Lectures 8, 9, 10,11:

Spatial Filtering

8. Linear Filters, Masks, Median Filter

- **Application of 1st and 2nd derivatives to Images**

9. The Laplacian to Image Enhancement

10. The Gradient

13. Fuzzy sets, membership function, operations, algorithms

Linear Filtering

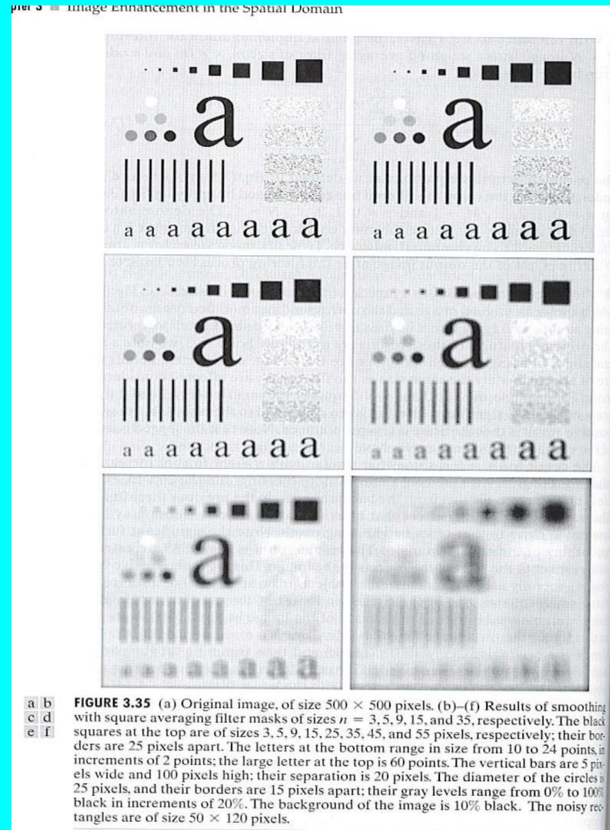


Figure 1. Smoothing with square averaging filter mask of size 3x3, 5x5, 9x9, 15x15, 35x35.

(Digital Image Processing, 2nd E, by Gonzalez, Richard, Prentice Hull, 2002).

