, Lee 2003).**

**REQUIREMENTS:**

- 1. To retrieve images in real time;**
- 2. To retrieve small amount of images;**
- 3. To use as little as possible prior information.**

# IMAGE DATABASE MANAGEMENT AND QUERYING



## Image DataBase Construction and Querying



## IMAGE SEGMENTATION 2D/3D

**Definition:** A central problem, called *segmentation*, is to distinguish objects from background.

Different types of *segmentation* by using:  
**texture, color, shape.**

### **Approaches used:**

**Texture** segmentation can be formulated as a constrained combinatorial optimization problem. Hoffmann, et al. (1996), have employed Gabor wavelet scale space representation and a sparse dissimilarity matrix. The primary disadvantage is its high dimensionality.

## IMAGE SEGMENTATION 2D/3D

### Shape Segmentation:

**Meyer (2000)** uses morphological hierarchical segmentation based on families of increasing flooding of a gradient image.

**A watershed transform**, used by D'Omellas, obtains the edges of the image regions without splitting the color channels.

**The main disadvantage** of the above methods is over-segmentation, which produces many irrelevant regions.

Another set of approaches, is based on **active models and PDE**, Chan (2003), Vase (2002) and Xu (1998) have emerged in the last decade.

**I propose a new shape segmentation approach based on the Heat DE.**

## Heat Differential Equation to Image Segmentation- THE 2D CASE

$$\frac{\partial C}{\partial t} = Pk\vec{N}$$

$$\frac{\vec{N}(p)}{\|\vec{N}(p)\|} = \frac{1}{\sqrt{x_p^2 + y_p^2}} \langle -y_p(p), x_p(p) \rangle$$

Where  $C$  is the evolving contour analogous to temperature,  $t$  is time,  $k$  is the heat of diffusivity, which is analogous to curvature and  $N$  is the inward normal vector and  $P$  is the penalty function.

The curve  $C$  is parameterized by  $t \in [0, \infty)$  and  $p \in [0, 2\pi]$ , where  $t$  is time and  $p$  is the space between each inward normal vector,  $\vec{N}$ , on the curve,  $C$ .

The image is first transformed to the domain of  $[-1,1] \times [-1,1]$ .

If  $t$  is fixed the curve  $C$  is parameterized by

$$r(p) = r(t, p) = X(p)i + Y(p)j$$

$$k(p) = \frac{|r_p(p) \times r_{pp}(p)|}{|r_p(p)|^3}$$

## The Algorithm for the 2D case

Employing central differences to approximate the 1st and 2<sup>nd</sup> derivative:

$$\frac{dx}{dp} \approx \frac{x(p+h) - x(p-h)}{2h}$$

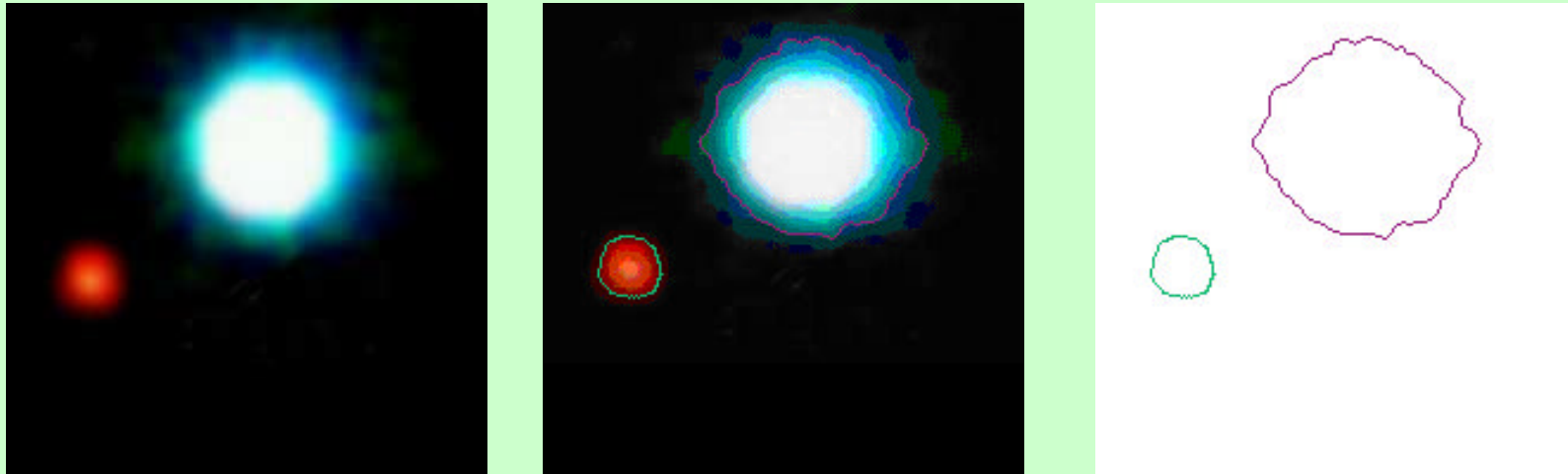
$$\frac{d^2x}{dp^2} \approx \frac{x(p+2h) - 2x(p) + x(p-2h)}{2h}$$