Averaging Mask

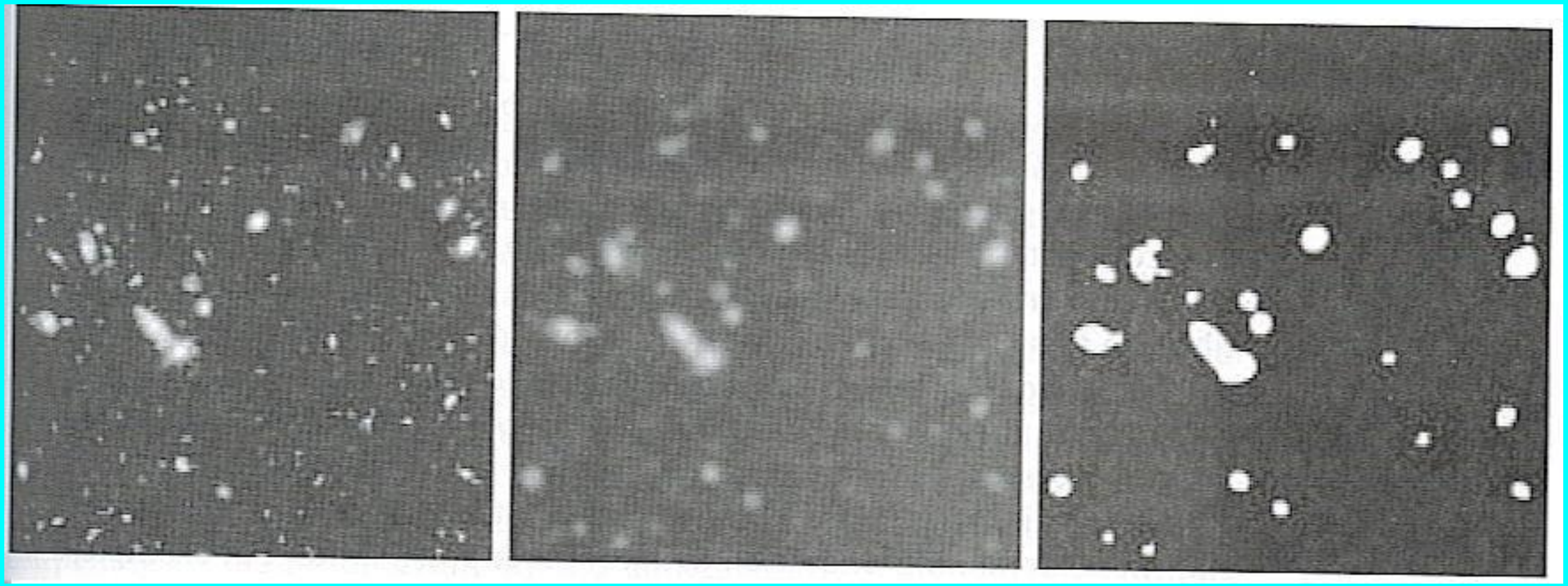
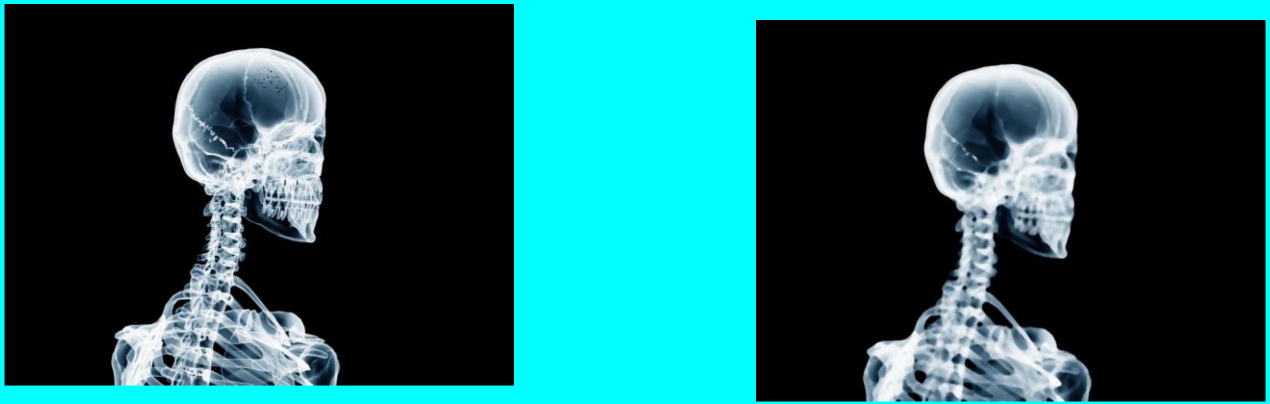


Figure 2. From left to the right: original image; processed by 15x15 averaging mask; processed by thresholding.

(Digital Image Processing, 2nd E, by Gonzalez, Richard, Prentice Hull, 2002).

Order Statistics Filters-Median Filter



a) b)

Figure 3. a) An X-ray image, where the skull is corrupted with random black noise**; b)** the cleaned image with a median filter**,** with a size of the mask 3x3. Available in Adobe Photoshop 5.5.

1st and 2nd derivatives on image

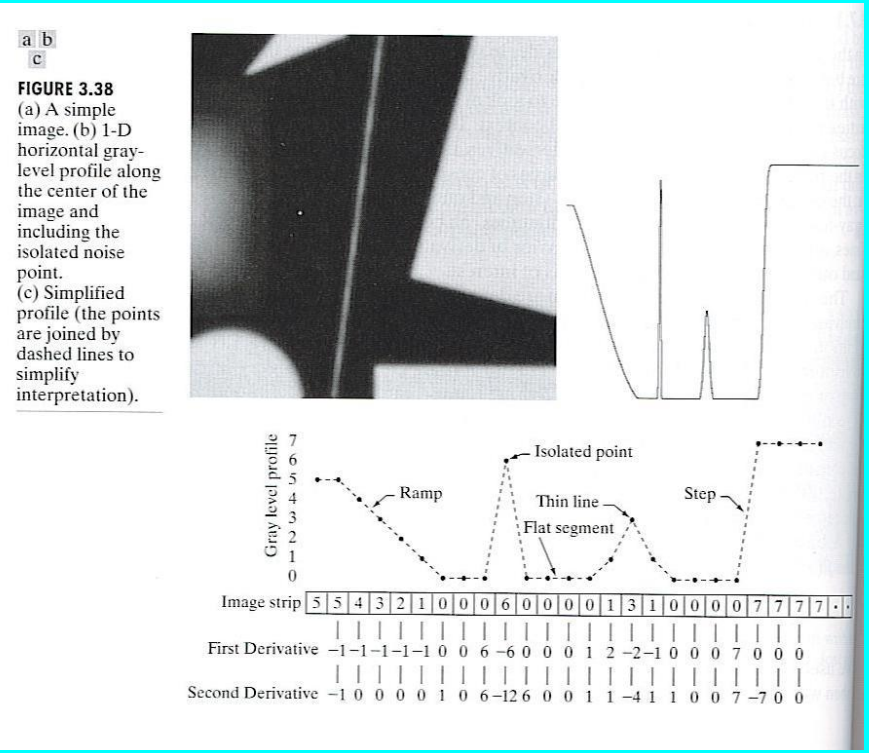


Figure 4. An Image and the results produced by 1st and 2nd derivatives. For more detailed explanation see the text on the left side of the image.

(Digital Image Processing, 2nd E, by Gonzalez, Richard, Prentice Hull, 2002).

Laplacian

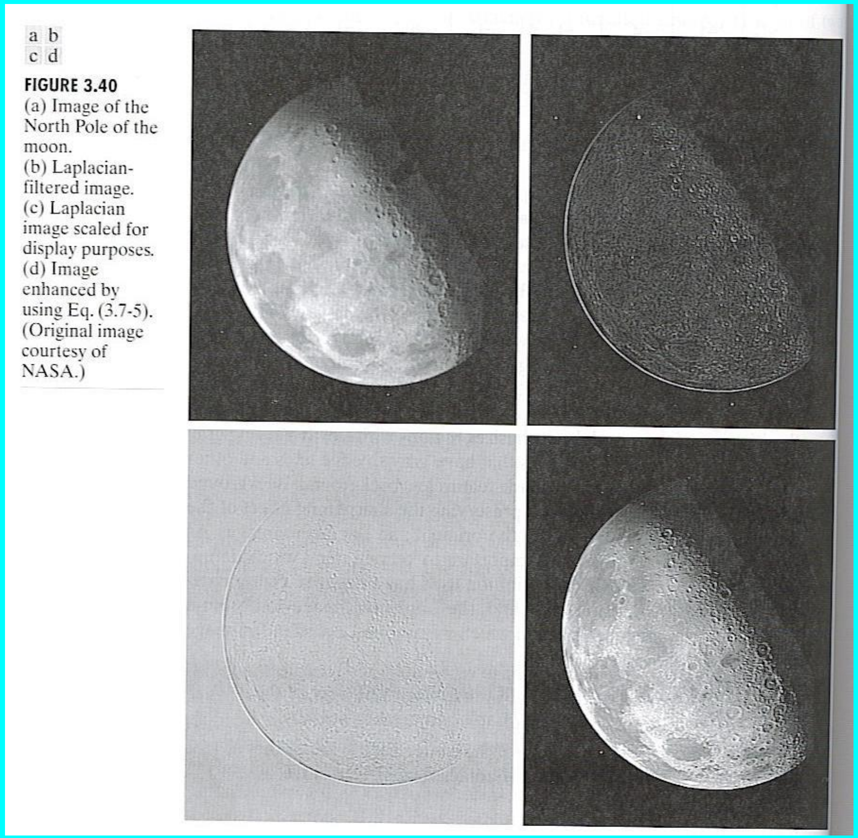


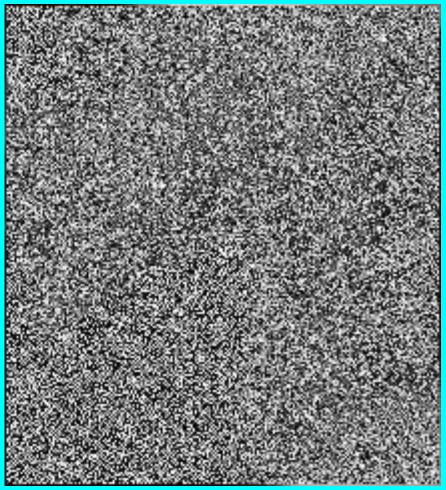
Figure 5. From left to the right, up down: original image; Laplacian filtered image; scaled Laplacian image; image obtained as a sum between the original and Laplaacian Images.