We transform the HDE to the following algorithm:

$$r(t+\delta, p) \approx r(t, p) + \delta Pk(t) \vec{N}(t, p) \quad r_i^{j+1} = r_i^j + \mathbf{d}_i^j k_i^j \vec{N}_i^j P_i^j$$

Where  $i=1, \dots, n$  space index that shows the number of arc segments (normal vectors) on the curve, and  $j=1, 2, 3, \dots$ , is the family iteration index over time.

## COLOR IMAGE SEGMENTATION

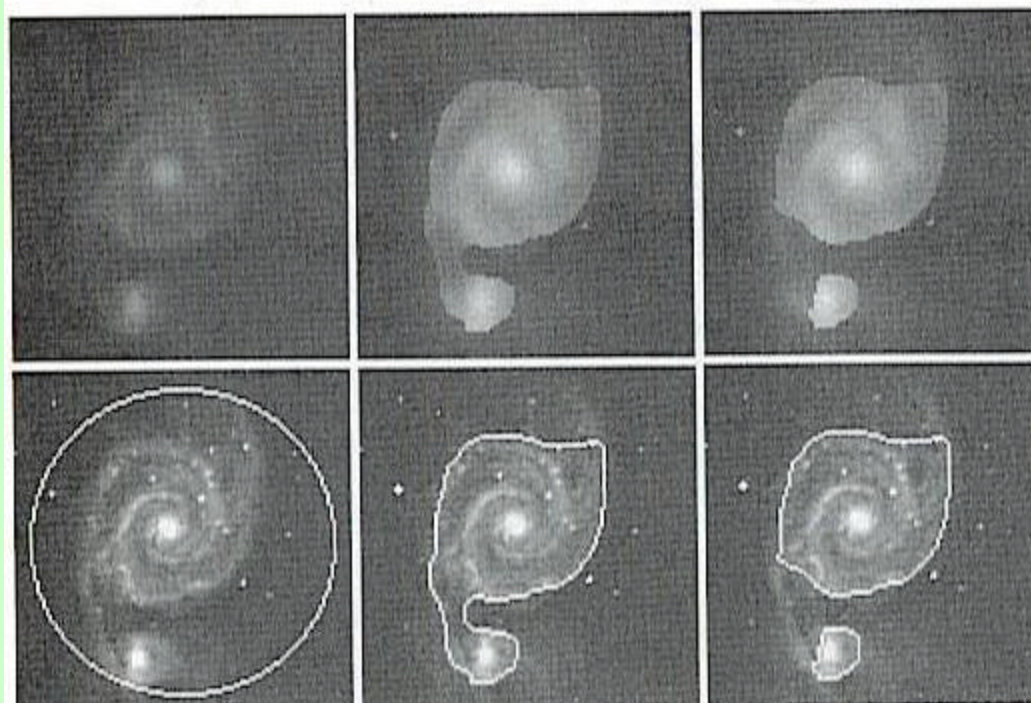


In April, a team of European and American astronomers used Yepun to detect a faint and very red point of light near the brown-dwarf star 2M1207. The astronomers believe they may have taken the first direct image of a planet circling another star.