(Digital Image Processing, 2nd E, by Gonzalez, Richard, Prentice Hull, 2002).

Laplacian

a)

b)



c)

Figure6. Results obtained by applying Laplacian operator, coded by Rohit Baxi in a team with Shannon Kratzmeyer – Spring 2005.

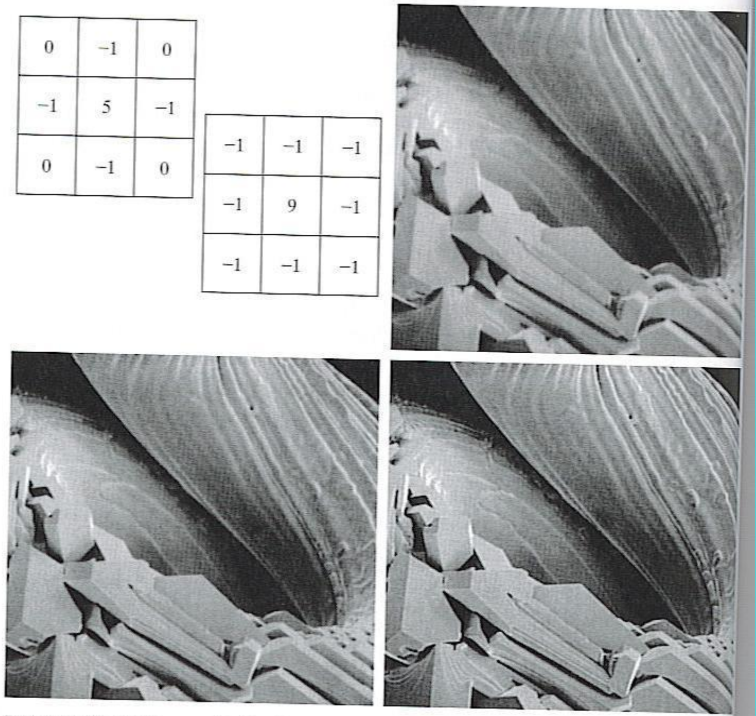
b) An x-ray image; b) The image after applying the $-\nabla^2 f(x, y)$ **the Laplacian with a positive center of the mask;**

c) after applying $-\nabla^2 f(x, y)$ **on the image given in Fig.2b).**

Laplacian

Figure7. The image from Fig.2a) after applying $-D-\nabla^2 f(x,y)$ **- the Laplacian and the directional derivatives with a positive center of the mask.**

Laplacian for sharpening



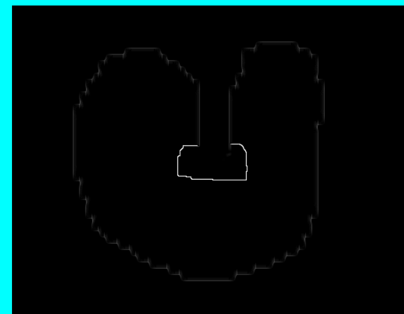
a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scannin electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Micha Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Figure 8. Image sharpening. Down left image is obtained by upper left Laplacian mask, Down right image is obtained by down right Laplacian mask.

(Digital Image Processing, 2nd E, by Gonzalez, Richard, Prentice Hull, 2002).

Laplacian for sharpening using $f_N(x,y) = Af(x,y) - \nabla^2 f(x,y) - Df(x,y)$



a) b) c)
d) e)