And Anne-Marie Lagrange "**Our discovery represents a first step towards opening a new field in astrophysics:**

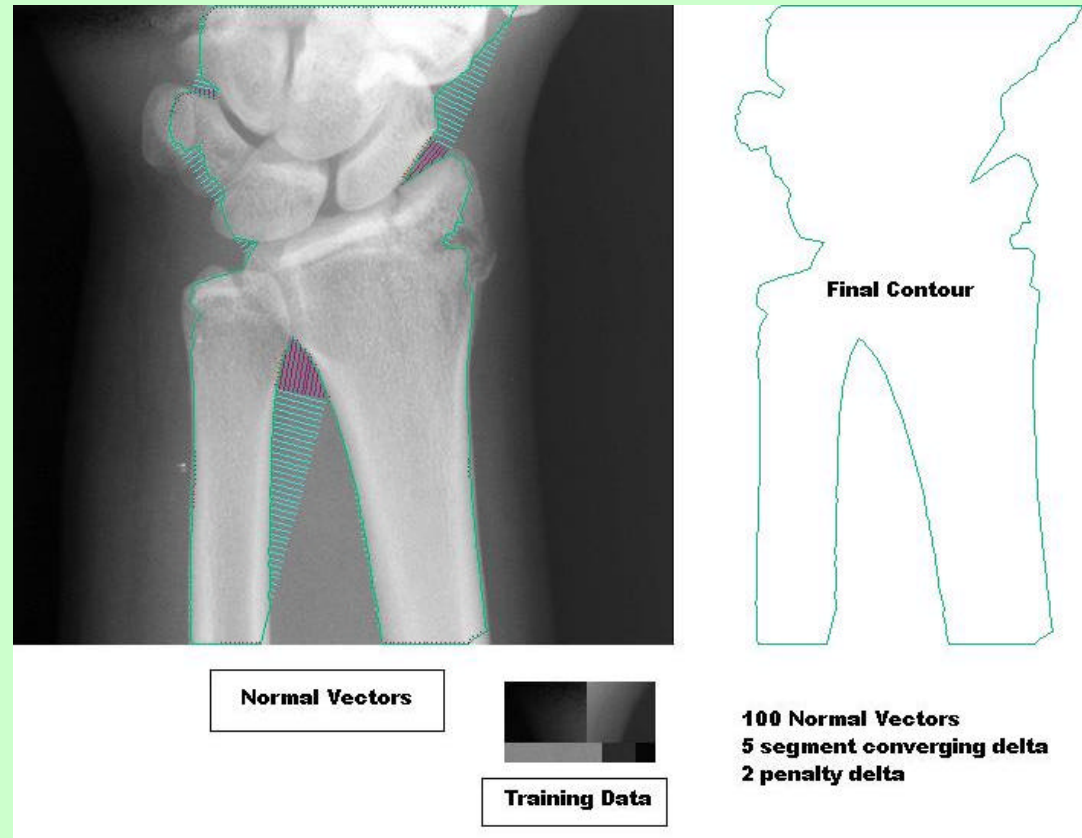
**the imaging and spectroscopic study of planetary systems.**

## SEGMENTATION



Result from the piecewise smooth Mumford-Shah level set algorithm with one level set function