−1	−1	−1
−1	A+8	−1
−1	−1	−1

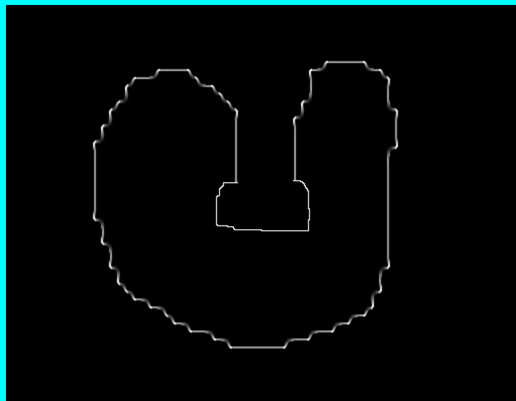
for A= 0

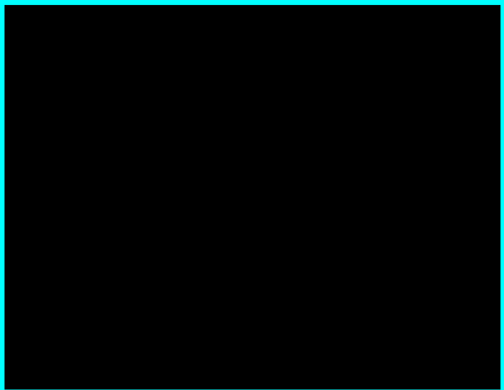
−1	−1	−1
−1	A+8	−1
−1	−1	−1

for A= 8

Figure 8. a) original synthetic image; **b)** the original image processed with the mask shown in d); **c)** the original image processed with the mask shown in e).

Laplacian for sharpening using $f_N(x,y) = Af(x,y) + \nabla^2 f(x,y) + Df(x,y)$





a) b) c)

d) e) f)

1	1	1
1	A- 8	1
1	1	1

for A= 0

1	1	1
1	A- 8	1
1	1	1

for A= 8

−1	−1	−1
−1	A+8	−1
−1	−1	−1

for A= -8

Figure 9. a) the original image from Fig.9 processed with the mask shown in d); **b)** the original image processed with the mask shown in e); **c)** the original image processed with the mask shown in f). This is useful for hiding images.

Laplacian and Gradient Operators

a) b)
c) d)

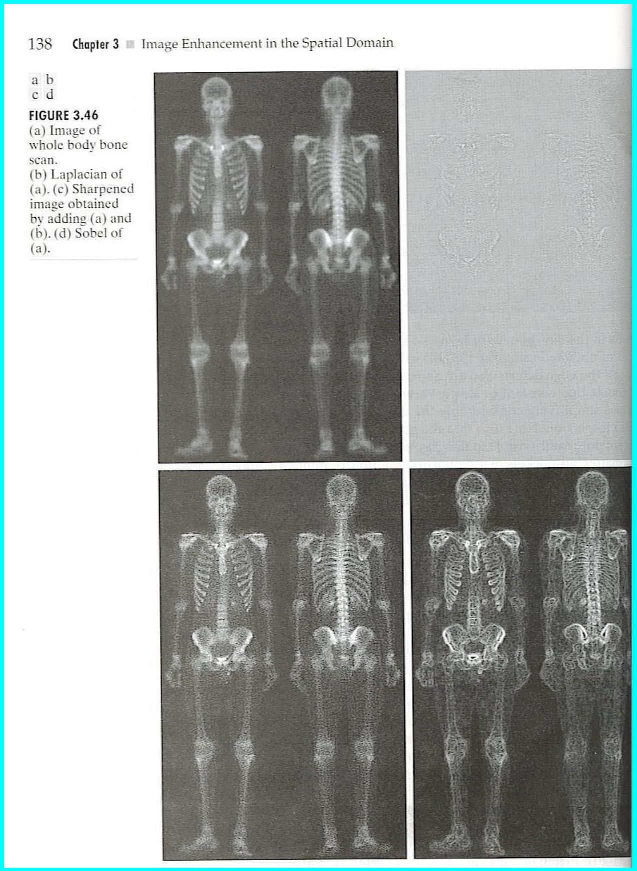


Figure 9.a) The original image; b) Laplacian of a); c) image obtained by adding a) and b) Sobel of a). (Digital Image Processing, 2nd E, by Gonzalez, Richard, Prentice Hull, 2002**).**

Sobel Operator

e) f)
g) h)

Figure 10.e) Sobel of a) **by mask 5x5; f) product of** c) **and** e); **g) sharpened image obtained by adding** a) **and** f)**; h) power low to** g).
(Digital Image Processing, 2nd E, by Gonzalez, Richard, Prentice Hull, 2002**).**

The Gradient

a) b) c)
d) e) f)

Figure11. Results obtained by employing different Gradient operators to an X-ray image of hand. The type of the operator is shown by the last two or four letters of the image title, give on the top of each image.

The Gradient – Continuation from slide 12

a) Gradient with horizontal (Gy)

-1	- 2	-1
1	2	1

b) vertical (Gx)

-1		1
- 2		2
-1		1

c) and Gx + Gy

-2	- 2	0
- 2	0	2
0	2	2

d) A mask with values 1, -2, 1 on the diagonal from the upper left to down right corner;

e) A mask with values 1, -2, 1 on the diagonal from the upper right to down left corner;

f) A mask with values 1, -2, 1 on both diagonals.

- **Fuzzy sets, membership function, operations, algorithms**

Fuzzy set, membership fuction, operations

$$\mu_3(z, v) = \min\{\mu_{red}(z), \mu_{mat}(v)\} \quad (3.8\text{-}1$$

a b
c d

FIGURE 3.49 (a) Shape of the membership function associated with the color red, and (b) corresponding output membership function. These two functions are associated by rule R_3. (c) Combined representation of the two functions. The representation is 2-D because the independent variables in (a) and (b) are different. (d) The AND of (a) and (b), as defined in Eq. (3.8-5).

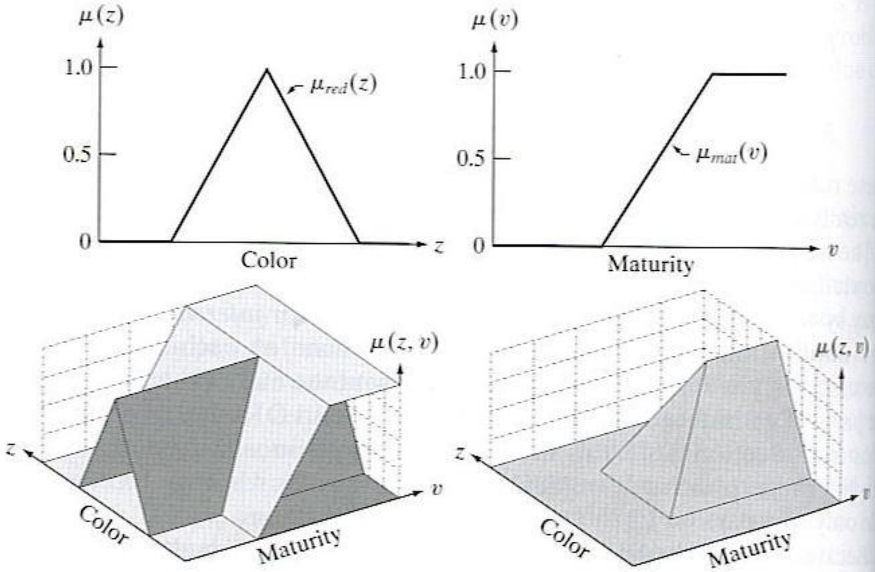


Figure 1. Input output membership functions.