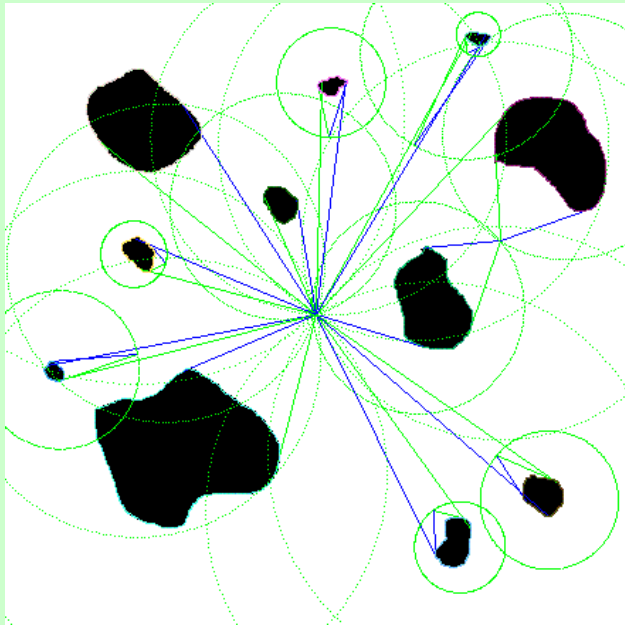
# SEGMENTATION



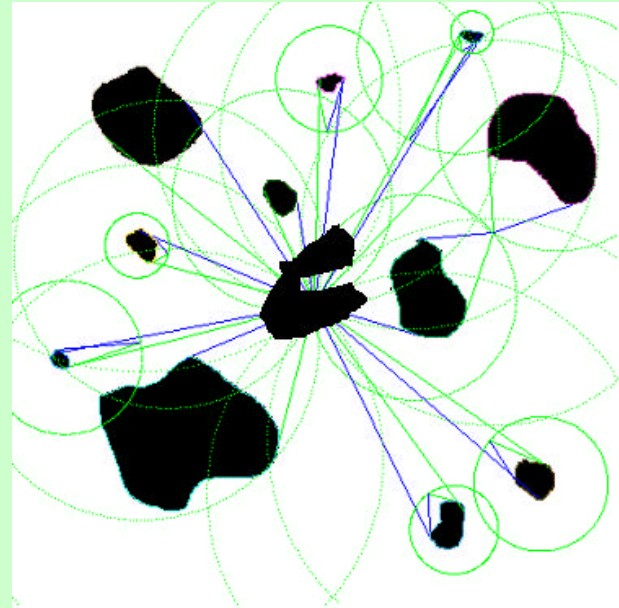
**Edge detection and medical X-Ray image segmentation,  
using curve, evolved according to the Heat DE**

## Open Problem

a)



b)



We are able to segment a), but not b).



## The 2D Convex Hull Algorithm

$$r^{j+1} = r^j + \mathbf{d}^j \partial \vec{\mathbf{t}}^j P^j - \vec{\mathbf{T}}^j s^j \quad (8)$$

where

$$r^j = \langle r_0^j, r_1^j, \dots, r_n^j \rangle \quad r_i^j = r^j(t, s_i^j) \quad , \text{ for } i=1, \dots, n \quad r_0^j = r_n^j$$

$$\vec{\mathbf{T}}^j = \langle \vec{\mathbf{T}}_1^j, \dots, \vec{\mathbf{T}}_n^j \rangle \quad \partial \vec{\mathbf{t}}^j = \langle \partial \mathbf{t}_1^j, \dots, \partial \mathbf{t}_n^j \rangle$$

$j$  denotes the time step (parameterization of the family),

$i$  represents the space step (parameterization of the curve);

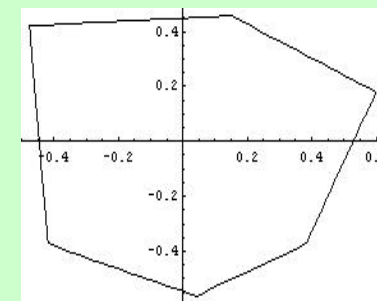
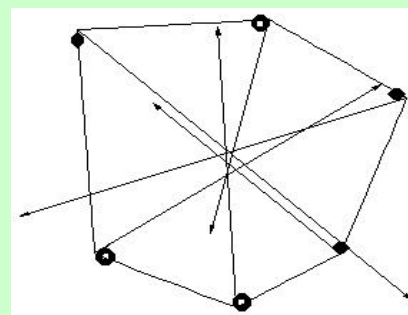
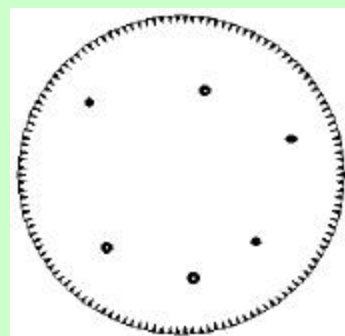
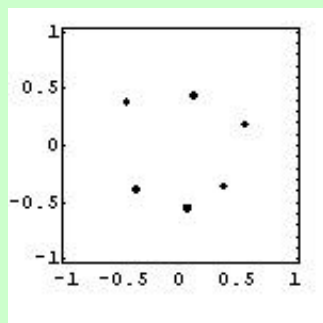
$s_i^j$  denotes the length of the  $i$ -th curve's arc segment at time  $j$ .

$P^j$  denotes the values of the penalty function in the form of an  $n \times n$  matrix:

# EXPERIMENTAL RESULTS



**Figure 7.** a) A vertebra in an image of size 64x64.  
b) The evolved curve after 500 iterations.  
c) The convex hull after 1000 iterations.



## Open Problems

- 2) To code the algorithm in C++;
- 3) To develop the theoretical base of the 3D case and code it by Mathematica or MathLab or C++.

## Heat Differential Equation to Image Segmentation-THE 3D CASE

The 3D model based on the 3D geometric heat DE

$$\frac{\partial S}{\partial t} = PH \frac{\vec{N}}{\|\vec{N}\|}$$

where  $S = \text{Family of surfaces}$

$H = \text{Mean curvature}$

$\vec{N} = \text{Inward normal vectors}$

$P = \text{Penalty Function}$

## Parameterization

The surface  $S$  is represented by the following vector function:

$$r(t, \mathbf{q}, \mathbf{f}) = \langle x(t, \mathbf{q}, \mathbf{f}), y(t, \mathbf{q}, \mathbf{f}), z(t, \mathbf{q}, \mathbf{f}) \rangle$$

where

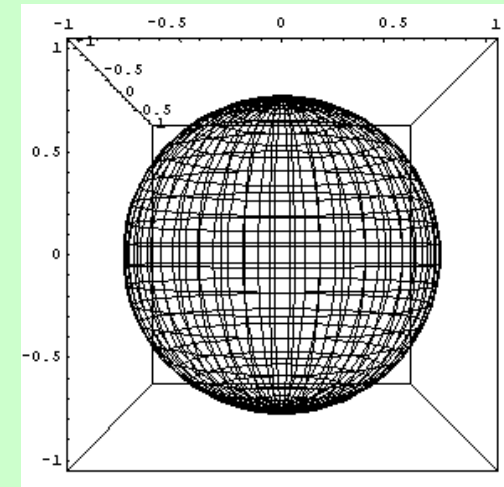
$t =$  Time parameter which parameterizes family  
of surfaces with  $t \in [0, \infty)$

$\mathbf{q} =$  Space parameter where  $\mathbf{q} \in [0, 2\mathbf{p}]$

$\mathbf{f} =$  Space parameter where  $\mathbf{f} \in [0, 2\mathbf{p}]$

The initial surface is defined for  $t=0$ :

$$r(0, \mathbf{q}, \mathbf{f}) = \langle \cos \mathbf{q} \sin \mathbf{f}, \sin \mathbf{q} \sin \mathbf{f}, \cos \mathbf{f} \rangle$$



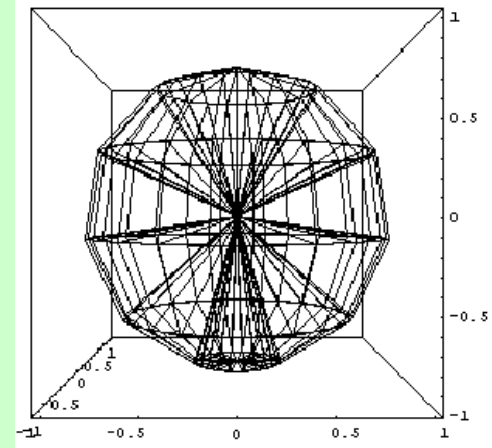
# THE 3D ALGORITHM

Using central differences we rewrite our theoretical model as an iterative process:

$$r_{ik}^{j+1} \approx r_{ik}^j + \mathbf{d}_{ik}^j P_{ik}^j H_{ik}^j \frac{\vec{N}_{ik}^j}{\|\vec{N}_{ik}^j\|}$$

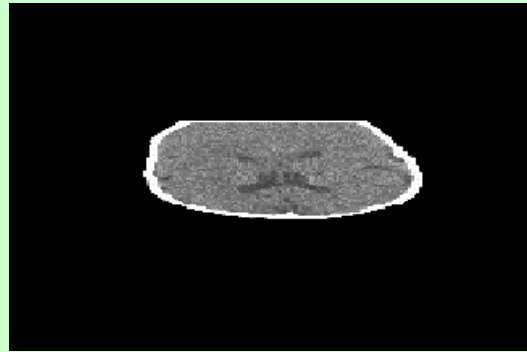
where

$$\left. \begin{array}{l} j = 1, 2, \dots \\ i = 1, 2, \dots, n \\ k = 1, 2, \dots, n \end{array} \right\} \begin{array}{l} \text{Time} \\ \text{Space} \end{array}$$