Digital Image Processing, 3rd Ed, by R.C. Gonzalez, Richard, Prentice Hull 2007

Fuzzy rule based system

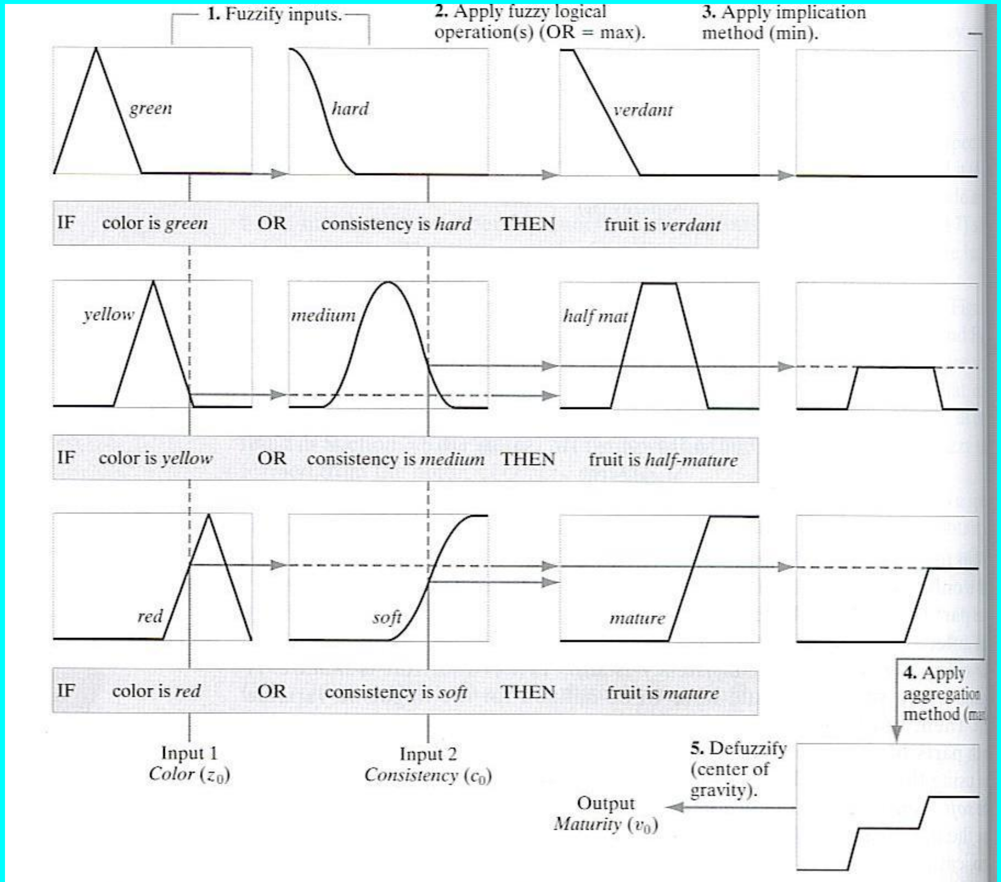


FIGURE 3.52 Example illustrating the five basic steps used typically to implement a fuzzy, rule-based syste
(1) fuzzification, (2) logical operations (only OR was used in this example), (3) implicatio
(4) aggregation, and (5) defuzzification.

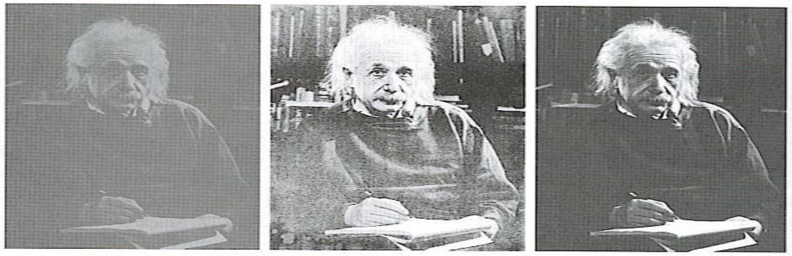
Figure 2. The basic steps of a fuzzy rule based system

Digital Image Processing, 3rd Ed, by R.C. Gonzalez, Richard, Prentice Hull 2007

The calculation complexity of the five steps approach, introduced above, includes fuzzifycation and defuzzifycation procedures, which are very time consuming. To speed up the algorithms a multi-variable single output function is often employed. As variables, this function may use multiple membership function. The results shown on the next slide are produced by using such a function which includes three membership functions, for:

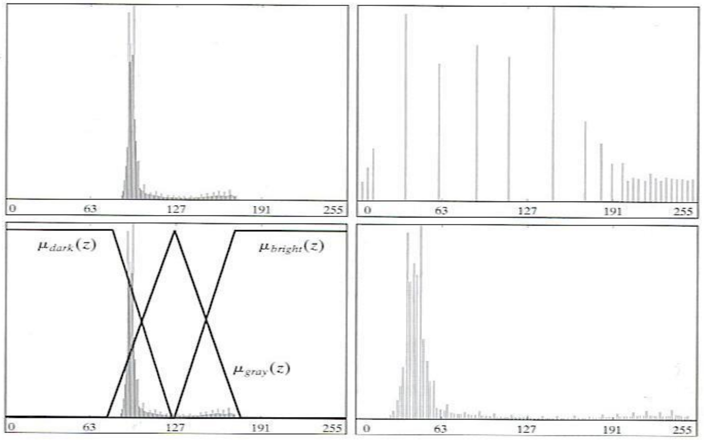
- **dark, gray and white** colors.

Fuzzy Image Enhancement- Results



a b c

FIGURE 3.54 (a) Low-contrast image. (b) Result of histogram equalization. (c) Result of using fuzzy, rule-based contrast enhancement.



a b
c d

FIGURE 3.55 (a) and (b) Histograms of Figs. 3.54(a) and (b). (c) Input membership functions superimposed on (a). (d) Histogram of Fig. 3.54(c).

Figure 3. Fuzzy contrast enhancement using a single output multi-variable function, which includes dark, bright and gray color membership functions. Digital Image Processing, 3rd Ed, by R.C. Gonzalez, Richard, Prentice Hull 2007