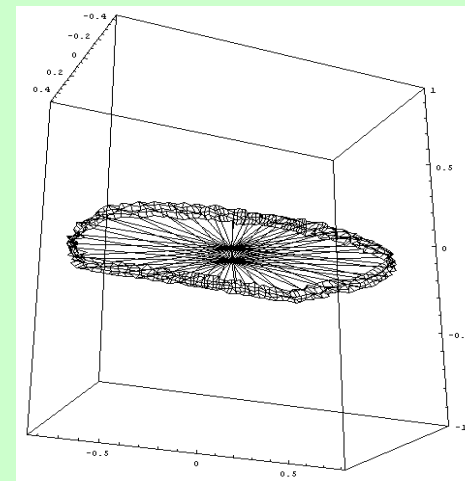
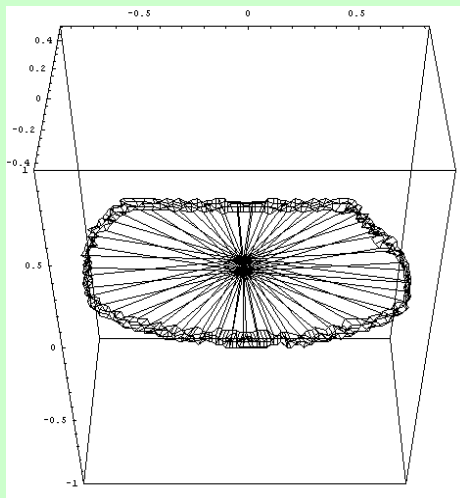


The vector field if  $n=12$

## Results



Brain's 2D section.



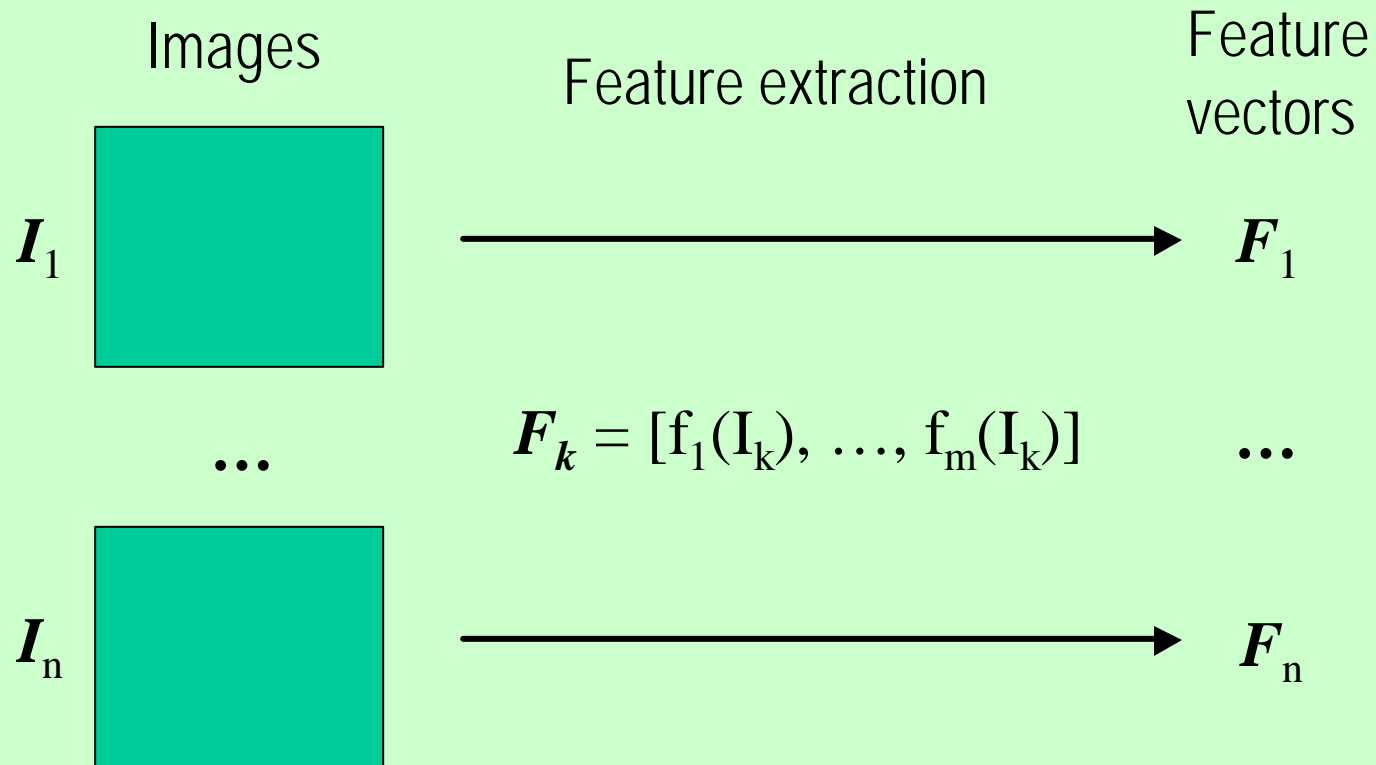
Brain edges from 5 2D sections.

## Open Problems

- 4) To code an algorithm for painting the reconstructed surface:
- 5) To code the algorithm in C++;
- 6) To make the 3D model working with more than one object and code it by Mathematica or MathLab or C++.

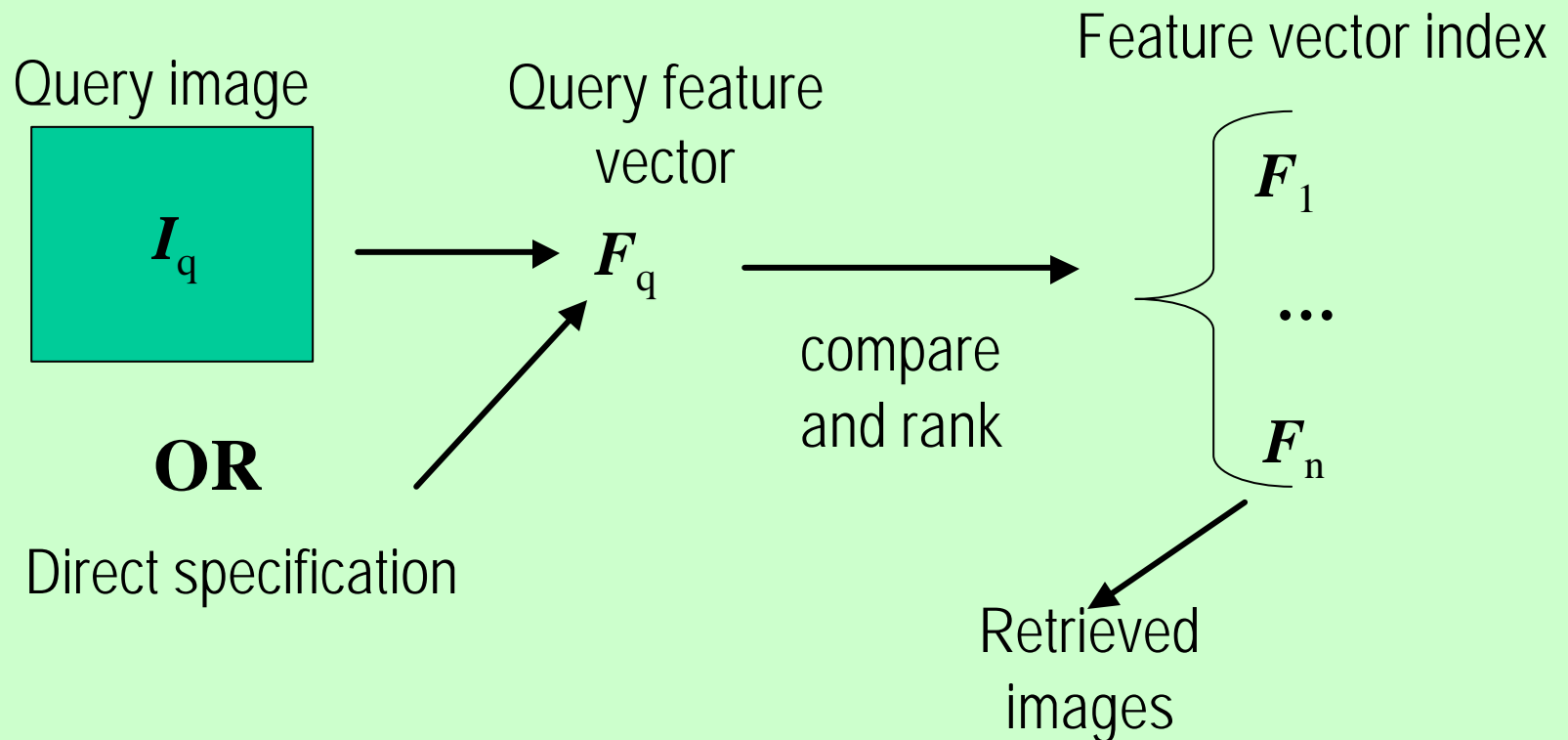


# Image Database and Features



# Image Database Query

Two common approaches:

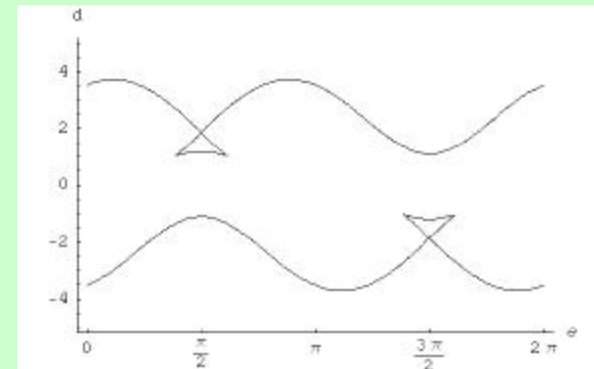
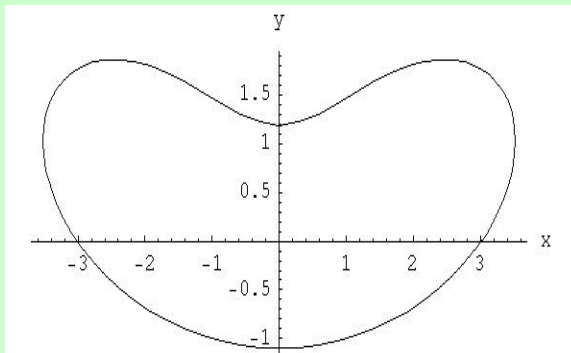
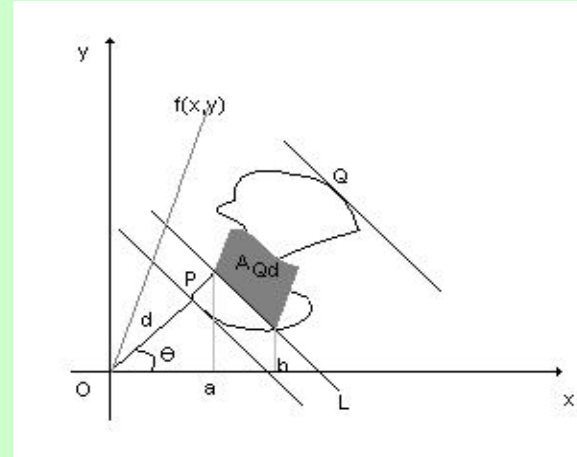


## Open Problem

- 7) An algorithm and tool for image database mining for shape features extraction and indexing:

# Concavities-Boundary Support

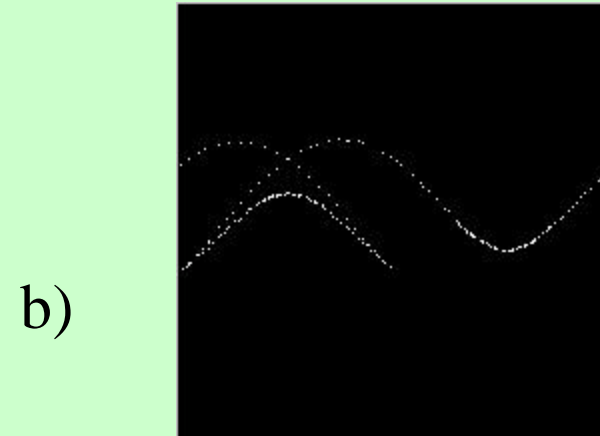
## Definition



a) An object

b) its boundary support

## Concavities-Boundary Support



- a) A section of an impermeable subsurface unit.
- b) Its boundary support obtained by C++ tool.

## Open Problems

- 8) To determine the correlation between the area of a boundary concavity and the area of the loop, that corresponds to the concavity, on the support.
- 9) C++ tool for matching objects by using boundary support.

## DEMONSTRATIONS

1. [BLOBWORLD](#)
2. [RetReg](#) ; [Results](#) .